Numerical Examples In PHYSICS

N. N. GHOSH

Numerical Examples in Physics

*[For I. Sc. students of Indian Universities, Bihar Intermediate Board,
Medical and Engineering Entrance Test, I. I. T., I. S. M.
(Dhanbad) and other Competitive Examinations]*

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[*With S. I. Units*]

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PREFACE TO THE FIRST EDITION

With the introduction of new courses of studies system of examinations in Indian Universities in general and Ranchi University in particular I have felt the necessity of writing a book on numerical examples in Physics introducing S. I. units. The courses of studies are so thoroughly revised and upgraded in all the universities that none of the books available in the market covers the entire course. Moreover those books are in C.G.S. and F.P.S. units. Though in writing this book I have graded the chapters according to Ranchi University courses of studies, I have not neglected the interest of those students who want to prepare themselves for various competitive examinations such as I.I.T., B.I.T., I.S.M. (Dhanbad), P.M.D.T. etc. entrance examinations. Those chapters not included in the courses of studies of R. U. are given in appendices I, II, III, IV and V.

My main aim in writing this books is to create keen interest, lively enthusiasm and confidence among students in tackling numerical problems in Physics. I shall deem my labour rewarded if the students and teachers really find it to their liking and usefulness.

This is an age of electronics. While students having 'electronic calculators' may freely use them to make arithmetical simplifications, students in general should learn use of log tables and make frequent use of them. For convenience of those students who do not know use of log tables, I have devoted one complete chapter (Chapter I) on 'Use of Log Tables'. Students are strongly advised never to indulge inordinary lengthly methods of arithmetical simplifications.

As the book had to be passed hurriedly through press, there is every likelihood of mistakes here and there. I shall be very much thankful if the students and teachers using this book draw my attention towards them.

Valuable suggestions for improvement will always be thankfully acknowledged.

July, 1979
St. Xavier's College, Ranchi **N. N. Ghosh**

PREFACE TO THE SECOND EDITION

In the first place I express my thanks to all teachers of Physics and students for their appreciation of the book. The contents of this edition remain the same; only they have been recast according to the new Intermediate Syllabus. Appendices of the first edition have been removed and the matters therein have been included in the appropriate chapters. Some new problems have been added and difficult problems have been 'star' marked. In the first attempt they may be left out. I hope the book as a supplement to 'Introductory Physics' will be helpful.

St. Xavier's College, Ranchi **N. N. Ghosh**

PREFACE TO THE THIRD EDITION

It is a great pleasure to prepare the third edition of this book. In this edition it has been my sincere endeavour to revise the book thoroughly well with few additions of problems illustrating new ideas. In compliance with the suggestions of some learned teachers every example in light has been solved in all conventions—*old convention*, *coordinate convention* and *new* (*modern*) *convention*. Realising the difficulties of the Biology students and average students the problems in exercises have been screened judiciously and easy problems have been sorted out for them in group A. Problems in group B are meant for competitive examinations only. I hope the book will now be more useful to the average students and also the students preparing for various competitive examinations.

St. Xavier's College, Ranchi **N. N. Ghosh**

PREFACE TO THE FOURTH EDITION

In this edition worked out examples have been increased in almost all the chapters and a few problems have been added in the exercises. In light old convention has been dropped and modern convention (with concept of R-side and V-side) and coordinate convention have been retained. A list of standard differentials and integrals and important trigonometric formula is a new attraction of this edition. A brief concept of finding maxima and minima of a function is also given in this list with illustrative examples referred to exercises in appropriate chapters.

September, 1988
St. Xavier's College, Ranchi **N. N. Ghosh**

IMPORTANT MATHEMATICAL RESULTS

1. Important trigonometric relations

(i) $\text{Sin}\,(A+B)=\sin A\cos B+\cos A\sin B$

(ii) $\cos(A+B)=\cos A\cos B-\sin A\sin B$

(iii) $\sin C+\sin D=2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(iv) $\sin C-\sin D=2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(v) $\cos C+\cos D=2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(vi) $\cos C-\cos D=-2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(vii) $\sin 2A=2\sin A\cos A$

(viii) $\cos 2A=\cos^2 A-\sin^2 A=1-2\sin^2 A=2\cos^2 A-1$

(ix) $\sin 3A=3\sin A-4\sin^3 A$

(x) $\cos 3A=4\cos^3 A-3\cos A$

2. Properties of a triangle

(i) sine property : $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$ where A, B and C are the angles and a, b and c are the sides opposite to A, B and C respectively.

(ii) cosine property : $c^2=a^2+b^2-2ab\cos C$; $a^2=b^2+c^2-2bc\cos A$ and $b^2=c^2+a^2-2ca\cos B$.

(iii) Area of a triangle $\triangle=\frac{1}{2}bc\sin A$ or $\frac{1}{2}ca\sin B$ or $\frac{1}{2}ab\sin C$.

3. Expansion formula

$$(1+x)^n=1+nx+\frac{n(n-1)}{1.2}x^2+\frac{n(n-1)(n-2)}{1.2.3.}x^3+\ldots\ldots$$

When x is very small, $(1+x)^n=1+nx$.

4. Importants differentials

(i) $y=x^n$	$dy=nx^{n-1}dx$
(ii) $y=e^{ax}$	$dy=ae^{ax}dx$

(iii) $y=\sin ax$	$dy=a\cos ax dx$
(iv) $y=\cos ax$	$dy=-a\sin ax dx$
(v) $y=\tan ax$	$dy=a\sec^2 ax dx$
(vi) $y=\cot ax$	$dy=-a\operatorname{cosec}^2 ax dx$
(vii) $y=\sec ax$	$dy=a\sec ax\tan ax dx$
(viii) $y=\operatorname{cosec} ax$	$dy=-a\operatorname{cosec} ax\cot ax dx$

5. Important Integrals

(i) $\int x^n dx=\frac{x^{n+1}}{n+1}$ when $n\neq -1$ and when $n=-1\int\frac{dx}{x}=\log_e x$

(ii) $\int e^{ax}dx=\frac{e^{ax}}{a}$

(iii) $\int \sin ax dx=-\frac{\cos ax}{a}$

(iv) $\int \cos ax dx=\frac{\sin ax}{a}$

(v) $\int \tan ax dx=-\frac{1}{a}\log_e \cos x$

(vi) $\int \cot ax dx=\frac{1}{a}\log_e \sin ax$

(vii) $\int \sec^2 ax dx=\frac{\tan ax}{a}$

(viii) $\int \operatorname{cosec}^2 ax dx=-\frac{\cot ax}{a}$

(ix) $\int \frac{dx}{a^2+x^2}=\tan^{-1}\frac{x}{a}$

(x) $\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}$

(xi) $\int \frac{dx}{\sqrt{x^2-a^2}}=\log_e\frac{1}{a}(x+\sqrt{x^2-a^2})$

6. Maxima and Minima

The condition for maximum or minimum of a function y with respect to a variable x is that $\frac{dy}{dx}=0$ and for maximum $\frac{d^2y}{dx^2}$ must be negative and for minimum $\frac{d^2y}{dx^2}$ must be positive. For illustration see Ex. 5 Chapter 4 and Ex. 7 Chapter 7.

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Some Physical Constants

Speed of light	c	$3 \times 10^{8}\ ms^{-1}$
Gravitational constant	G	$6{\cdot}67 \times 10^{-11}\ Nm^{2}kg^{-2}$
Universal gas constant	R	$8{\cdot}31\ J\ mol^{-1}\ K^{-1}$
Permeability constant	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Permittivity constant	ϵ_0	$8{\cdot}85 \times 10^{-12}\ Fm^{-1}$
Avogadro's constant	N	$6{\cdot}02 \times 10^{23}\ mol^{-1}$
Boltzmann constant	k	$1{\cdot}38 \times 10^{-23}\ JK^{-1}$
Planck's constant	h	$6{\cdot}63 \times 10^{-34}\ Js$
Elementary charge	e	$1{\cdot}60 \times 10^{-19}\ C$
Electron rest mass	m_e	$9{\cdot}11 \times 10^{-31}\ kg$
Electron magnetic moment	μ_e	$9{\cdot}27 \times 10^{-24}\ JT^{-1}$
Acceleration due to gravity (standard)	g	$9{\cdot}80\ ms^{-2}$
Standard atmospheric pressure	atm	$1{\cdot}01 \times 10^{5}\ Nm^{-2}$
Density of water at 4°C		$1000\ kg\ m^{-3}$
Rydberg Constant	R	$1{\cdot}097 \times 10^{7}\ m^{-1}$

* * *

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CHAPTER 1

UNITS AND DIMENSIONS

1. Unit : To measure a physical quantity and express its measure we require a standard of that physical quantity. This 'standard' is called unit of that physical quantity. Measure of any physical quantity $= nu$ where $n =$ numerical value of the measure of the quantity, $u =$ unit of the quantity.

2 System of units : The common systems of units are (i) Foot Pound Second (F.P.S.) system, (ii) Centimetre-Gramme-Second (C.G.S.) system, (iii) Metre-Kilogramme-Second (M.K.S. or Giorgi) system, (iv) recently developed Systeme' Internationale (S.I.) d' Unities.

3. Base units : In Science there are certain physical quantities which are very fundamental in nature. There is no way out but to define 'Standard or Units' for such quantities. Units of all other quantities can be derived from the units of these quantities. Such quantities are called basic or fundamental quantities. The units of these quantities are called 'base units' of the system. In mechanics three base units are required. These are units of length, mass and time. In heat and thermodynamics besides these three we require standards for two more quantities, namely, temperature and amount of substance. In electricity and magnetism we require besides the three base units in mechanics, a standard for current. In light a standard for luminous intensity is required. Thus in all in any self-consistent and well developed system of measurements 'seven' base units are required. Recently there is an all round attempt to develop such a system of measurements. As a result of such attempt there has emerged a new system known as the Systeme' Internationale d' Unities abbreviated as S.I. system of units.

The seven base units in S.I. are :

(i) the metre (m), the standard of length

(ii) the kilogramme (kg), the standard of mass

(iii) the second (s), the standard of time

(iv) the ampere (A), the standard of electric current

(v) the kelvin (K), the standard of temperature
(vi) the candela (cd), the standard of luminous intensity and
(vii) the mole (mol), the standard of amount of substance.

Definitions of base units :

The metre (m) :

This is defined as 1650763·73 of the wavelength, in vacuo of the orange light emitted by ${}_{36}K^{86}$ in transition from $2p_{10}$ to $5d_5$.

The kilogramme (kg) :

This is defined as the mass of a platinum-iridium cylinder kept at Sevres.

The second (s) :

This is the time taken by 9192631770 cycles of the radiation from the hyperfine transition in caesium when unperturbed by external fields.

The ampere (A) :

This is defined as the constant current which, if maintained in each of two infinitely long straight parallel wires of negligible cross-section placed 1 metre apart, in vacuum, will produce between the wires a force of 2×10^{-7} newton per metre length of the wires.

The kelvin (K) :

In SI, temperatures are measured on the thermodynamic scale with absolute zero as zero and the triple point of water (i.e. the temperature at which ice, water and water vapour are in equilibrium) as the upper fixed point. The interval is divided into 273·15 divisions and each division is taken as unit temperature. This unit is called kelvin.

The candela (cd) :

This is defined as the luminous intensity in the perpendicular direction of a surface of 1/600000 square metre of a full radiator at the temperature of freezing platinum under a pressure of 101325 newton per square metre.

The mole (mol) :

The mole is the amount of substance which contains as many elementary entities as there are atoms in ·012 kg of the carbon isotope ${}_{6}C^{12}$.

4. Prefixes and Multiplication Factors and their names in SI :

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga (billion in America)	G	10^{-6}	micro	μ
			10^{-9}	nano	n
10^{6}	mega (million in America)	M	10^{-12}	pico	p
			10^{-15}	femto	f
10^{3}	kilo		10^{-18}	atto	a

5. Derived units : Units of all other quantities may be obtained by combining base units. These units are called derived units. Often derived units are given names. For example, the unit of force in SI is kg m s^{-2}. It has been given the name 'newton' (N). The unit of power is kg m^2 s^{-3}. It has been given the name watt (W).

6. Some important derived units : Through common usage, certain multiples and submultiples of the fundamental units have been given names. We will use them here and there but it is to be borne in mind that these are not recognised in SI.

Micron (μm) $= 10^{-6}$ m

Are (a) $= 100$ m^2

Angstrom (Å) $= 10^{-10}$ m

Barn (b) $= 10^{-28}$ m^2

Farmi (fm) $= 10^{-15}$ m

7. Dimensions : The dimensions of a physical quantity are the powers of the fundamental quantities to which they are to be raised to represent a unit of that physical quantity. The dimensions of fundamental quantities are expressed as (i) that of length by L, (ii) that of mass by M, (iii) that of time by T, (iv) that of current by I, (v) that of temperature by K.

Symbolically dimension of a physical quantity is written by putting that physical quantity within brackets such as this : $[A]$ and it is read as 'dimension of A'.

8. Uses of dimensions : (i) To change values of a physical quantity from one system into another.

(ii) To test the correctness of the results already arrived at.

(iii) To determine exact relation among physical quantities which are likely to be interlinked.

Some important physical quantities and their SI units with standard abbreviations :

Physical quantity	**SI unit**	**Abbreviation**
Mass	kilogramme	kg
Length	metre	m
Time	second	s
Force	newton	N
Energy or work	joule	J
Power	watt	W
Current	ampere	A
Charge	coulomb	C
P.D.	volt	V
Resistance	ohm	Ω (omega)
Capacitance	farad	F
Magnetic flux	weber	wb
Magnetic induction field	tesla	T

Physical quantity	**SI unit**	**Abbreviation**
Conductance	siemens	S
Temperature	kelvin or celsius	K °C
Amount of substance	mole	mol
Luminous intensity	candela	cd
Illuminance	lux	lx
Luminous flux	lumen	lm
Frequency	hertz	Hz
Pressure	pascal	Pa

EXAMPLES

Ex. 1. *Find the dimensions of the following quantities : (i) Velocity, (ii) Acceleration, (iii) Force, (iv) Angle, (v) Angular velocity, (vi) Density, (vii) Pressure, (viii) Kinetic energy, (ix) Couple, (x) Constant of gravitation, (xi) Coefficient of viscosity, (xii) Permeability of a medium.*

Sol. (i) $\text{Velocity} = \dfrac{\text{Distance}}{\text{Time}}$; $\therefore$ $[\text{Velocity}] = \dfrac{[\text{Distance}]}{[\text{Time}]} = \dfrac{L}{T}$

$$= LT^{-1}**.$$

(ii) $\text{Acceleration} = \dfrac{\text{Velocity}}{\text{Time}}$; $\therefore$ $[\text{Acceleration}] = \dfrac{[\text{Velocity}]}{[\text{Time}]} = \dfrac{LT^{-1}}{T}$

$$= LT^{-2}**.$$

(iii) $\text{Force} = \text{Mass} \times \text{Acceleration}$; $\therefore$ $[\text{Force}] = [\text{Mass}]\,[\text{Acceleration}]$

$$= MLT^{-2}**.$$

**To commit to memory.

(iv) Angle $= \dfrac{\text{Distance}}{\text{Distance}}$; $\therefore$ Angle is dimensionless.

(v) Angular velocity $= \dfrac{\text{Angle}}{\text{Time}}$; $\therefore$ [Angular velocity] $= \dfrac{[\text{Angle}]}{[\text{Time}]}$
$= T^{-1}$.

(vi) Density $= \dfrac{\text{Mass}}{\text{Volume}}$; $\therefore$ [Density] $= \dfrac{[\text{Mass}]}{[\text{Volume}]} = \dfrac{M}{L^3} = ML^{-3}$.

(vii) Pressure $= \dfrac{\text{Force}}{\text{Area}}$; $\therefore$ [Pressure] $= \dfrac{[\text{Force}]}{[\text{Area}]} = \dfrac{MLT^{-2}}{L^2}$
$= ML^{-1}T^{-2}$.

(viii) Kinetic energy $= \frac{1}{2}$ Mass $\times$ Velocity^2;
$\therefore$ [Kinetic energy] = [Mass][Velocity]^2 $= ML^2T^{-2}$.

(ix) Couple = Force × distance;
$\therefore$ [Couple] = [Force][Distance] $= MLT^{-2}.\ L = ML^2T^{-2}$.

(x) Constant of Gravitation occurs in Newton's law of gravitation i.e., in the formula, $F = G\,\dfrac{m_1 m_2}{d^2}$;

$$\therefore\quad [G] = \frac{[F]\,[d^2]}{[m_1]\,[m_2]} = \frac{MLT^{-2}L^2}{M\,M} = M^{-1}L^3T^{-2}.$$

(xi) Coefficient of viscosity occurs in formula

$$F = \eta\frac{Av}{x};\ \therefore\ [\eta] = \frac{[F]\,[x]}{[A]\,[v]} = \frac{MLT^{-2}L}{L^2LT^{-1}} = ML^{-1}T^{-1}.$$

(xii) Permeability occurs in Ampere's law of force, namely

$$\Delta F = \mu\frac{(i_1\Delta l_1)(i_2\Delta l_2)\sin\theta}{r^2};$$

$$\therefore\ [\mu] = \frac{[\Delta F][r^2]}{[i_1\Delta l_1][i_2\Delta l_2]} = \frac{MLT^{-2}L^2}{IL\,IL} = MLT^{-2}I^{-2}.$$

Illustration of change from one system into another :

Ex. 2. *The value of a force acting on a body is* 20 *newton (newton is unit of force in SI), What is the value of this force in cgs ?*

Sol. Dimension of force $= MLT^{-2}$.
$\therefore$ Unit of force in SI = kg m s^{-2}
and ,, ,, in cgs = gm cm s^{-2}.

Let n be the numerical value of the force in cgs.

$\therefore$ 20 kg m s^{-2} = n gm cm s^{-2};

$$\therefore \quad n=20\left(\frac{\text{kg}}{\text{gm}}\right)\left(\frac{\text{m}}{\text{cm}}\right)\left(\frac{\text{s}}{\text{s}}\right)^{-2}=20\left(\frac{1000 \text{ gm}}{\text{gm}}\right)\left(\frac{100 \text{ cm}}{\text{cm}}\right)$$

or $n=20\times1000\times100=2\times10^6$;

$\therefore$ 20 newton $=2\times10^6$ dyne. Ans.

Illustration of checking correctness of an equation by the method of dimensions :

Ex. 3. *Check by the method of dimensions whether the following relations are true—*

(*i*) $t=2\pi\sqrt{\frac{l}{g}}$, (*ii*) $v=\sqrt{\frac{P}{D}}$ *where* v = *velocity of sound,* P = *pressure and* D = *density of medium.*

(iii) $n=\frac{1}{2l}\sqrt{\frac{F}{m}}$ *where* n = *frequency of vibration of a string,* l = *length of the string,* F = *stretching force and* m = *mass per unit length of the string.*

Sol. (i) $[\text{R.H.S.}]=\sqrt{\frac{[l]}{[g]}}=\sqrt{\frac{L}{LT^{-2}}}=T.$

$[\text{L.H.S.}]=[t]=T.$ Hence the relation is correct.

(ii) $[\text{R.H.S.}]=\sqrt{\frac{[P]}{[D]}}=\sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}}=LT^{-1}.$

$[\text{L.H.S.}]=[v]=LT^{-1}.$

Hence the relation is true.

(iii) $[\text{R.H.S.}]=\frac{1}{[l]}\sqrt{\frac{[F]}{[m]}}=\frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}}=\frac{1}{L}LT^{-1}=T^{-1}.$

$$[\text{L.H.S.}]=\frac{[\text{A number}]}{[\text{Time}]}=\frac{1}{T}=T^{-1}.$$

Hence the result is correct.

Illustration of the third use of dimensions :

Ex. 4. *Assuming that critical velocity of flow of a liquid through a narrow tube depends on the radius of the tube, density of the liquid and viscosity of the liquid, find an expression for critical velocity.*

Sol. Suppose, $v=kr^a\rho^b\eta^c$ where r=radius of the tube, ρ=density of liquid and η=coefficient of vicosity; a, b and c are the unknown powers to be determined.

$$\therefore \quad [v]=[r]^a[\rho]^b[\eta]^c.$$

or $$LT^{-1}=L^a(ML^{-3})^b(ML^{-1}T^{-1})^c=M^{b+c}L^{a-3b-c}T^{-c}.$$

Equating powers of M, L and T^{-1} we have

$$b+c=0$$

$$a-3b-c=1$$

and $$-c=-1.$$

Hence $c=1$, $b=-1$ and $a=-1$.

$$\therefore \quad v=kr^{-1}\rho^{-1}\eta^1 \quad \text{or} \quad v=k\eta/r\rho. \text{ Ans.}$$

EXERCISES

(A)

1. Obtain the dimensions of the following physical quantities—(i) momentum, (ii) moment, (iii) impulse, (iv) power, (v) frequency, (vi) angular acceleration, (vii) velocity gradient, (viii) surface tension, (ix) moment of inertia, (x) ϵ, permittivity of a medium, (xi) thermal conductivity, (xii) stress (xiii) strain, (xiv) Young's modulus.

[**Ans.** (i) MLT^{-1}, (ii) ML^2T^{-2}, (iii) MLT^{-1}, (iv) ML^2T^{-3}, (v) T^{-1}, (vi) T^{-2}, (vii) T^{-1}, (viii) ML^0T^{-2}, (ix) ML^2, (x) $M^{-1}L^{-3}T^4I^2$, (xi) $MLT^{-3}K^{-1}$, (xii) $ML^{-1}T^{-2}$, (xiii) No dimension, (xiv) $ML^{-1}T^{-2}$.]

2. Acceleration due to gravity in F. P. S. system is 32·2. What is the value in SI? (**Ans.** 9·82 m s^{-2})

3. The value of coefficient of viscosity in cgs is 12. What is the value of the same in SI? (**Ans.** 1·2 N s m^{-2})

4. Surface tension of water in cgs is 72 dyne per cm. What is the value in SI? (**Ans.** ·072 N m^{-1}),

5. The value of acceleration due to gravity is 980 cm/sec². What will be its value if the unit of length is kilometre and that of time is an hour?

(**Ans.** $1{\cdot}27\times10^5$ km hour^{-2})

6. Obtain the formula $t=k\sqrt{\dfrac{l}{g}}$ by the method of dimensions.

7. Assuming that the largest mass that can be moved by a flowing river depends on velocity of flow, density of river water and on acceleration due to gravity, show that the mass varies as the sixth power of velocity of flow.

8. The velocity of sound in a gas depends on its pressure and density. Obtain the relation among velocity, pressure and density,

$\left(\textbf{Ans.}\quad V=k\sqrt{\dfrac{P}{D}}\right)$

9. The viscous force on a spherical body when it moves through a viscous liquid depends on radius of the body, coefficient of viscosity of the liquid and the velocity of the body. Find an expression for the viscous force.

(**Ans.** $F=k\eta rv$)

10. The rate of volume flow of water through a canal is to found to be a function of the area of cross-section of the canal and velocity of water. Show that the rate of volume flow is proportional to the velocity of flow of water.

(B)

11. Show that the following are of the dimensions of energy—

(i) mc^2 where m=mass and c is velocity of light.

(ii) $\frac{mP}{\rho}$ where P=pressure and ρ=density of liquid and m=mass.

(iii) mB where m=magnetic moment and B=magnetic induction field.

12. Show that RC, where R is the resistance and C is the capacitance, is of the dimension of time.

13. Using force (F), length (L) and time (T) as base quantities, find dimension of (i) mass, (ii) surface tension, and (iii) Young's modulus.

[**Ans.** (i) $FL^{-1}T^2$, (ii) FL^{-1}, (iii) FL^{-2}]

14. If the units of length and force be increased three times, show that the unit energy is increased nine times.

15. A gas bubble, from an explosion under water, oscillates with a period proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion. Find the values of a, b and c.

(I.I.T. 1981)

(**Ans.** $a=-\frac{5}{6}$, $b=\frac{1}{2}$, $c=\frac{1}{3}$)

16. The time period (t) of vibration of a liquid drop depends on surface tension (s), radius (r) of the drop and density (ρ) of the liquid. Find t.

(**Ans.** $t=k\sqrt{\frac{\rho r^3}{s}}$)

* * *

CHAPTER 2

VECTORS AND SCALARS

1. All physical quantities fall under two categories—scalars and vectors.

Scalars :

A scalar quantity is that one which requires only magnitude for its complete specification e.g. mass, time, volume, temperature, speed, energy, etc.

Vectors :

A vector quantity is that one which requires both magnitude and direction for its complete specification e.g. displacement, velocity, force, acceleration, magnetic field, current density, etc.

2. Geometrical representation of vectors :

Geometrically a vector is represented by 'directed line segment' i.e. by a line of definite length proportional to the magnitude of the vector and to which a direction has been assigned with an arrow-head.

To represent a vector geometrically first select a suitable 'scale', say, $1\ \text{cm} = a$ units of the vector. Then draw a line parallel to the direction of the vector, cut a length OP from it on that scale and finally put an arrow on the line along the direction of the vector. Now this directed line segment, namely, OP represents the vector. It is written as $\overrightarrow{OP}$. 'O' is called the 'initial point' and P, the terminal point. The vector $\overrightarrow{OP}$ is also written as $\vec{r}$, i.e., we also write $\vec{r} = \overrightarrow{OP}$.

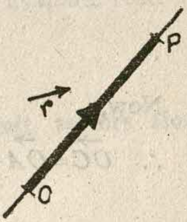

Fig. 2-1

3. Definitions :

(i) **Modulus of a vector** (in short 'mod' of a vector) :

The modulus of a vector is simply the magnitude of the vector. It is written as $|\vec{r}|$ and read as 'mod of r'. Obviously mod of a vector is a scalar.

(ii) Unit vector : A vector whose 'mod' is unity is called unit vector.

(iii) Null vector : A vector whose 'mod' is zero is called null vector.

(iv) Proper vector : A vector whose 'mod' is not zero is called a proper vector.

(v) Constant vector : A vector whose components are constants is called a constant vector.

4. Addition of vectors :

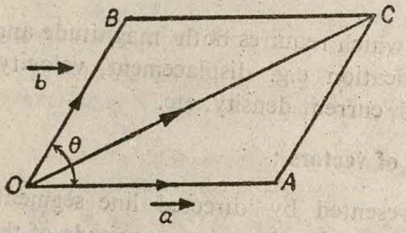

Fig. 2-2

Let $\vec{a}$ and $\vec{b}$ be any two given vectors at inclination θ to each other and suppose that we have to add them.

Take $\vec{OA}=\vec{a}$ and $\vec{OB}=\vec{b}$. Complete the prallelogram $OACB$. Then by theorem of parallelogram of vectors, the resultant (vector sum) of $\vec{OA}$ and $\vec{OB}$ is $\vec{OC}$.

$$\therefore \qquad \vec{OC}=\vec{OA}+\vec{OB}=\vec{a}+\vec{b}.$$

Now,
$$\vec{AC}=\vec{OB}=\vec{b}.$$

$$\therefore \quad \vec{OC}=\vec{OA}+\vec{AC} \text{ and } |\,\vec{OC}\,| = \sqrt{a^2+b^2+2ab\cos\theta}.$$

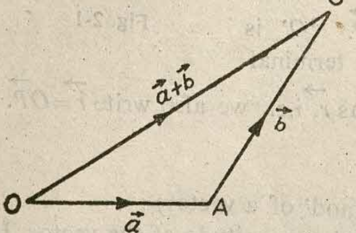

Fig. 2-3

Thus parallelogram is not needed to find the sum of the two vectors. The triangle OCA is sufficient to find the sum of the vectors. In the triangle OAC, $\vec{OA}=\vec{a}$ with 'O' as the initial point and A as the terminal point. $\vec{AC}=\vec{b}$ with A as the initial point, and C as the terminal point. Then $\vec{OC}$ is the sum of $\vec{a}$ and $\vec{b}$.

We can now extend this to 'summing up' of a number of vectors. For this represent the vectors in order by taking 'terminal point' of one as the 'initial point' of the next. Then the line joining the initial point of the first vector with the terminal point of the last vector gives the sum of the vectors.

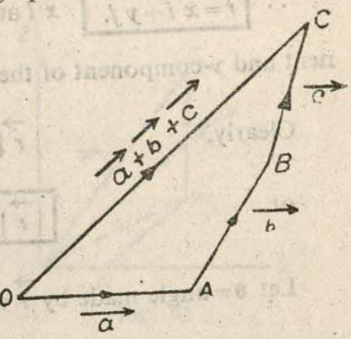

Fig. 2-4

In the figure

$$\vec{OC}=\vec{OA}+\vec{AB}+\vec{BC}$$
$$=\vec{a}+\vec{b}+\vec{c}.$$

Representation of a vector in terms of unit vector along the vector :

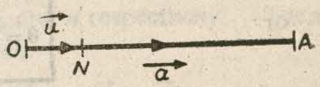

Fig. 2-5

Suppose $\vec{OA}=\vec{a}$ and $\vec{ON}=\vec{u}$ is a unit vector along $\vec{a}$. Let $|\vec{a}|=a$.

It is quite obvious that the vector $\vec{OA}$ is '*a*' times the vector $\vec{ON}$.

$$\therefore\quad \vec{OA}=a.\,\vec{ON}$$

or $$\boxed{\vec{a}=a\,\vec{u}}.$$

Usually unit vector along vector *a* is written as $\hat{a}$ and read as '*a* hat'.

$$\therefore\quad \boxed{\vec{a}=a\,\hat{a}}.$$

5. Representation of a vector in terms of unit vectors along co-ordinate axes :

(i) *In two dimensions :*

Suppose $\vec{i}$ and $\vec{j}$ are unit vectors along OX and OY respectively and $\vec{OP}=\vec{r}$ is any vector.

Draw perpendicular PN from P on OX.

We have,

$$\vec{OP}=\vec{ON}+\vec{NP}.$$

But $$\vec{ON}=ON\,\vec{i}=x\,\vec{i}$$
$$\vec{NP}=NP\,\vec{j}=y\,\vec{j}.$$

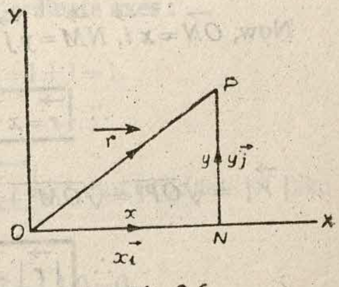

Fig. 2-6

$\therefore \boxed{\vec{r} = x\,\vec{i} + y\,\vec{j}.}$ $x\,\vec{i}$ and $y\,\vec{j}$ are respectively called x-component and y-component of the vector.

Clearly, $$|\vec{r}| = OP = \sqrt{ON^2 + NP^2}$$

or $$\boxed{|\vec{r}| = \sqrt{x^2 + y^2}}.$$

Let θ = angle made by $\vec{r}$ with x-axis;

then $$\tan\theta = \frac{PN}{ON}, \text{ or } \tan\theta = \frac{y}{x}$$

or $$\boxed{\theta = \tan^{-1}\frac{y}{x}}.$$

(ii) *In three dimensions* :

Suppose $\vec{i}, \vec{j}$ and $\vec{k}$ are the unit vectors along co-ordinate axes. Draw perpendicular PM from P on XY plane and then MN from M on OX.

Then we have,

$$\vec{OP} = \vec{OM} + \vec{MP}$$

$$= (\vec{ON} + \vec{NM}) + \vec{MP}$$

$$(\because \text{ from } \triangle ONM,\ \vec{OM} = \vec{ON} + \vec{NM}).$$

Now, $\vec{ON} = x\,\vec{i}$, $\vec{NM} = y\,\vec{j}$ and $\vec{MP} = z\,\vec{k}$.

$$\therefore \boxed{\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}}.$$

$$|\vec{r}| = \sqrt{OP^2} = \sqrt{OM^2 + MP^2} = \sqrt{ON^2 + NM^2 + MP^2}.$$

$$\therefore \boxed{|\vec{r}| = \sqrt{x^2 + y^2 + z^2}}.$$

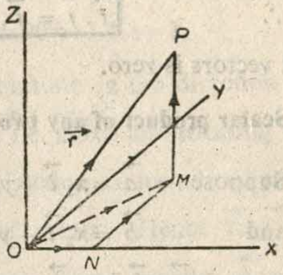

Fig. 2-7

Let θ_z = angle made by $\vec{r}$ with Z-axis.

$$\tan\theta_z = \frac{OM}{PM} = \frac{\sqrt{x^2+y^2}}{z}$$

or $\theta_z = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$.

Similarly, $\theta_x = \tan^{-1}\frac{\sqrt{y^2+z^2}}{x}$

and $\theta_y = \tan^{-1}\frac{\sqrt{x^2+y^2}}{y}$.

The cosines of θ_x, θ_y and θ_z are called 'direction cosines' of the vector and they are denoted by l, m, and n respectively. Then $x = r\cos\theta_x = rl$. Similarly, $y = rm$ and $z = rn$.

$$\therefore\quad \vec{r} = rl\,\vec{i} + rm\,\vec{j} + rn\,\vec{k} = r(l\,\vec{i} + m\,\vec{j} + n\,\vec{k})$$

and $\hat{r} = \frac{\vec{r}}{r} = (l\,\vec{i} + m\,\vec{j} + n\,\vec{k})$.

6. Scalar product of two vectors :

Def. If $\vec{a}$ and $\vec{b}$ are two vectors, then their scalar product is defined as $|\vec{a}|\ |\vec{b}|\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$ and is written as $\vec{a}\,.\,\vec{b}$.

Therefore, by definition, $\vec{a}\,.\,\vec{b} = |\vec{a}|\ |\vec{b}|\cos\theta$,

It follows easily from definition, $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$.

Scalar product of unit vectors along co-ordinate axes :

$$\vec{i}\,.\,\vec{i} = |\vec{i}|\ |\vec{i}|\ \cos 0 = 1.1.1 = 1.$$

Similarly, $\vec{j}\,.\,\vec{j} = 1$ and $\vec{k}\,.\,\vec{k} = 1$.

$\therefore$ $\boxed{\vec{i}\,.\,\vec{i} = \vec{j}\,.\,\vec{j} = \vec{k}\,.\,\vec{k} = 1}$ i.e. dot product of like unit vectors is 1.

$$\vec{i}\,.\,\vec{j} = |\vec{i}|\ |\vec{j}|\cos 90^\circ = 1.1.0 = 0.$$

$\therefore \quad \boxed{\vec{i}\,.\,\vec{j}=\vec{j}\,.\,\vec{k}=\vec{k}\,.\,\vec{i}=0}$ i.e. dot product of unlike unit vectors is zero.

Scalar product of any two vectors :

Suppose $\vec{a}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$

and $\vec{b}=x_2\vec{i}+y_2\vec{j}+z_2\vec{k};$

then

$$\vec{a}\,.\,\vec{b}=(x_1\vec{i}+y_1\vec{j}+z_1\vec{k})(x_2\vec{i}+y_2\vec{j}+z_2\vec{k})$$

$$=x_1x_2\vec{i}\,.\,\vec{i}+x_1y_2\vec{i}\,.\,\vec{j}+x_1z_2\vec{i}\,.\,\vec{k}+y_1x_2\vec{j}\,.\,\vec{i}+y_1y_2\vec{j}\,.\,\vec{j}$$
$$+y_1z_2\vec{j}\,.\,\vec{k}+z_1x_2\vec{k}\,.\,\vec{i}+z_1y_2\vec{k}\,.\,\vec{j}+z_1z_2\vec{k}\,.\,\vec{k}$$

or

$$\boxed{\vec{a}\,.\,\vec{b}=x_1x_2+y_1y_2+z_1z_2}$$

$(\because\ \vec{i}\,.\,\vec{i}=\vec{j}\,.\,\vec{j}=\vec{k}\,.\,\vec{k}=1$ and $\vec{i}\,.\,\vec{j}=\vec{j}\,.\,\vec{k}=\vec{k}\,.\,\vec{i}=0)$.

7. Cross product of vectors :

Def. Cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined as a vector $\vec{c}$ whose magnitude is $|\vec{a}|\ |\vec{b}|\sin\theta$ where θ is the smaller angle between the vectors and whose direction is in the direction of a screw perpendicular to both $\vec{a}$ and $\vec{b}$ and rotating from $\vec{a}$ to $\vec{b}$ through the smaller angle. It is written as $\vec{a}\times\vec{b}$.

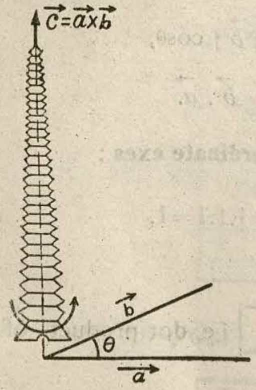

Fig. 2-8

Therefore, by definition,

$$|\vec{a}\times\vec{b}|=|\vec{a}|\ |\vec{b}|\sin\theta$$
$$=ab\sin\theta.$$

It easily follows from definition that $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}.$

Cross-product of unit vectors :

By definition we have

$$|\vec{i}\times\vec{i}|=|\vec{i}|\ |\vec{i}|\sin 0$$
$$=1.1.0=0;$$

$$\therefore\quad \boxed{\vec{i}\times\vec{i}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}=0}$$

i.e. cross product of like unit vectors is zero.

By definition we have

$$|\vec{i}\times\vec{j}| = |\vec{i}|\,|\vec{j}| \ \sin 90^\circ = 1.1.1 = 1.$$

Thus $\vec{i}\times\vec{j}$ is a vector of unit magnitude in the direction of a screw perpendicular to $\vec{i}$ i.e., x-axis and $\vec{j}$ i.e. $\vec{y}$-axis and rotating from $\vec{i}$ to $\vec{j}$ i.e. from x-axis to y-axis. The direction of motion of such a screw is along the positive direction of z-axis. Hence $\vec{i}\times\vec{j}$ is a vector of unit magnitude along the z-axis. This is exactly $\vec{k}$.

$$\left.\begin{array}{ll} \therefore & \vec{i}\times\vec{j}=\vec{k}. \\ \text{Similarly,} & \vec{j}\times\vec{k}=\vec{i} \\ \text{and} & \vec{k}\times\vec{i}=\vec{j} \end{array}\right\}$$

Cross product of any two unlike unit vectors taken in order (order being $\vec{i}$, $\vec{j}$ and $\vec{k}$) is equal to the third one.

8. Cross product of any two vectors :

Suppose $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$ and

$$\vec{b}=x_2\vec{i}+y_2\vec{j}+z_2\vec{k}.$$

$$\begin{aligned}\therefore\quad \vec{a}\times\vec{b}=\vec{c} &=(x_1\vec{i}+y_1\vec{j}+z_1\vec{k})\times(x_2\vec{i}+y_2\vec{j}+z_2\vec{k})\\ &=(x_1x_2\vec{i}\times\vec{i}+x_1y_2\vec{i}\times\vec{j}+x_1z_2\vec{i}\times\vec{k})\\ &\quad+(y_1x_2\vec{j}\times\vec{i}+y_1y_2\vec{j}\times\vec{j}+y_1z_2\vec{j}\boxtimes\vec{k})\\ &\quad+(z_1x_2\vec{k}\times\vec{i}+z_1y_2\vec{k}\times\vec{j}+z_1z_2\vec{k}\times\vec{k}).\end{aligned}$$

We have as properties of unit vectors along the co-ordinate axes that

$$\vec{i}\times\vec{i}=\vec{j}\boxtimes\vec{j}=\vec{k}\times\vec{k}=0$$

$$\text{and}\quad \vec{i}\times\vec{j}=\vec{k},\ \ \vec{j}\times\vec{k}=\vec{i} \text{ and } \vec{k}\times\vec{i}=\vec{j}.$$

$$\begin{aligned}\therefore\quad \vec{a}\times\vec{b}&=x_1y_2\vec{k}+x_1z_2(-\vec{j})+y_1x_2(-\vec{k})+y_1z_2\vec{i}\\ &\quad+z_1x_2\vec{j}+z_1y_2(-\vec{i})\end{aligned}$$

$$\text{or}\quad \vec{a}\times\vec{b}=(y_1z_2-z_1y_2)\vec{i}+(z_1x_2-x_1z_2)\vec{j}+(x_1y_2-y_1x_2)\vec{k}$$

$$\text{or} \quad \vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

and $|\vec{c}|$ or $|\vec{a}\times\vec{b}|$

$$=\sqrt{(y_1z_2-y_2z_1)^2+(z_1x_2-z_2x_1)^2+(x_1y_1-x_2y_1)^2}$$

EXAMPLES

Ex. 1. *If* $\vec{a}=3\,\vec{i}-2\,\vec{j}$ *and* $\vec{b}=2\,\vec{i}+3\,\vec{j}$, *calculate*

(i) $\vec{a}+\vec{b}$, (ii) $\vec{a}-\vec{b}$, (iii) $\vec{b}-\vec{a}$.

Sol. (i) $\vec{a}+\vec{b}=3\,\vec{i}-2\,\vec{j}+2\,\vec{i}+3\,\vec{j}=5\vec{i}+\vec{j}$;

$\therefore \quad |\vec{a}+\vec{b}| = \sqrt{5^2+1^2}=\sqrt{26}=5{\cdot}1$;

$\tan\theta=\frac{1}{5}$ or $\theta=\tan^{-1}(\frac{1}{5})=11{\cdot}18°$.

(ii) $\vec{a}-\vec{b}=3\,\vec{i}-2\,\vec{j}-2\,\vec{i}-3\,\vec{j}$

$=\vec{i}-5\,\vec{j}=1\,\vec{i}+(-5)\,\vec{j}$;

$\therefore \quad |\vec{a}-\vec{b}| = \sqrt{1^2+(-5)^2}=\sqrt{26}=5{\cdot}1$;

$$\tan\theta=\frac{-5}{1} \text{ or } \theta=\tan^{-1}\left(\frac{-5}{1}\right)=281°\ 18'.$$

(iii) $\vec{b}-\vec{a}=2\,\vec{i}+3\,\vec{j}-3\,\vec{i}+2\,\vec{j}$

$=-\vec{i}+5\,\vec{j}$;

$\therefore \quad |\vec{b}-\vec{a}| = \sqrt{(-1)^2+5^2}=\sqrt{26}=5{\cdot}1$;

$$\tan\theta=\frac{5}{-1} \text{ or } \theta=\tan^{-1}\left(\frac{5}{-1}\right)=101°\ 18'.$$

Ex. 2. *Calculate the angle between the vectors*

$\vec{a}=3\,\vec{i}+2\,\vec{j}$ *and* $\vec{b}=2\,\vec{i}+\vec{j}$.

Sol. Method I : Let $\vec{a}$ and $\vec{b}$ make angles θ_1 and θ_2 with x-axis. Then $\tan\theta_1=\frac{2}{3}$ and $\tan\theta_2=\frac{1}{2}$.

$$\therefore \quad \tan(\theta_1-\theta_2)=\frac{\tan\theta_1-\tan\theta_2}{1+\tan\theta_1\tan\theta_2}=\frac{\frac{2}{3}-\frac{1}{2}}{1+\frac{2}{3}\cdot\frac{1}{2}}=\frac{1}{8}.$$

$\therefore \quad \theta_1-\theta_2=\tan^{-1}\left(\frac{1}{8}\right)=7^\circ\ 8'$. Ans.

Method II : $\vec{a}\cdot\vec{b}=(3\vec{i}+2\vec{j})\cdot(2\vec{i}+\vec{j})=6+2=8$

$$|\vec{a}|=\sqrt{3^2+2^2}=\sqrt{13} \text{ and } |\vec{b}|=\sqrt{2^2+1^2}=\sqrt{5}.$$

Now, $\quad \vec{a}\cdot\vec{b}=|\vec{a}|\ |\vec{b}|\cos\theta; \quad \therefore \quad 8=\sqrt{13}\sqrt{5}\cos\theta$

or $\quad \cos\theta=\dfrac{8}{\sqrt{65}}=\cdot9922; \quad$ or $\quad \theta=7^\circ\ 8'$. Ans.

Ex. 3. *If* $\vec{A}+\vec{B}=\vec{A}-\vec{B}$ *then* (i) $\vec{A}=0$, (ii) $\vec{B}=0$, (iii) $\vec{A}$ *and* $\vec{B}$ *are simultaneously zero,* (iv) $\vec{A}+\vec{B}=0$.

Sol. We have, $\quad \vec{A}+\vec{B}=\vec{A}-\vec{B}$

or $\quad \vec{A}+\vec{B}-\vec{A}+\vec{B}=0$ or $2\vec{B}=0$, or $\vec{B}=0$.

Therefore (ii) is the correct answer.

Ex. 4. *Show that the vectors* $\vec{a}=2\vec{i}+3\vec{j}$ *and* $\vec{b}=6\vec{i}-4\vec{j}$ *are at right angles to each other.*

Sol. $\quad \vec{a}\cdot\vec{b}=(2\vec{i}+3\vec{j})\cdot(6\vec{i}-4\vec{j})$

$$=12-12=0.$$

But $\vec{a}\cdot\vec{b}=ab\cos\theta$ where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$.

$\therefore \quad ab\cos\theta=0$

or $\quad \cos\theta=0$

or $\quad \theta=\dfrac{\pi}{2}$. Proved.

Ex. 5. *Find the dot and cross product of the vectors*

$$\vec{a}=2\vec{i}-3\vec{j}+\vec{k} \textit{ and } \vec{b}=-\vec{i}+3\vec{j}+\vec{k}.$$

Sol. $\quad \vec{a}\cdot\vec{b}=(2\vec{i}-3\vec{j}+\vec{k})\cdot(-\vec{i}+3\vec{j}+\vec{k})$

$$=\{2\vec{i}+(-3)\vec{j}+\vec{k}\}\cdot\{(-1)\vec{i}+3\vec{j}+\vec{k}\}$$

$$=2(-1)+(-3).\ 3+1.1=-2-9+1=-10. \text{ Ans.}$$

$$\vec{a}\times\vec{b}=(2\vec{i}-3\vec{j}+\vec{k})\times(-\vec{i}+3\vec{j}+\vec{k})$$

$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & -3 & 1\\ -1 & 3 & 1\end{vmatrix}=\vec{i}(-3-3)+\vec{j}(-1-2)+\vec{k}(6-3)$$

$$=-6\,\vec{i}-3\,\vec{j}+3\,\vec{k}.\ \text{Ans.}$$

Ex. 6. *Show that the magnitude of a vector product gives numerically the area of the parallelogram formed by the two component vectors as sides.*

Sol. We have, by definition of vector product,

$$|\vec{a}\times\vec{b}|=|\vec{a}|\ |\vec{b}|\sin\theta=ab\sin\theta.$$

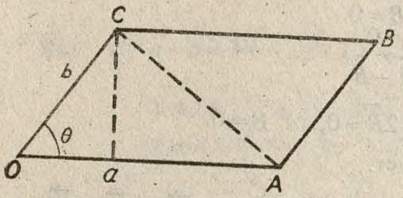

Fig. 2-9

Now draw a parallelogram *OABC* of sides *a* and *b* and θ as included angle between *a* and *b*.

Now, area of parallelogram

$=2\times$area of OAC

$=2.\frac{1}{2}\times\text{base}\times\text{height}$

$=ab\sin\theta.$

$\therefore\ |\vec{a}\times\vec{b}|=$area of parallelogram formed by *a* and *b* as sides. Proved.

Ex. 7. *The co-ordinates of the initial point of a vector are* (2, 3) *and those of the terminal point are* (10, 6). *Find the vector in magnitude and direction.*

Sol. Suppose *A* is the initial point and *B* is the terminal point of the vector. *O* is the origin.

Then, $\vec{OA}=2\,\vec{i}+3\,\vec{j}$

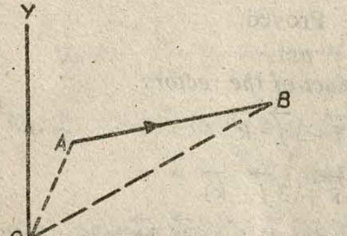

Fig. 2-10

and $\vec{OB}=10\,\vec{i}+6\,\vec{j}.$

We have, $\vec{OA}+\vec{AB}=\vec{OB}$

$\therefore\ \vec{AB}=\vec{OB}-\vec{OA}$

$=10\,\vec{i}+6\,\vec{j}-2\,\vec{i}-3\,\vec{j}$

$=8\,\vec{i}+3\,\vec{j}.$

$$\therefore \quad |\overrightarrow{AB}| = \sqrt{8^2+3^2} = \sqrt{73}$$
$$= 8{\cdot}54$$
$$\tan\theta = \frac{y}{x} = \frac{3}{8} = {\cdot}375$$
$$\therefore \quad \theta = 20°33'.$$

Ans. The required vector is of 8·54 unit along the direction making an angle 20°33′ with 2*ta*-axis.

Ex. 8. *If* $\vec{a} + \vec{b} = \vec{c}$ *and* $a+b=c$, *what is the angle between* $\vec{a}$ *and* $\vec{b}$ *?*

Sol. We have, $|\vec{a} + \vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}$.

$$\because \quad \vec{a} + \vec{b} = \vec{c},$$
$$\therefore \quad |\vec{a}+\vec{b}| = |\vec{c}| \quad \text{or} \quad |\vec{a}+\vec{b}|^2 = |\vec{c}|^2$$

or $\quad a^2+b^2+2ab\cos\theta = c^2 = (a+b)^2 = a^2+b^2+2ab.$

or $\quad 2ab(\cos\theta-1) = 0$ or $\cos\theta = 1$

or $\quad \theta = 0°$. Ans.

Ex. 9. *A particle undergoes three successive displacements in a plane; the first by* 4 m *south west, the second by* 5 m *east and the third by* 6 m *in a direction* 60° *north of east. Draw the vector diagram and determine the total displacement from the starting point.*

Sol. $\overrightarrow{OA} = 4\cos(180°+45°)\,\vec{i} + 4\sin(180°+45°)\,\vec{j}$

$$\overrightarrow{OA} = -2\sqrt{2}\,\vec{i} - 2\sqrt{2}\,\vec{j}$$
$$\overrightarrow{AB} = 5\cos 0°\,\vec{i} = 5\,\vec{i}$$
$$\overrightarrow{BC} = 6\cos 60°\,\vec{i} + 6\sin 60°\,\vec{j}$$
$$= 3\,\vec{i} + 3\sqrt{3}\,\vec{j}$$
$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$$
$$= (-2\sqrt{2}\,\vec{i} - 2\sqrt{2}\,\vec{j}) + 5\,\vec{i} + (3\,\vec{i} + 3\sqrt{3}\,\vec{j})$$
$$= (-2\sqrt{2}+5+3)\,\vec{i} + (-2\sqrt{2}+3\sqrt{3})\,\vec{j}$$
$$= +5{\cdot}17\,\vec{i} + 2{\cdot}37\,\vec{j}.$$

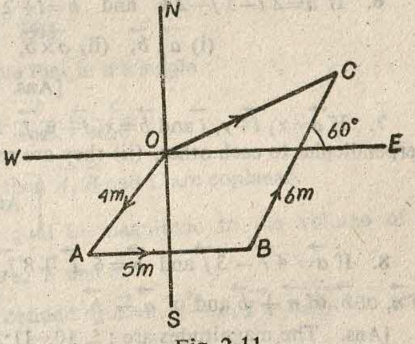

Fig. 2-11

$$|\vec{OC}|=\sqrt{5{\cdot}17^2+2{\cdot}37^2}=\sqrt{26{\cdot}63+5{\cdot}62}=\sqrt{32{\cdot}25}=5{\cdot}68\text{ m}$$

$$\tan\theta=\frac{2{\cdot}37}{5{\cdot}17}={\cdot}458 \quad\text{or}\quad \theta=24°36'.$$

Ans. Total displacement 5·68 m in the direction 24°36′ north of east.

EXERCISES

(A)

1. $\vec{a}+\vec{a}=\vec{a}$ is (i) always possible, (ii) never possible, (iii) possible if $\vec{a}$ is a null vector, (iv) possible if $\vec{a}$ is a unit vector. [**Ans.** (iii)]

2. $\vec{a}+\vec{b}=\vec{c}$ and $|\vec{a}|=|\vec{b}|=|\vec{c}|$.

This is (i) impossible, (ii) possible when angle between $\vec{a}$ and $\vec{b}$ is 60°, (iii) possible when angle between $\vec{a}$ and $\vec{b}$ is 120°, (iv) always possible. [**Ans.** (iii)]

3. $\vec{a}+\vec{b}=\vec{c}$ and $c=\sqrt{a^2+b^2}$. What is the angle between $\vec{a}$ and $\vec{b}$? [**Ans.** 90°]

4. The coordinates of the initial pt. of a vector are (1, 2) and those of terminal pt. are (5, 9). Find the vector in magnitude and direction.

[**Ans.** $\vec{r}=4\vec{i}+7\vec{j}$; $|\vec{r}|=\sqrt{65}$; $\theta=\tan^{-1}\frac{7}{4}$]

5. If $\vec{a}=2\vec{i}+3\vec{j}$ and $\vec{b}=\vec{i}+\vec{j}$, find (i) $\vec{a}\cdot\vec{b}$, (ii) $\vec{a}\times\vec{b}$, (iii) $\vec{b}\times\vec{a}$.

[**Ans.** (i) 5, (ii) $-\vec{k}$, (iii) $\vec{k}$]

6. If $\vec{a}=2\vec{i}-3\vec{j}+2\vec{k}$ and $\vec{b}=\vec{i}+2\vec{j}+\vec{k}$, find

(i) $\vec{a}\cdot\vec{b}$, (ii) $\vec{a}\times\vec{b}$, (iii) $\vec{b}\times\vec{a}$.

[**Ans.** (i) -2, (ii) $-7\vec{i}+7\vec{k}$, (iii) $7\vec{i}-7\vec{k}$]

7. If $\vec{a}=x_1\vec{i}+y_1\vec{j}$ and $\vec{b}=x_2\vec{i}+y_2\vec{j}$, find the condition that (i) they are perpendicular to each other, (ii) they are parallel to each other.

[**Ans.** (i) $y_1y_2=-x_1x_2$, (ii) $\dfrac{x_1}{y_1}=\dfrac{x_2}{y_2}$]

8. If $\vec{a}=4\vec{i}-3\vec{j}$ and $\vec{b}=6\vec{i}+8\vec{j}$, find the magnitude and direction of $\vec{a}$, of $\vec{b}$, of $\vec{a}+\vec{b}$ and of $\vec{a}-\vec{b}$.

[**Ans.** The magnitudes are : 5, 10, 11·2 and 11·2. The angles made with the x-axis are : 323°, 53·1°, 26·5° and 260°]

9. Three coplanar vectors with respect to a certain rectangular coordinate system are given by

$$\vec{a}=4\vec{i}-\vec{j};\ \vec{b}=-3\vec{i}+2\vec{j}\text{ and }\vec{c}=-3\vec{j}.$$

Find (i) $\vec{a}+\vec{b}+\vec{c}$, (ii) $\vec{a}+\vec{b}-\vec{c}$.

[**Ans.** (i) Magnitude is $\sqrt{5}$ and the angle made by it with the positive x-axis, measured counterclockwise from that axis, is 297°, (ii) Magnitude $=\sqrt{17}$. Direction$=75° \ 59'$ with the positive x-axis in counterclockwise direction.]

10. Find the angle between the vectors given by

$$\vec{a}=3\vec{i}+3\vec{j}-3\vec{k} \text{ and } \vec{b}=2\vec{i}+\vec{j}+3\vec{k}.$$

[*Hint* : Use dot product of vectors.] [**Ans.** $\pi/2$)

11. A car is driven eastward for a distance of 50 km, then northward for 30 km, and then in a direction 30° east of north for 25 km. Draw the vector diagram and determine the total displacement of the car from the starting point.

[**Ans.** 81 km; 39·5° north of east]

12. A golfer takes his ball into the hole by three strokes. The first stroke displaces the ball 4 m north, the second stroke 2 m south east, and the third stroke 1 m south west. What displacement was needed to get the ball into the hole on the first stroke ?

[**Ans.** 2·01 m; 69° 18′ north of east]

13. Four forces of magnitude P, $2P$, $3P$ and $4P$ act along the four sides of a square $ABCD$ in cyclic order. Use the vector method to find the resultant force.

[**Ans.** $2\sqrt{2}\,P$, 225° with AB in the antcilockwise direction]

14. Find the direction cosines and unit vector along the vector $\vec{r}=2\hat{i}+\hat{j}+3\hat{k}$.

[**Ans.** $\dfrac{2}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}$; $\hat{r}=\dfrac{2}{\sqrt{14}}\,\hat{i}+\dfrac{1}{\sqrt{14}}\,\hat{j}+\dfrac{3}{\sqrt{14}}\,\vec{k}$.]

(B)

15. Prove by the method of vectors that in a triangle

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

16. Show that if $\vec{A}.(\vec{B}\times\vec{C})=0$, then $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar.

17. Show that $\vec{a}.(\vec{b}\times\vec{c})$ is equal in magnitude to the volume of the parallelopiped formed by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

18. Prove that for a vector $\vec{a}$ defined by $\vec{a}=a_x\,\vec{i}+a_y\,\vec{j}+a_z\,\vec{k}$, the scalar components are given by $a_x=\vec{i}.\,\vec{a}$; $a_y=\vec{j}.\,\vec{a}$ and $a_z=\vec{k}.\,\vec{a}$.

19. If $\vec{b}$ and $\vec{c}$ be the intersecting face diagonals of a cube of edge a in the planes XOY and YOZ respectively with respect to a frame of reference erected at the edge of intersection of the vectors and edges of the cube as the axes, find

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(a) the components of the vector, where $\vec{d}=\vec{b}\times\vec{c}$ and (b) the values of $\vec{b}.\vec{c}$ and $\vec{d}.\vec{c}$. [**Ans.** (a) $d_x=d_z=a^2$; $d_y=-a^2$; (b) $\vec{b}.\vec{c}=a^2$; $\vec{d}.\vec{c}=0$]

20. Show that the position vector of a place on the surface of the earth having latitude and longitude α°N and β°E respectively is $\vec{r}=R\cos\alpha\,\sin\beta\,\hat{i}+R\cos\alpha\,\cos\beta\,\hat{j}+R\sin\alpha\,\hat{k}$, where R is the radius of the earth. The frame of reference is erected at the centre of the earth with the polar radius as z-axis and the intersection of the equatorial plane and the meridian plane through Greenwich as y-axis.

21. A man flies from Washington to Manila. Find the displacement vector of the man (magnitude only) if the latitude and longitude of the two cities are 39° N, 77°W and 15° N, 121°E respectively.

Radius of the earth$=6{\cdot}4\times10^6$ m. [**Ans.** $11{\cdot}8\times10^3$ km]

22. Two particles, 1 and 2, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their radius vectors are equal to $\vec{r}_1$ and $\vec{r}_2$. How must these four vectors be interrelated for the particles to collide ?

[*Hint* : Unit vector along $(\vec{r}_2-\vec{r}_1)$ must be the same as the unit vector along $(\vec{v}_2-\vec{v}_1)$]

[**Ans.** $\dfrac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|}=\dfrac{\vec{v}_2-\vec{v}_1}{|\vec{v}_2-\vec{v}_1|}$]

23. A force $\vec{F}=5\vec{i}+2\vec{j}+\vec{k}$ displaces a body from a point of co-ordinates (1, 1, 1) to another point of co-ordinates (2, 0, 3). Calculate the work done by the force. All are in SI units.

[*Hints* : $W=\vec{F}\,.\,\vec{S}$]

[**Ans.** 5 joule]

* * *

CHAPTER 3

KINEMATICS

1. Instantaneous and uniform velocity :

The velocity is the *rate of change of displacement*.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{\vec{ds}}{dt}.$$

The velocity at an instant is called the instantaneous velocity. If the instantaneous velocities are the same at all instants, the velocity is said to be uniform.

The *average velocity* during an interval Δt is given by

$$\vec{v}_{average} = \frac{\vec{\Delta s}}{\Delta t}.$$

$$\text{That is, average velocity} = \frac{\text{Displacement during an interval}}{\text{interval}}.$$

Similarly, instantaneous acceleration is the acceleration at an instant and if the instantaneous accelerations are the same at all instants, the acceleration is said to be uniform.

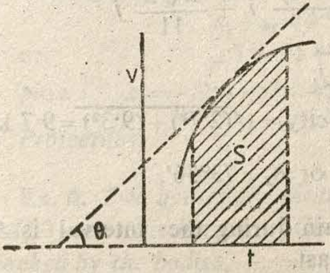
$$\text{That is,} \quad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}, \text{ and } \vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t}.$$

2. Kinematics :

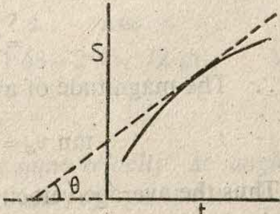
For a uniformly accelerated body we have the following equations describing final velocity and distance described (i) $v = u + ft$, (ii) $s = ut + \frac{1}{2} ft^2$, (iii) $v^2 - u^2 = 2fs$ where u = initial velocity, v = final velocity, f = uniform acceleration, s = distance described and t = time. Graphically *the distance described is given by the area of the plot of v against t whatever be the kind of motion. The slope of the plot at any point gives the acceleration. Similarly the slope of the plot of distance s against t gives the velocity.* Since $v = \frac{ds}{dt}$ (in one dimension), $s = \int v dt$.

Thus graphically *displacement = area between the* $v - t$ *curve and t-axis.*

Note. In one dimension distance covered and displacement are equal but not necessarily always.



tanθ=slope= acceleration



tanθ=slope=velocity

EXAMPLES

Ex. 1. *A train moving at an essentially constant speed to* 72 kph *moves eastward for* 40 minutes, *then in direction* 45° *east of north for* 20 minutes, *and finally westward for* 50 minutes. *What is the average velocity of the train during this run* ?

Sol. $$72 \text{ kph} = \frac{72 \times 1000}{3600} = 20 \text{ ms}^{-1}.$$

Let $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ be successive displacements of the train. Let us take the eastward direction as the positive direction of x-axis and the northward direction as y-axis.

Then $$|\vec{s}_1| = 20 \times 40 \times 60 = 48000 \text{ m}$$
$$\doteq 48 \text{ km}$$

and $$\vec{s}_1 = 48\,\vec{i};$$
$$|\vec{s}_2| = 20 \times 20 \times 60 = 24000 \text{ m} = 24 \text{ km}$$

and $$\vec{s}_2 = 24 \cos 45°\,\vec{i} + 24 \sin 45°\,\vec{j} = 12\sqrt{2}\,\vec{i} + 12\sqrt{2}\,\vec{j};$$
$$|\vec{s}_3| = 20 \times 50 \times 60 = 60000 \text{ m} = 60 \text{ km}$$
$$\vec{s}_3 = -60\,\vec{i}.$$

The net displacement $= \vec{s} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3$
$$= 48\,\vec{i} + (12\sqrt{2}\,\vec{i} + 12\sqrt{2}\,\vec{j}) - 60\,\vec{i}$$

or $\vec{s} = (12\sqrt{2} - 12)\,\vec{i} + 12\sqrt{2}\,\vec{j}.$

Total time that elapsed $= 40 + 20 + 50$
$$= 110 \text{ minutes} = \tfrac{110}{60} = \tfrac{11}{6} \text{ hours.}$$

$$\therefore \text{ Average velocity} = \frac{12\sqrt{2} - 12}{\frac{11}{6}}\,\vec{i} + \frac{12\sqrt{2}}{\frac{11}{6}}\,\vec{j}$$

$$= \frac{12 \times 6(\sqrt{2} - 1)}{11}\,\vec{i} + \frac{72\sqrt{2}}{11}\,\vec{j}$$

$$= 2{\cdot}7\,\vec{i} + 9{\cdot}3\,\vec{j}.$$

$\therefore$ The magnitude of average velocity $= \sqrt{(2{\cdot}7^2) + (9{\cdot}3^2)} = 9{\cdot}7$ kph

$$\tan \theta_x = \frac{9{\cdot}3}{2{\cdot}7} = 3{\cdot}4 \text{ or } \theta_x = 73°36'.$$

Thus the average velocity of the train during the interval is 9·7 kph in the direction 73° 36′ north of east.

Ex. 2. *A car moving with constant acceleration covers the distance between two points* 60 m *apart in* 6 s. *Its speed as it passes the second point is* 15 ms^{-1}. (a) *What is its speed at the first point ?* (b) *What is its acceleration ?* (c) *At what prior distance from the first point was the car at rest ?*

Sol. We have $15=v_0+a.6$ where v_0 is the velocity at the first point and a is the acceleration of the car in the direction from the first point to the second point.

Also $$60=v_0.6+\tfrac{1}{2}.a.6^2=6\ v_0+18\ a.$$

Solving these two equations for v_0 and a,
$$a=\tfrac{5}{8}\ ms^{-2} \quad\text{and}\quad v_0=5\ ms^{-1}.$$

Let the car be at rest at a distance s prior to the first point.

Then $$5^2=2\ .\ \tfrac{5}{8}\ .\ s$$
or $$s=7{\cdot}5\ \text{m}.$$ **Ans.**

Ex. 3. *An elevator (lift) ascends with an upward acceleration of* 1·2 ms^{-2}. *At the instant its upward speed is* 2·4 ms^{-1}, *a loose bolt drops from the ceiling of the elevator* 2·7 m *from the floor. Calculate* (a) *the time of flight of the bolt from ceiling to floor and* (b) *the distance it has fallen relative to the elevator shaft.*

Sol. Let s_1 be the *downward* displacement of the bolt relative to the shaft and s_2 be the upward displacement of the lift just before the bolt reaches the floor of the lift. Then $s_1+s_2=2{\cdot}7$.

The initial velocity of the bolt is 2·4 ms^{-1} upward and its acceleration is $g=9{\cdot}8\ ms^{-2}$ downward.

$$\therefore \quad s_1=-2{\cdot}4\ t+\tfrac{1}{2}\times 9{\cdot}8\ t^2=-2{\cdot}4\ t+4{\cdot}9\ t^2$$

where t is the time of flight of the bolt.

The initial velocity, acceleration and displacement of the lift during the same time are all in the same direction viz. upward.

$$\therefore \quad s_2=2{\cdot}4\ t+\tfrac{1}{2}\times 1{\cdot}2\ t^2=2{\cdot}4\ t+{\cdot}6t^2.$$

Adding the two, we have
$$s_1+s_2=4{\cdot}9\ t^2+{\cdot}6\ t^2=5{\cdot}5\ t^2$$
or $$2{\cdot}7=5{\cdot}5\ t^2 \quad\text{or}\quad t={\cdot}7\ \text{s}.$$ **Ans.**

Now $$s_1=-2{\cdot}4\times{\cdot}7+4{\cdot}9\times{\cdot}7^2=-1{\cdot}68+2{\cdot}4={\cdot}72\ \text{m}.$$ **Ans.**

Projectiles.

Ex. 4. *Two bodies are thrown with the same velocity at angles* α *and* $90-\alpha$ *to the horizon. Calculate the ratio of maximum heights reached by the bodies.*

Sol. Let v_0 be the velocity of projection. Then initial horizontal and vertical velocities are $v_0\cos\alpha$ and $v_0\sin\alpha$ respectively. At the highest point the vertical velocity is reduced to zero. Applying the first kinematic ($v=u+ft$) equation we have

$$0=v_0\sin\alpha-gt \quad \text{or} \quad t=\frac{v_0\sin\alpha}{g}.$$

Applying second kinematic equation ($s=ut+\frac{1}{2}ft^2$) to the vertical motion we have

$$H=v_0\sin\alpha t-\tfrac{1}{2}gt^2$$

$$\text{or} \quad H=v_0\sin\alpha\frac{v_0\sin\alpha}{g}-\frac{1}{2}g\left(\frac{v_0\sin\alpha}{g}\right)^2=\frac{v_0{}^2\sin^2\alpha}{2g}.$$

$$\therefore \quad \frac{H_1}{H_2}=\frac{\dfrac{v_0{}^2\sin^2\alpha}{2g}}{\dfrac{v_0{}^2\sin^2(90-\alpha)}{2g}}=\frac{\sin^2\alpha}{\cos^2\alpha}=\tan^2\alpha. \qquad \textbf{Ans.}$$

Ex. 5. *A body is thrown at an angle* α *to the horizontal with the initial velocity* v_0. *Assuming air drag to be negligible, find* (i) *the time of motion,* (ii) *the maximum height of ascent and the horizontal range,* (iii) *the angle of projection at which they are equal,* (iv) *the angle of projection at which the horizontal range is maximum and the angle at which the vertical range is maximum.*

Sol. In the horizontal direction displacement is R called the 'range' but the vertical displacement is zero.

Time is the same for both horizontal and vertical motion.

Applying the second kinematic equation to the horizontal and vertical motion we have

$$R=v_0\cos\alpha t \quad \text{and} \quad 0=v_0\sin\alpha t-\tfrac{1}{2}gt^2$$

$$\text{or} \quad t=\frac{2v_0\sin\alpha}{g} \quad \text{and} \quad R=\frac{2v_0{}^2\sin\alpha\cos\alpha}{g}=\frac{v_0{}^2\sin2\alpha}{g}.$$

Time to reach the maximum height $=\dfrac{t}{2}=\dfrac{v_0\sin\alpha}{g}$.

$$\therefore \quad H=v_0\sin\alpha.\frac{v_0\sin\alpha}{g}-\frac{1}{2}g\left(\frac{v_0\sin\alpha}{g}\right)^2=\frac{v_0{}^2\sin^2\alpha}{2g}. \qquad \textbf{Ans.}$$

When $R=H$ we have $\dfrac{v_0{}^2\sin2\alpha}{g}=\dfrac{v_0{}^2\sin^2\alpha}{2g}$

or $2\sin2\alpha=\sin^2\alpha$ or $2.2\sin\alpha\cos\alpha=\sin^2\alpha$.

Since $\alpha \neq 0$, $\sin\alpha \neq 0$ and hence $4\cos\alpha = \sin\alpha$

$$\text{or} \quad \tan\alpha = 4$$

$$\text{or} \quad \alpha = \tan^{-1}4 = 76^\circ. \quad \text{Ans.}$$

The horizontal range is maximum when $\sin 2\alpha$ is maximum, that is, $\sin 2\alpha = 1$ or $2\alpha = 90^\circ$ or $\alpha = 45^\circ$. Ans.

$$H = \frac{v_0^2 \sin^2\alpha}{2g} = \frac{v_0^2}{4g}(1 - \cos 2\alpha).$$

The vertical range is maximum when $\cos 2\alpha$ is minimum, that is,

$$\cos 2\alpha = -1 \quad \text{or} \quad 2\alpha = 180^\circ \quad \text{or} \quad \alpha = 90^\circ. \quad \text{Ans.}$$

EXERCISES

(A)

1. A body describes first half of the total distance with velocity v_1 and the second half with velocity v_2. Calculate the average velocity.

$\left(\textbf{Ans.} \ \frac{2v_1v_2}{v_1+v_2}\right)$

2. A particle moves along a horizontal line and its velocity changes with time as shown in the figure.

Calculate (i) initial acceleration, (ii) total distance described.

[**Ans.** (i) 8 ms^{-2}, (ii) 160 m]

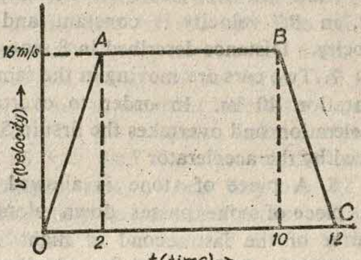


Fig. 3-1

3. A parachutist after bailing out falls 50 metres freely. Then his parachute opens and he falls with deceleration of 2 ms^{-2}. When he reaches the ground his speed is reduced to only 3 metres per second. At what height did he bail out ? For how long was he in air ? (**Ans.** 293 metres, 17 s)

4. A block is released from rest at the top of a frictionless inclined plane 10 m long. It reaches the bottom in 5 seconds. A second block is projected up the plane from the bottom at the instant the first block is released and the two return to the bottom simultaneously. (a) Find the acceleration of each block on the incline. (b) What is the initial velocity of the second block ? (c) How far up the inclined plane does it travel ?

[**Ans.** (a) ·8 ms^{-2}. (b) 2 ms^{-1}, (c) 2·5 m]

5. A plot of velocity versus time for a body in rectilinear motion is as shown in the figure. Indicate whether acceleration is positive, negative, zero, variable or constant in the region *OA*, *AB*, *BC* and *CD*. Calculate the distance described up to the point *C*. What is the displacement up to the point *C* ?

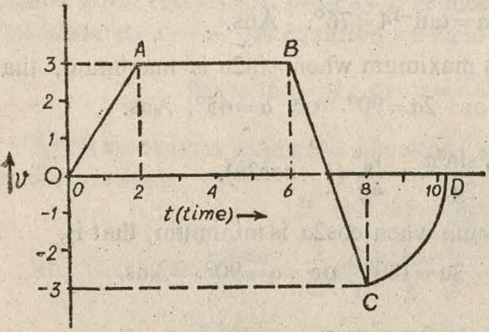


Fig. 3-2

[*Hint* : Slope of line= acceleration; Distance described=area of plot v versus t]

(**Ans.** In *OA* acceleration is +ve and constant, in *AB* acceleration is zero, in *BC* acceleration is −ve and constant, in *CD* acceleration is positive but variable. Distance described=18 m, Displacement=15 m)

6. If the same plot is of displacement (x) versus time t, indicate whether body is at rest or in motion and that too with uniform velocity or variable velocity in the region *OA*, *AB*, *BC* and *CD*. Find the distance described in 8 sec. What is the displacement of the body in 8 s ?

(**Ans.** In *OA* velocity is constant, in *AB* velocity is zero i.e., body is at rest, in *BC* velocity is constant and negative and in *CD* body has variable velocity. Distance described in 8 sec=9 m. Displacement in 8 s=−3 m)

7. Two cars are moving in the same direction with the same speed with separation 40 m. In order to overtake the driver of the rear car puts on the acceleration and overtakes the first in 5 seconds. What is the acceleration produced by the accelerator ? (**Ans.** $3{\cdot}2\ \text{ms}^{-2}$)

8. A piece of stone is allowed to fall from a balloon floating in air. This piece of stone passes down along a tower 30 m in height in the last quarter of the last second of flight. Find the height of stone at the instant the stone was dropped and the velocity of the piece of stone as it reaches the ground. ($g=9{\cdot}8\ \text{m s}^{-2}$)

(**Ans.** 749·54 m; 121·21 m s^{-1})

9. A body travels half of its total path in the last second of its fall from rest, find the time and height of its fall. (**Ans.** 3·41 s, 57·1 m)

(B)

10. A point traverses half a circle of radius R during the interval τ. Calculate the following quantities averaged over that time :

(a) the mean velocity, that is, speed $<v>$;

(b) the modulus of the mean velocity vector $<\vec{v}>$.

(**Ans.** $<v>=\frac{\pi R}{\tau}$; $|\overrightarrow{<v>}|=\frac{2R}{\tau}$)

11. A point traversed half of the distance with velocity v_0. The remaining part of the distance was covered with velocity v_1 for half of the remaining time, and with velocity v_2 for the other half of the remaining time. Find the mean velocity of the point averaged over the whole time of motion.

$$\left[\textbf{Ans.}\quad \langle v\rangle = \frac{2v_0(v_1+v_2)}{(2v_0+v_1+v_2)}\right]$$

12. The driver of a train moving at a speed v_1 sights another train a distance d ahead of the first on the same track moving in the same direction with a slower speed v_2. He puts on the brake and gives his train a constant deceleration a. Show that if $d > \frac{(v_1-v_2)^2}{2a}$ there will be no collision and if $d < \frac{(v_1-v_2)^2}{2a}$ there will be collision.

13. A particle travels with constant speed on a circle of radius 3 m and completes one revolution in 20 s. Starting with the lowest point as origin, find (a) the magnitude and direction of the displacement vectors 5 s, 7·5 s and 10 s later; (b) the magnitude and direction of the displacement in 5 s interval from fifth to the tenth second; (c) average velocity vector in this interval; (d) the instantaneous velocity vector at the beginning and at the end of the interval.

[**Ans.** (a) 4·2 m at 45°, 5·5 m at 68° and 6 m at 90°; (b) 4·2 m at 135°; (c) ·85 m s^{-1} at 135°; (d) ·94 m s^{-1} at 90°, ·94 m s^{-1} at 180°]

14. Two trains, each having a speed of 50 kph are headed at each other on the same track. A bird that can fly at the rate of 100 kph flies off one train when they are 100 km apart and heads directly to the other train. On reaching the other train it flies back to the first train, and so forth. (a) How many trips can the bird make from one train to the other before they crash? (b) What is the total distance the bird travels? (**Ans.** Infinite; 100 km)

15. Two trains, one travelling at 96 kph and the other at 128 kph are headed towards one another along a straight track. When they are 3·2 km apart, the drivers of the two trains simultaneously see the other's train and apply their brakes. If the brakes decelerate each train at the rate of 9 ms^{-2}, determine whether there is a collision. (**Ans.** No)

16. A steel ball is dropped from the roof of a building. An observer standing in front of a window 1·2 m high notes that the ball takes $\frac{1}{8}$ s to fall from the top to the bottom of the window. The ball continues to fall, makes a completely elastic collision with a horizontal sidewalk, and reappears at the bottom of the window 2 s after passing it on the way down. How tall is the building? [*Hint*: The ball will return with the same speed after collision.]

(**Ans.** 20·43 m)

17. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and ultimately comes to rest. If the total time-lapse is t, what is the total distance described and maximum velocity reached? (I. I. T. 1980)

$$[\textbf{Ans.}\quad \left(\frac{1}{2}\cdot\frac{\alpha\beta t^2}{\alpha+\beta},\ \frac{\alpha\beta t}{\alpha+\beta}\right)$$

18. A body begins to move with an initial velocity 2 ms^{-1} and continues to move at a constant acceleration a. Ten seconds later a second body begins to move from the same point with an initial velocity 12 ms^{-1} in the same direction with the same acceleration. What is the maximum acceleration at which the latter can overtake the former ? (**Ans.** 1 ms^{-2})

19. Every 10 minutes one car starts simultaneously from two points A and B 60 km apart at 60 kph. Find graphically how many cars will be met by a passenger in one of the cars between A and B. (**Ans.** 11)

20. Two equal masses are connected by a light string passing over a fixed frictionless pulley. They are set in motion with uniform speed u. The mass going down is momentarily stopped and then released immediately. Calculate the time that will elapse before the string again becomes tight.

(**Ans.** u/g)

[*Hint* : The height through which one goes up must be equal to the height through which the other goes down.]

Projectiles.

21. A ball starts falling with zero initial velocity on to a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time ? (**Ans.** $8\,h \sin\alpha$)

[*Hint* : Take the frame of reference along and perpendicular to the plane and resolve g along these directions.]

22. A cannon and a target are 5·0 km apart on the same horizontal plane. How soon will the shell launched with velocity 240 ms^{-1} reach the target ? $g=10$ ms^{-2}. (**Ans.** 41·5 s or 34 s)

23. A cannon fires successively two shells with velocity 250 ms^{-1} at angles 60° and 45° with the horizontal in the same vertical plane one after the other. What should be the interval between the firings so that they may hit the same target ? (**Ans.** 11 s)

24. A body projected with the same velocity at two different angles covers the same horizontal distance R. If t and t' are the two times of flight, prove that $R=\frac{1}{2}\,gtt'$.

25. A body projected vertically upwards from A, the top of a tower reaches the ground in t_1 seconds. If it is projected vertically downwards from A with the same velocity, it reaches the ground in t_2 seconds. If it falls freely from rest at A, in what time does it reach the ground ? (**Ans.** $\sqrt{t_1 t_2}$)

26. A particle projected with velocity $2\sqrt{ag}$ so that it just clears two walls, of equal height a, which are at a distance $2a$ from each other. Show that the velocity of projection is 60° and that the latus-rectum of the path is $2a$.

[*Hint* : The coefficient of the linear term in the equation of a parabola represents latus-rectum.]

□□□

CHAPTER 4

PARALLELOGRAM OF VELOCITIES AND FORCES : RESOLUTION OF VELOCITIES AND FORCES : RELATIVE VELOCITY

(i) Parallelogram of Forces or Velocities : If a particle possesses two velocities or acted upon by two forces given in magnitude and direction by the adjacent sides of a parallelogram then they are equivalent to a single velocity or force called their resultant and is given in magnitude and direction by the diagonal of that parallelogram passing through their point of intersection.

$$w=\sqrt{u^2+v^2+2uv\cos\theta}.$$

Let w (resultant velocity) make an angle ψ with u then

$$\tan\psi=\frac{v\sin\theta}{u+v\cos\theta}.$$

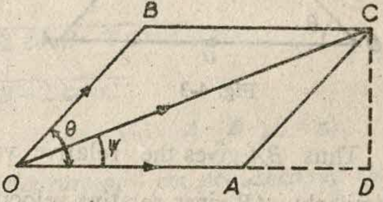


Fig. 4-1

It is to be noted carefully that if we take ψ as angle made by w with v then u and v will exchange their positions in the formula. Also note that reference velocity occurs in the denominator without being multiplied by $\sin\theta$ or $\cos\theta$.

(ii) Magnitude of u and v in terms of w and their inclinations with w : Suppose u and v make angles α and β with w. Then it follows from the figure,

$$\frac{OA}{\sin\beta}=\frac{AC}{\sin\alpha}=\frac{OC}{\sin(\alpha+\beta)}$$

$$\text{or } u=\frac{w\sin\beta}{\sin(\alpha+\beta)}$$

$$\text{and } v=\frac{w\sin\alpha}{\sin(\alpha+\beta)}.$$

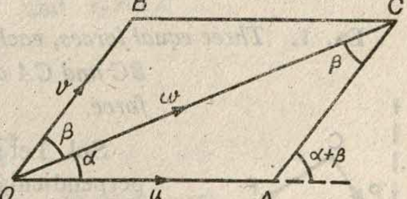


Fig. 4-2

(iii) Resolution of Velocities or Forces : When components of a velocity or force are specially considered in two mutually perpendicular directions, they are called resolved parts. Putting $\alpha+\beta=90^\circ$ we have resolved parts, $u=w\cos\alpha$ and $v=w\sin\alpha$.

(iv) **Relative Velocity** : The relative velocity of a body B with respect to A is *the rate of change of displacement of B relative to A, as if A were at rest.* If $\overrightarrow{OA}$ is the actual velocity of A and $\overrightarrow{OB}$ is the actual velocity of B then $\overrightarrow{OB}-\overrightarrow{OA}$ is the relative velocity of B with respect to A and $\overrightarrow{OA}-\overrightarrow{OB}$ is the relative velocity of A with respect to B.

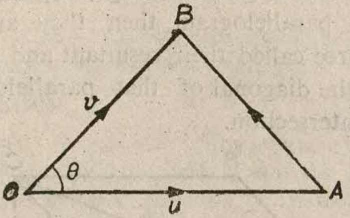


Fig. 4-3

In the figure from triangle law of velocities we have

$$\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BO}=0$$

or $$\overrightarrow{OA}-\overrightarrow{OB}=\overrightarrow{BA}$$

$$(\because\ \overrightarrow{OB}=-\overrightarrow{BO}$$

and $$\overrightarrow{AB}=-\overrightarrow{BA}).$$

Thus $\overrightarrow{BA}$ gives the relative velocity of A with respect to B. Similarly, $\overrightarrow{AB}$ gives relative velocity of B with respect to A.

From property of a triangle, $|\overrightarrow{BA}|$ or $|\overrightarrow{AB}|$ is magnitude of relative velocity $=\sqrt{u^2+v^2-2uv\cos\theta}$ where θ is the angle between u and v.

Vectorially, $\overrightarrow{V_{AB}}$ (relative velocity of A relative to B) $=\overrightarrow{V_A}-\overrightarrow{V_B}$ and $\overrightarrow{V_{BA}}=\overrightarrow{V_B}-\overrightarrow{V_A}$.

EXAMPLES

Ex. 1. *Three equal forces, each of magnitude P, act along AB, BC and CA of a triangle. Calculate the resultant force.*

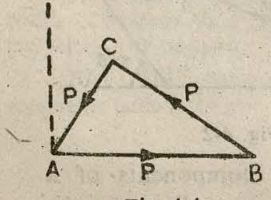


Fig. 4-4

Sol. Let us take AB as x-axis and the line perpendicular to it at A as y-axis.

The first force P_1 along AB is of absolute value P.

$$\therefore\quad \overrightarrow{P_1}=P\,\overrightarrow{i}.$$

The second force P_2 along BC is of absolute value P. Its resolved parts are $P \cos B$ in the negative direction of x-axis and $P \sin B$ in the positive direction of y-axis.

$$\therefore \quad \vec{P_2}=P \cos B\,(-\vec{i})+P \sin B\,\vec{j}.$$

The third force P, along CA is of absolute value P. Its resolved parts are $P \cos A$ in the negative direction of x-axis and $P \sin A$ in the negative direction of y-axis.

$$\therefore \quad \vec{P_3}=P \cos A\,(-\vec{i})+P \sin A\,(-\vec{j}).$$

The resultant force $\vec{R}$ is given by

$$\vec{R}=\vec{P_1}+\vec{P_2}+\vec{P_3}=P\,(1-\cos A-\cos B)\,\vec{i}+P\,(\sin B-\sin A)\,\vec{j}$$

$$\therefore |\vec{R}| = P\sqrt{(1-\cos A-\cos B)^2+(\sin B-\sin A)^2}$$

$$= P\sqrt{3+2 \cos A \cos B-2 \cos B-2 \cos A-2 \sin A\ \mathrm{si\ n}}$$

$$\text{or } |\vec{R}| = P\sqrt{3+2 \cos(A+B)-2 \cos B-2 \cos A}$$

$$\text{or } |\vec{R}| = P\sqrt{3-2 \cos A-2 \cos B-2 \cos C}$$

$$(\because \quad A+B+C=\pi).$$

Ex. 2. *A ship is sailing north at the rate of* 4 *cm per second, the current is taking it east at the rate of* 3 *cm per second, and a sailor is climbing a vertical pole at the rate of* 2 *cm per second. Find the resultant velocity of the sailor in space.*

Sol. Let us take the eastward direction as x-axis, northward direction as y-axis and the vertically upward direction as z-axis. The sailor will simultaneously have velocity 4 cm s^{-1} in the positive direction of y-axis, 3 cm^{-1} along the x-axis and 2 cms^{-1} along the z-axis.

Therefore $\quad \vec{v_x}=3\,\vec{i}\,; \ \vec{v_y}=4\,\vec{j}$ and $\vec{v_z}=2\,\vec{k}$

The resultant velocity is

$$\vec{v}=\vec{v_x}+\vec{v_y}+\vec{v_z}$$

$$\text{or} \qquad \vec{v}=3\,\vec{i}+4\,\vec{j}+2\,\vec{k}$$

$$\therefore \quad |\vec{v}| = \sqrt{3^2+4^2+2^2}=\sqrt{29}=5{\cdot}4 \text{ cms}^{-1}.$$

Let the resultant velocity make angles θ_x, θ_y and θ_z with x-, y- and z-axis. Then

$$\tan\theta_x = \frac{\sqrt{y^2+z^2}}{x} = \frac{\sqrt{4^2+2^2}}{3} = \frac{\sqrt{20}}{3},$$

$$\tan\theta_y = \frac{\sqrt{z^2+x^2}}{y} = \frac{\sqrt{2^2+3^2}}{4} = \frac{\sqrt{13}}{4} \text{ and}$$

$$\tan\theta_z = \frac{\sqrt{x^2+y^2}}{z} = \frac{\sqrt{3^2+4^2}}{2} = \frac{\sqrt{25}}{2};$$

$$\therefore \quad \cos\theta_x = \frac{3}{\sqrt{29}}, \quad \cos\theta_y = \frac{4}{\sqrt{29}}, \quad \cos\theta_z = \frac{2}{\sqrt{29}}.$$

Thus the velocity of the sailor is 5·4 cms^{-1} along the direction of direction-cosines

$$\left(\frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right).$$

Ex. 3. *To a man walking at the rate of* 3 *kph the rain appears to fall vertically: when he increases his speed to* 6 *kph it appears to meet him at an angle of* 45° *with the vertical from the front. Find the real direction and speed of rain.*

Sol. Let us take the horizontal *direction to the right as x-axis* and the vertically *upward direction* as *y*-axis.

Let us take $\quad \vec{v}_{rain} = a\,\vec{i} + b\,\hat{j}.$

In the first case $\quad \vec{v}_{man} = 3\,\vec{i}.$

$\therefore \quad \vec{v}_{rain-man}$ (relative velocity of rain relative to man)

$= \vec{v}_{rain} - \vec{v}_{man}$ or $\vec{v}_{rain-man} = (a-3)\,\vec{i} + b\,\vec{j}.$

Let θ be the angle made by $v_{rain-man}$ with x-axis.

Then $\quad \tan\theta = \frac{b}{a-3}.$

According to the question $\theta = 270°$ ($\because$ angle is *always counted anticlockwise from the x-axis in vector notations*).

$$\therefore \quad \tan 270° = \frac{b}{a-3} \quad \text{or} \quad \propto = \frac{b}{a-3} \text{ or } a-3=0 \text{ or } a=3.$$

$\therefore \quad \vec{v}_{rain} = 3\,\vec{i} + b\,\vec{j}.$

In the second case, $\vec{v}_{man} = 6\,\vec{i}.$

$\therefore \quad \vec{v}_{rain-man} = \vec{v}_{rain} - \vec{v}_{man} = -3\,\vec{i} + b\,\vec{j}.$

$\therefore$ Now, $\quad \tan\theta = \frac{b}{-3}.$

According to the question $\theta=180°+45°$;

$$\therefore \quad \tan(180°+45°)=\frac{b}{-3} \quad \text{or} \quad 1=\frac{b}{-3} \quad \text{or} \quad b=-3;$$

$$\therefore \quad \vec{v}_{rain}=3\vec{i}-3\vec{j};$$

$$|\vec{v}_{rain}|=\sqrt{3^2+3^2}=\sqrt{18}=3\sqrt{2}\text{ kph.}$$

Let θ be the angle made by $\vec{v}_{rain}$ with x-axis.

$$\therefore \quad \tan\theta=\frac{-3}{+3}.$$

Since here base is positive and perpendicular is negative, θ is an angle in the fourth quadrant.

$$\therefore \quad \theta=270°+45°.$$

Thus the true velocity of rain is $3\sqrt{2}$ kph making $270°+45°$ with the direction of motion of the man counted in the anticlockwise direction.

Ex. 4. *The wind appears to blow from north to a man moving in the north-east direction. When he doubles his velocity the wind appears to move in the direction* $\cot^{-1} 2$ *east of north. Find the actual direction of the wind.*

Sol. Let v be the absolute value of the velocity of the man. We take as usual eastward direction as x-axis and northward direction as y-axis. Then

$$\vec{v}_{man}=\frac{v}{\sqrt{2}}\vec{i}+\frac{v}{\sqrt{2}}\vec{j}.$$

Let $\vec{v}_{wind}=a\vec{i}+b\vec{j}$.

Then $$\vec{v}_{wind-man}=\left(a-\frac{v}{\sqrt{2}}\right)\vec{i}+\left(b-\frac{v}{\sqrt{2}}\right)\vec{j}.$$

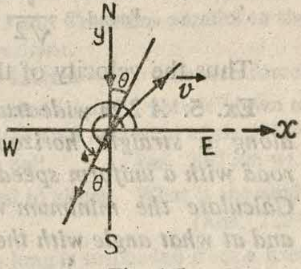


Fig. 4-5

$$\therefore \quad \tan\theta=\frac{b-\frac{v}{\sqrt{2}}}{a-\frac{v}{\sqrt{2}}}=\tan 270° \quad (\because \ \theta=270° \text{ by the question})$$

or $$a=\frac{v}{\sqrt{2}}.$$

$$\therefore \quad \vec{v}_{wind}=\frac{v}{\sqrt{2}}\,\vec{i}+b\vec{j}.$$

When the man doubles his speed, we have

$$\vec{v}_{man}=\frac{2v}{\sqrt{2}}\,\vec{i}+\frac{2v}{\sqrt{2}}\,\vec{j}=\sqrt{2}\,v\,\vec{i}+\sqrt{2}\,v\,\vec{j}$$

and $\vec{v}_{wind\text{-}man}=\left(\frac{v}{\sqrt{2}}-\sqrt{2}\,v\right)\vec{i}+(b-\sqrt{2}\,v)\,\vec{j}.$

$$\therefore \quad \tan\theta=\frac{b-\sqrt{2}\,v}{\frac{v}{\sqrt{2}}-\sqrt{2}\,v}=-\frac{(\sqrt{2}\,b-2v)}{v}.$$

Here $\theta=(270^\circ-\cot^{-1}2).$

$$\therefore \quad \tan(270^\circ-\cot^{-1}2)=-\left(\frac{\sqrt{2}\,b-2v}{v}\right)$$

$$\cot(\cot^{-1}2)=-\frac{\sqrt{2}\,b-2v}{v}$$

or $2=-\left(\frac{\sqrt{2}\,b-2v}{v}\right)$

or $2v=-\sqrt{2}\,b+2v$

or $b=0.$

$$\therefore \quad \vec{v}_{wind}=\frac{v}{\sqrt{2}}\,\vec{i}.$$

Thus the velocity of the wind is to the east. **Ans.**

Ex. 5. *A 2 m wide truck is moving with uniform speed of 8 ms^{-1} along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4 m away from him. Calculate the minimum value of v so that he can cross the road safely and at what angle with the road should he start walking.*

Sol. Let the man start crossing the road at an angle θ with the road. For safe crossing the condition is that the man must cross the road by the time the truck describes the distance $(4+2\cot\theta)$.

$$\therefore \quad \frac{4+2\cot\theta}{8}=\frac{2}{v\sin\theta}$$

or $$v=\frac{8}{2\sin\theta+\cos\theta}.$$

For minimum v, $\frac{dv}{d\theta}=0.$

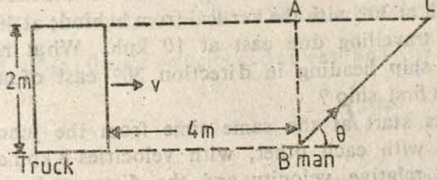


Fig. 4-6

Now $\frac{dv}{d\theta}=\frac{-8(2\cos\theta-\sin\theta)}{(2\sin\theta+\cos\theta)^2}.$

$\therefore$ For minimum v, $2\cos\theta-\sin\theta=0$

or $\tan\theta=2$ or $\theta=\tan^{-1}2=63{\cdot}4°$. **Ans.**

$$v_{min}=\frac{8}{2\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}}=\frac{8}{\sqrt{5}}=3{\cdot}57\ \text{ms}^{-1}.$$ **Ans.**

EXERCISES

(A)

1. The greatest and least resultant of two forces acting at a point are 13 newtons and 5 newtons. Calculate each force. (**Ans.** 9 newtons and 4 newtons)

2. A particle is acted on by forces 1, 2, 3 and 4 newtons parallel to the sides of a rectangle taken in order. Find their resultant.

(**Ans.** $2\sqrt{2}$ N at 45° inclination to 4 newton force)

3. A pith ball of mass 1·5 gm and suspended by a silk fibre is blown to one side by a horizontal current of air so that the fibre makes an angle 30° to the vertical. Find the force of air current on the ball. (**Ans.** ·867 gm wt.)

4. Two equal velocities have resultant equal to either. What is the angle between them ? (**Ans.** 120°)

5. A uniform rod of mass 3 kg and 1 m long is suspended from a fixed point by means of two strings of lengths ·6 m and ·8 m and attached to the free ends of the rod. Find the tensions in the string. (**Ans.** 1·8 kg f; 2·4 kg f)

6. A river is ½ km wide and flows at the rate of 3 kph. A man, who can swim in still water at the rate of 5 kph wishes to reach the other bank of the river. In what direction should he start swimming to cross the river (a) along the shortest route, (b) in the shortest possible time ? Calculate how much time he will take to cross the river in the two cases.

[**Ans.** (a) In direction 126° 52′ to the current, (b) in direction at 90° to the current. Time 7 min 30 s and 6 min respectively)

7. To a man walking at 5 kilometres per hour (kph) rain appears to fall vertically, and when he doubles his speed it appears to make an angle of 30° with the vertical. Find the actual velocity of the rain. In what direction will rain appear to fall if the man suddenly turns about in the first case ?

(**Ans.** 10 kph at 30° with the vertical from behind; at 49°6 from the front)

8. A ship is travelling due east at 10 kph. What must be the velocity of a second ship heading in direction 30° east of north, if it is always due north from the first ship ? (**Ans.** 20 kph)

9. Two cyclists start at the same time from the junction of two roads making angle 60° with each other, with velocities 8 kph and 13 kph respectively. Find their relative velocity and the distance between them after 10 minutes. (**Ans.** 11·36 kph; 1·89 km)

10. A man walking on a road with velocity of 5 km per hour encounters rain falling vertically with a velocity 25 km per hour. At what angle should he hold his umbrella now in order to protect himself from the rain ?

(**Ans.** 11° 19′ with the vertical)

11. The resultant of two forces P and $2P$ acting at a point is perpendicular to P. Find the angle between the forces. (**Ans.** 120°)

12. Two cars are moving in the same direction with same speed 30 kph. They are separated by a distance 5 km. What is the speed of a car moving in the opposite direction if it crosses the two cars at an interval of 4 minutes ?

(**Ans.** 45 kph)

13. A man wants to cross a river 500 m wide. His rowing speed in still water is 3 kph. The river flows at a speed of 2 kph. If the man's walking speed on the shore is 5 kph, (a) find the path he should take to get to the point directly opposite to his starting point in the shortest time. (b) How long does it take ? (**Ans.** (a) He should head the boat at 115·4° with the stream. (b) 12·7 min)

14. Find the speed of two objects f, when they move uniformly towards each other, they get 4 m closer in every second, and, when they move uniformly in the same direction with the original speeds, they get 4 m closer in every 10 s.

(**Ans.** 2·2 ms^{-1}; 1·8 ms^{-1})

15. The resultant of two forces 5 newtons and 3 newtons trisects the angle between them. Calculate the angle between them. (**Ans.** 100° 40′)

16. Two bodies were thrown simultaneously from the same point : one, straight up, and, the other, at an angle of $\theta=60°$ to the horizontal. The initial velocity of each body is equal to $v_0=25\ ms^{-1}$. Neglecting the air drag, find the distance between the bodies $t=1{\cdot}7$ s later.

[*Hint* : Find the relative velocity at time t. Distance between the bodies =relative velocity×time.]

(**Ans.** $s=v_0t\sqrt{2(1-\sin\theta)}=22$ m)

17. Two particles, 1 and 2, move with constant velocities v_1 and v_2 along two mutually perpendicular straight lines towards the intersection O. At the moment $t=0$ the particles were located at the distances x_1 and x_2 from the point O. How soon will the distance between the particles become the smallest ? What is it equal to ?

$$\left(\textbf{Ans. } t_{min}=\frac{v_1x_1+v_2x_2}{v_1^2+v_2^2}\ ;\ s_{min}=\frac{x_1v_2-x_2v_1}{\sqrt{v_1^2+v_2^2}}\right)$$

18. A pilot flies due east from A to B. The velocity of the aeroplane in air is v and the velocity of the air relative to the ground is u. The distance between A and B is l and the aeroplane's air speed v is constant. In still air the time for the round trip is, obviously, $t_0=\frac{2l}{v}$. Calculate the time for round trip in terms of t_0, u and v if (a) the air velocity is due east and (b) the air velocity is due north. $\left(\textbf{Ans.}\ \frac{{}_0}{1-\frac{u^2}{v^2}};\ \frac{t_0}{\sqrt{1-\frac{u^2}{v^2}}}\right)$

19. Two boats, A and B, move away from a buoy anchored in the middle of a river. A moves along the river and B at right angles to it. Having moved off equal distances from the buoy the boats returned. Find the ratio of the times of motion of the boats if the velocity of each boat with respect to water is n times greater than the velocity of the river. $\left(\textbf{Ans.}\ \frac{n}{\sqrt{n^2-1}}\right)$

20. A boat moves relative to still water with a velocity which is $n=2$ times less than the river flow velocity. At what angle to the stream direction must the boat move to minimise drifting ? (**Ans.** $\theta=\sin^{-1}\frac{1}{n}+90^\circ=120^\circ$ for $n=2$)

* * *

CHAPTER 5

MOMENTUM : NEWTON'S LAWS OF MOTION : FORCE : CONSERVATION OF MOMENTUM : D' ALEMBERT'S PRINCIPLE : FREE-BODY DIAGRAM : COLLISION OF BODIES

(i) Momentum of a body :

Momentum of a body = mass × velocity kg ms^{-1}.

Vectorially, $\vec{p} = m\vec{v}$.

(ii) According to Newton's second law of motion,

$$F = ma \text{ (Force} = \text{mass} \times \text{acceleration)}$$

$$= \frac{mv - mu}{t}$$

or $F.\,t = mv - mu$. The product $F.\,t$ is called *impulse of force*. Unit, newton second (Ns).

If two bodies (two separate bodies or two parts of the same body) numbered 1 and 2 exert force on each other when they are in contact or separated by a distance (magnetic force, electric force, gravitational force are such forces) then according to Newton's third law,

$\vec{F}_{1\,2} = -\vec{F}_{2\,1}$ where $\vec{F}_{1\,2}$ is the force by 1 on 2 and $\vec{F}_{2\,1}$ is the force by 2 on 1.

We have $|\vec{F}_{1\,2}| = |\vec{F}_{2\,1}| = F$ (say) but their direction is opposite and they act on different bodies. This is why action and reaction though equal and opposite they cannot produce equilibrium.

(iii) Conservation of momentum : Under the mutual action and reaction of two or more than two bodies, free from external forces, the algebraic sum of their momenta in each and every direction remains conserved i.e. remains unchanged.

(iv) Collision of bodies : (a) When two bodies collide and no force acts on them from outside, their momenta in any direction is always conserved. There are two types of collision : (i) elastic, (ii) inelastic. *When collision takes place in such a way that kinetic energy before and after is the same, it is said to be elastic. When kinetic energy is not conserved, collision is inelastic. Further if after collision the two bodies stick together, collision is said to be completely inelastic.*

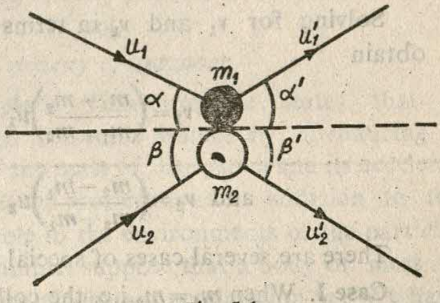


Fig. 5-1

Suppose two bodies moving with velocities u_1 and u_2 at angles α and β with any direction collide elastically and after collision they move on with velocities u_1', and u_2' making angles α' and β' with the same line. According to principle of conservation of momentum along and perpendicular to the line, we have

$$m_1u_1 \cos \alpha + m_2u_2 \cos \beta = m_1u_1' \cos \alpha' + m_2u_2' \cos\beta'$$

and $$m_2u_2 \sin \beta - m_1u_1 \sin \alpha = m_1u_1' \sin \alpha' - m_2u_2' \sin \beta';$$

and from consideration of conservation of kinetic energy,

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1u_1'^2 + \tfrac{1}{2}m_2u_2'^2.$$

(b) One-dimensional elastic collision : Let two bodies of masses m_1 and m_2 move with velocities u_1 and u_2 along a line before collision and with velocities v_1 and v_2 along the same line after collision. We take the positive direction of the momentum and velocity to be to the right. Then from conservation of momentum, we obtain

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

Because the collision is elastic, kinetic energy is conserved and we obtain $$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2.$$

The momentum equation can be written as

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{.. (i)}$$

and the energy equation can be written as

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{.. (ii).}$$

Dividing (i) by (ii) and assuming that $u_1 \neq v_1$ and $u_2 \neq v_2$, we obtain

$$u_1 + v_1 = u_2 + v_2$$

and, after rearrangement,

$$(u_1 - u_2) = -(v_1 - v_2).$$

This tells us that in an elastic one-dimensional collision *relative velocity after collision is equal and opposite to the relative velocity before collision* or, say, velocity of approach = velocity of separation.

Solving for v_1 and v_2 in terms of initial velocities u_1 and u_2, we obtain

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$\text{and} \quad v_2 = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)u_2 + \left(\frac{2m_1}{m_2 + m_1}\right)u_1.$$

There are several cases of special interest.

Case I. When $m_1 = m_2$ i.e. the colliding particles have the same mass, then $v_1 = u_2$ and $v_2 = u_1$. That is, in a one-dimensional elastic collision of two particles of equal mass, the particles simply *exchange* velocities during collision.

Case II. Another case of interest is that in which one particle (m_2) is initially at rest. Then u_2 equals zero and

$$v_2 = u_1 \text{ and } v_1 = 0.$$

The first particle is 'stopped dead', and the second one '*takes off*' with the velocity the first one originally had.

Case III. If m_2 is very large such as a wall, then

$$u_2 = 0 \text{ and } m_2 = \infty.$$

$$v_1 = -u_1$$

$$v_2 = 0.$$

That is, when a light particle collides with a massive body at rest, the particle is returned with the same speed.

Case IV. If the collision takes place from a moving wall, that is,

$$m_2 = \infty \text{ and } u_2 \neq 0,$$

$$v_1 = -u_1 + 2u_2 = -(u_1 - 2u_2).$$

That is, the light particle is returned with its own velocity minus twice the velocity of the wall.

If the wall moves towards the in-coming particle then $v_1 = -(u_1 + 2u_2)$, that is, the particle is returned with its own velocity plus twice the velocity of the wall.

According to Newton's experimental law on collisions of two bodies, the relative velocity of any one of the bodies relative to the other after impact is proportional to the relative veloctiy of the same body before impact but opposite in direction. The constant of proportionality is called 'coefficient of restitution'. V_{AB} (after impact) $= -e \times V_{AB}$ (before impact), where V_{AB} stands for relative velocity of A relative to B. Remember that only the velocity perpendicular to the plane is subject to Newton's law and not the one parallel to the plane. The velocity parallel to the plane remains unaltered. In other words we may say,

Velocity of separation $= e \times$ *velocity of approach.*

(v) D' Alembert's principle : This principle states that a dynamic system can be reduced to a static system by introducing a force, equal to the product of the mass of the system and its acceleration and directed oppositely to the accelerations in addition to the real forces (forces accountable to the environments of the particle) acting on the system. For example, suppose that a body of mass m is moved vertically upward with a constant acceleration 'a'. We have to find the tension of the rope. We can solve for tension in two ways : one by 'consideration of dynamics' of the body. The real forces acting on it are mg downward and the tension T upward. Under the action of these two forces the body is going up with acceleration a.

$$\therefore \quad T - mg = ma \text{ or } T = mg\left(1 + \frac{a}{g}\right).$$

The same can be solved by D' Alembert's principle. According to this principle we may consider the body to be at rest by introducing a force ma downwards in addition to the real forces T (upward) and mg (downward). Since the body is at 'rest' now, the net upward or downward force must be zero. That is,

$$T - mg - ma = 0 \text{ or } T = mg + ma = mg\left(1 + \frac{a}{g}\right).$$

Find the usefulness of this principle in problem 11.

(vi) Free-body diagram : In solving problems when several forces are involved, the following steps are recommended.

(a) First, fix up the system whose motion you want to consider to solve a given problem.

(b) Next, turn your attention to the *objects which are the immediate environments* of the system. These will exert forces on the system.

(c) Now make a separate diagram of the system alone, showing all the forces acting *on* the system, and the frame of reference. This is called '*free-body*' diagram.

(d) Finally apply Newton's second law to each component of force and acceleration.

Illustrations :

(1) Motion in an elevator (or a lift) :

Consider a lift moving vertically upward with acceleration a. We wish to find the force exerted by a man on the floor of the lift.

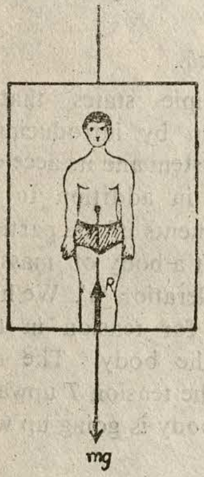


Fig. 5-2

Two forces always act on the body whether the lift moves or not. These forces are (i) its weight mg vertically downward, (ii) R, reaction of the floor on the man. When the lift is going up with acceleration, the man is also going up with the same acceleration.

∴ From second law of motion,

$$R-mg=ma, \text{ or } R=mg+ma.$$

If the man stands on the platform of a spring balance placed on the floor of a lift, it will indicate a force $(mg+ma)$ and hence this is the apparent weight of the man in the lift.

$$\text{Apparent weight}=(mg+ma) \text{ newton}$$
$$=mg\,(1-a/g) \text{ newton}=m\,(1+a/g) \text{ kg}.$$

(2) Motion of connected bodies : Suppose two bodies m and m' are connected by an inextensible weightless string passing over a frictionless light pulley and $m>m'$. Then m will go down and m' will go up with the same acceleration. Considering motion of m and m',

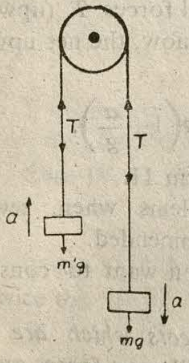


Fig. 5-3

$$mg-T=ma.$$

$T-m'g=m'a'$ where T is the tension of the string. Since the string is inextensible, $a=a'$. Adding, we have

$$a=\frac{mg-m'g}{m+m'}=\frac{m-m'}{m+m'}\times g.$$

Putting value of a in any of the equations, $T=\dfrac{2mm'}{m+m'}g.$

(vii) **Systems of variable mass** : There are many systems in which the mass of the system is changing while we are observing it such as rocket, conveyor belt etc.

Let M be the instantaneous mass and v (to the *right*) be the instantaneous velocity of a system of variable mass.

Let ΔM be the mass detached from it in Δt second.

By the second law of motion,

$$F_{ext}=\frac{dp}{dt}.$$

We can write this for a finite interval Δt,

$$F_{ext}\cong\frac{\Delta p}{\Delta t}.$$

If the detached mass moves to the *left* with velocity u, then

$$\Delta p=p_{initial}-p_{initial}=\{(M-\Delta M).(v+\Delta v)-\Delta Mu\}-Mv$$

$$\therefore\quad F_{ext}\cong M\frac{\Delta v}{\Delta t}+[-u-(v+\Delta v])\frac{\Delta M}{\Delta t}.$$

In the limit $\Delta t\to 0$, $F_{ext}=M\dfrac{dv}{dt}-(u+v)\dfrac{dM}{dt}$

or $$M\frac{dv}{dt}=F_{ext}+(u+v)\frac{dM}{dt}.$$

We have $$u_{rel}=u-(-v)=v+u.$$

$$\therefore\quad M\frac{dv}{dt}=F_{ext}+u_{rel}\frac{dM}{dt}.$$

The last term represents the rate at which the momentum is being transferred *into* (or out of) the system *by* the mass that the system ejects (or collects). It can be interpreted as a force on the system by the *leaving* (or entering mass). For a rocket, this term is called the *thrust*.

$$\therefore\quad M\frac{dv}{dt}=F_{ext}+F_{reaction}.$$

Thus $F_{reaction}$ (i.e. thrust generated by the leaving mass)

$$=u_{rel}\frac{dM}{dt}.$$

EXAMPLES

Ex. 1. *A body of mass* 2 kg *falls* 60 m *from rest and is then brought to rest penetrating* ·6 m *into some sand. Find the average thrust of the sand on it.*

Sol. While penetrating into the sand the forces acting on the body are : its weight 2g downward and the average thrust 'T' of the sand upward.

$\therefore\ T-2g=2a$ where a is the retardation of the body through the sand.

$$\text{Now}\quad v^2=2\times 9{\cdot}8\times 60$$
$$\text{also}\quad v^2=2.a\times{\cdot}6$$
$$\therefore\quad 2a\times{\cdot}6=2\times 9{\cdot}8\times 60$$
$$\text{or}\quad a=980\ \text{ms}^{-2}$$
$$\therefore\quad T-2\times 9{\cdot}8=2\times 980$$
$$\text{or}\quad T=2\times 989{\cdot}8=1979{\cdot}6\ \text{newtons.}\ \textbf{Ans.}$$

Ex. 2. *A body of* 2 kg *mass makes an elastic collision with another body at rest and afterwards continues to move in the original direction but with one-fourth of its original speed. What is the mass of the struck body ?*

Sol. Let v be the velocity of 2 kg mass before collision. The body at rest is of mass m (say) and let V be its velocity after collision in the original direction of motion of 2 kg mass. From the principle of conservation of momentum,

$$2\times v+0=MV+2\times v/4$$
$$\text{or}\quad 3v/2=MV.$$

Since the collision is elastic, the kinetic energy is conserved.

$$\therefore\ \tfrac{1}{2}2v^2=\tfrac{1}{2}\,MV^2+\tfrac{1}{2}.2\frac{v^2}{16}$$

$$\text{or}\quad v^2=\tfrac{1}{2}MV^2+\frac{v^2}{16}$$

$$\text{or}\quad \frac{30v^2}{16}=MV^2.$$

Solving for M we have $M=1{\cdot}2$ kg. **Ans.**

Ex. 3. *A ball of* 1 kg *drops vertically on to the floor with a speed of* 25 ms^{-1}. *It rebounds with an initial velocity of* 10 ms^{-1}. (a) *What impluse acts on the ball during contact ?* (b) *If the ball is in contact for* ·02 s, *what is the average force exerted on the floor ?*

Sol. Change in momentum of the ball $=1\times(-10)-1\times25$

$=-35$ kg ms^{-1} (*downward*).

Hence impulse on the ball during contact is -35 kg ms^{-1} (downward) or 35 kg ms^{-1} (upward).

(b) Force of the body $=\dfrac{\text{Change in momentum}}{\text{time}}=\dfrac{-35}{\cdot02}$

$=-1750$ newtons (downward)

$=1750$ newtons (upward).

$\therefore$ Average force on the floor $=1750$ newtons (downward)

(by Newton's third law). **Ans.**

Ex. 4. *A rocket weighs* 13,500 kg *when fueled up on launching pad. It is fired vertically upward. Gases are exhausted at the rate of* 145 kg *per second with velocity* 1500 ms^{-1} *relative to the rocket. What is the 'thrust' on the rocket ?*

Sol. The forward thrust in a rocket $=u_{rel}\dfrac{dM}{dt}$ where u_{rel} is the relative velocity of leaving gas relative to the rocket.

$\therefore$ Forward thrust $=1500\times145$

$=217500$ newtons. **Ans.**

Ex. 5. *A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. A force P is applied to the other end of the rope.* (a) *Find the acceleration of the block and the rope.* (b) *Find the force that the rope exerts on the block M in terms P, M and m.*

Sol. Make a free-body diagram of the rope and the body.

Considering vertical and horizontal motion of the block, we have

N (normal reaction) $=Mg$

and $T=Ma$ where a is the acceleration of the block.

Again considering horizontal motion of the rope, $P-T=ma$.

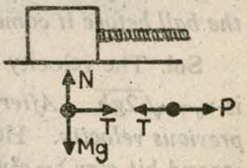


Fig. 5-4

Adding, we have $P=(m+M)a$ or $a=\dfrac{P}{m+M}$. **Ans.**

Putting the value of a in any one of the equations, we have

$$T=\frac{P.M}{m+M}. \quad \textbf{Ans.}$$

Ex. 6. *A gun (of mass M) whose barrel makes an angle α with the horizontal fires a shell (of mass m). The gun is mounted on a frictionless track, so that recoil takes place with no resistive forces. Find the recoil velocity V of the gun, and the absolute velocity of the shell in magnitude and direction if the muzzle velocity of the shell relative to the barrel is v.*

Sol. Since there is no horizontal force on the system (gun + shell + explosives) the horizontal momentum is conserved. Horizontal relative velocity of the shell $= v\cos\alpha$. If v_{abs} be the absolute horizontal velocity of the shell, then $v\cos\alpha = v_{abs}-(-V)$

or $$v_{abs}=v\cos\alpha - V.$$

By conservation of linear momentum

$$m(v\cos\alpha - V)-MV=0 \quad \text{or} \quad V=\frac{mv\cos\alpha}{m+M}.$$

The components of the absolute velocity of the shell are

$$(v\cos\alpha - V) \text{ and } v\sin\alpha.$$

Thus, absolute velocity of shell $=\sqrt{(v\cos\alpha - V)^2+(v\sin\alpha)^2}$

$$=v\sqrt{1-\frac{m(m+2M)}{(m+M)^2}\cos^2\alpha}. \quad \textbf{Ans.}$$

$$\tan\beta=\frac{v\sin\alpha}{v\cos\alpha - V}=\left(1+\frac{m}{M}\right)\tan\alpha. \quad \textbf{Ans.}$$

Inelastic collision.

Ex. 7. *A spherical ball falls from a height h on to a floor of coefficient of restitution e. Calculate the total distance described by the ball before it comes to rest.*

Sol. The velocity acquired by the ball before hitting the ground is $v_0=\sqrt{2gh}$. *After each hit its velocity is reduced e time of its previous velocity.* Hence velocity after first hit is ev_0, velocity after second hit $e(ev_0)=e^2v_0$, that after third hit $e(e^2v_0)=e^3v_0$ and so on. After nth hit velocity becomes e^nv_0.

Height reached by the ball after *n*th hit $=\dfrac{(e^n v_0)^2}{2g}=\dfrac{v_0^2}{2g}e^{2n}$.

Putting $n=1, 2, 3, \ldots$ we have

H (total distance described)

$$=h+2\left(\frac{v_0^2}{2g}e^2+\frac{v_0^2}{2g}e^4+\ldots\right)$$

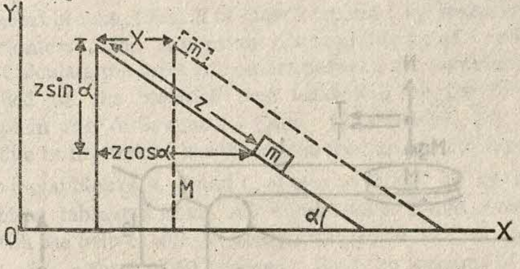
or
$$H=h+\frac{v_0^2}{g}.e^2(1+e^2+e^4+\ldots)$$

$$=h+\frac{2gh}{g}e^2.\frac{1}{1-e^2}=h\left(1+\frac{2e^2}{1-e^2}\right)$$

or
$$H=h\times\frac{1+e^2}{1-e^2}.\quad \textbf{Ans.}$$

Ex. 8. *A heavy mass m slides down a triangular prism of mass M. The prism lies on a horizontal surface and can move along it without friction. What will be the direction of motion of m if the inclination of the hypotenuse of the prism is α with the horizontal?*

Sol. Consider any displacement of *m* by *z* along the prism. Let us use *x* and *y* to denote the actual rightward displacement and downward displacement of *m* respectively and *X* and *Y* to denote actual leftward and vertical displacement of the prism. Obviously $Y=0$ because the prism cannot have downward displacement.



Relative rightward displacement of $m=z\cos\alpha$

$$=x-(-X)=x+X$$

or $\dot{z}\cos\alpha=\dot{x}+\dot{X}$ where dot represents $\dfrac{d}{dt}$

or
$$v_{rel}\cos\alpha=v_x+V_x \quad \ldots \text{(i)}.$$

Relative downward displacement $= z\sin\alpha$

$$= y - 0 = y$$

or $\dot{z}\sin\alpha = \dot{y}$ or $v_{rel}\sin\alpha = v_y$ or $v_{rel} = \dfrac{v_y}{\sin\alpha}$ .. (ii).

Conserving the momentum along the horizontal because the system is free from external horizontal force,

$$mv_x = MV_x \quad \text{.. (iii).}$$

Combining (i), (ii) and (iii),

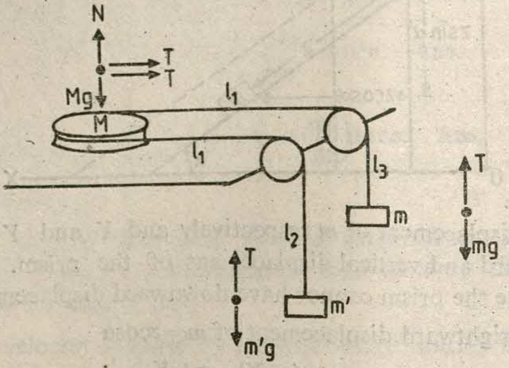
$$\therefore \quad \frac{v_y}{\sin\alpha}\cos\alpha = v_x + \frac{m}{M}v_x; \quad \frac{v_y}{v_x} = \left(1+\frac{m}{M}\right)\tan\alpha.$$

If the direction of motion of m makes angle β with the horizontal then

$$\tan\beta = \frac{v_y}{v_n} = \left(1+\frac{m}{M}\right)\tan\alpha. \quad \textbf{Ans.}$$

Ex. 9. *A smooth pulley of mass M is lying on a smooth table. A light string passes round the pulley and has masses m and m′ attached to its ends, the two portions of the string being perpendicular to the edge of the table so that the masses hang vertically. Calculate the acceleration of the pulley.*

Sol. Let the lengths of the portions of the string be l_1, l_2, l_3. Let M move to the right by x on the table, m' go down by y and m go up by z.



Then l_1 becomes $l_1 - x$, l_2 becomes $l_2 + y$ and l_3 becomes $l_3 - z$. Since the length of the string must remain the same,

$$2l_1 + l_2 + l_3 = 2(l_1 - x) + l_2 + y + l_3 - z$$

or $2x=y-z$ or $2\ddot{x}=\ddot{y}-\ddot{z}$ where 'double dots' represent $\frac{d^2}{dt^2}$

or $$2a_M=a_m'-a_m.$$

By the free body diagram of M, m' and m we have

$$2T=Ma_M,$$

$$m'g-T=m'a_m' \quad \text{and} \quad T-mg=ma_m.$$

Substituting the value of a_M, a_m' and a_m,

$$2.\,\frac{2T}{M}=\left(g-\frac{T}{m'}\right)-\left(\frac{T}{m}-g\right)$$

$$=2g-T\,\frac{m+m'}{mm'} \quad \text{or} \quad T=\frac{2Mmm'g}{4mm'+M(m+m')};$$

$$\therefore \quad a_M=\frac{2}{M}\times T=\frac{4mm'g}{4mm'+M(m+m')}. \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. A constant force acts on a body of mass 10 gm moving with a velocity of 500 ms^{-4}. The body is found to move with the same velocity but in opposite direction after 10 s. Calculate the magnitude of the force. [**Ans.** (i) 1 newton]

2. Two metal blocks A and B of mass 2 kg and 1 kg respectively are in contact on a frictionless table. A constant horizontal force of 3 newtons is applied to block A. Calculate the force of contact between the two blocks. If the same force is applied to the block B then what will be the force of contact? Can you explain the difference? [**Ans.** (i) 1 newton, (ii) 2 newtons. The difference is due to inertial resistance to the transmission of force.]

3. Three metal blocks A, B nnd C of masses 10 kg, 20 kg and 30 kg rest on a frictionless table in a line. A is connected to B and B to C by two pieces of string. With the help of a third string attached to C, the system is pulled to the right with a force of 60 newtons. Find the tensions of first and second string. (**Ans.** 10 newtons and 30 newtons)

4. Two blocks of masses m and m' are connected by a light spring on a horizontal frictionless table. The spring is compressed and then released. Find the ratio of acceleration of masses. (I. I. T. 1975)

$$\left(\textbf{Ans.}\ \frac{a}{a'}=\frac{m'}{m}\right)$$

5. A constant force acts on a body of mass 1 kg for 5 s and then ceases to work. In the next 5 seconds the body describes 100 m. Calculate the force that acted on the body. (**Ans.** 4 newtons)

6. A uniform rope of length L, resting on a frictionless horizontal table, is pulled at one end by a force F. What is the tension in the rope at a distance x from the end where the force is applied ? (I. I. T. 1978)

(*Hint* : Consider motion of entire rope and motion of x length of rope using $P=ma$ formula and third law of motion.) $\left[\text{Ans. } T=F\left(1-\frac{x}{L}\right)\right]$

7. A shell of mass 100 kg moving with velocity 500 m/s suddenly bursts into two pieces one of which of mass 70 kg moves with a velocity of 30 m/s. What is the increase in kinetic energy by explosion ?

(**Ans.** $2{\cdot}577\times10^{7}$ joule)

8. A block of mass 3 kg on smooth inclined plane of angle 30° to the horizontal is connected by a cord over a small frictionless pulley to a second of mass 2 kg hanging vertically. What is the acceleration of each body and what is the tension of the cord ? ($g=9{\cdot}8$ ms^{-2}) (**Ans.** ·98 ms^{-2}; 1·8 kg wt.)

9. How can you lower a 100 kg body from the roof of a house using a cord with a breaking strength of 80 kgf (kilogramme force) without breaking the rope ?

(**Ans.** By allowing the weight to go down with acceleration $g/5$)

10. A machine gun fires 50 gm bullets at a speed of 1000 metre per second. The gunner, holding the machine in his hands, can exert an average force of 150 newtons against the gun. Calculate the maximum number of bullets he can fire in one minute. (**Ans.** 180)

11. A block slides down a frictionless incline at the floor of an elevator. Find acceleration of the block relative to the incline when (a) elevator descends at constant speed v, (b) elevator ascends at constant speed v, (c) elevator descends with acceleration a, (d) elevator ascends with acceleration a, (e) elevator cable breaks. The inclination of the incline with the horizontal is θ.

(*Hint* : Apply *D*' Alembert's principle.)

[**Ans.** (a) $g\sin\theta$, (b) $g\sin\theta$, (c) $(g-a)\sin\theta$, (d) $(g+a)\sin\theta$, (e) zero]

12. A railroad flat car of mass M is moving with v along a straight horizontal track. A man of mass m who was initially standing on the car now runs in the direction opposite to the car with velocity v_{rel} relative to the car. What is the change of velocity of car when the man jumps off the car ?

[*Hint* : $v_{rel}=u-(-v')=u+v'$ where u is the velocity of the man to the left and v' is the velocity of the car to the right after he jumps off.]

$\left(\text{Ans. } \frac{mv_{rel}}{M+m}\right)$

13. A box is put on the platform of a spring balance which is adjusted to zero when the box is empty. A stream of pebbles is then poured into the box from a height h above its bottom at a constant rate n pebbles per second. Find the scale reading t seconds after the pebbles begin to fill the box assuming that the collisions between pebbles and box are completely inelastic.

(**Ans.** $mngt+mn\sqrt{2gh}$)

14. A scale is adjusted to zero. Particles fall from a height 2·5 m before colliding with the balance pan of the scale; the collisions are elastic. If each

particle has a mass of 100 mg and collisions occur at the rate 1000 particles per second, what is the scale reading in kg ? (Ans. ·143 kg)

15. An electron of mass m is involved in head-on collision with an atom of mass M, initially at rest. As a result of the collision certain fixed amount of energy E is stored internally in the atom. What is the minimum initial velocity that the electron must have ?

(*Hint* : Since energy is stored in the atom, collision is essentially inelastic. $mv=mv'+Mv''$ and $\frac{1}{2}mv^2=E+\frac{1}{2}mv'^2+\frac{1}{2}Mv''^2$. Solve for v' and apply the condition for real values of v'.) $\left(\text{Ans. } \sqrt{\frac{2(m+M)E}{mM}}\right)$

16. Two masses m and M are kept slightly separated. A third 'm' coming from left with velocity v makes head-on *elastic* colision with m. Show that if $m \geqslant M$ there are exactly two collisions and find all final velocities and if $m \leqslant M$ there are three collisions and find all final velocities.

[**Ans.** The left mass comes to rest, the next mass moves to the right with a speed $v\left(\frac{m-M}{m+M}\right)$ and the third mass (M) moves to the right with a speed $\frac{2vm}{m+M}$. In the second case the left mass moves to the left with speed $\frac{v(M-m)}{M+m}$. The central mass comes to rest and the right mass moves to the right with speed $\left(\frac{2vm}{m+M}\right)$.]

17. A steel ball weighing 1 kg is fastened to a cord 1 m long and is released when the cord is horizontal. At the bottom of its path the ball strikes a 5 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find the speed of the ball and speed of the block just after collision. ($g=9{\cdot}8$ ms^{-2})

[*Hint* : Apply principle of conservation of momentum and conservation of kinetic energy as the collision is elastic.)

(**Ans.** The block of 5 kg moves forward with speed 1·472 m/s and the ball moves backward with speed 2·95 m/s)

18. An elevator (a lift) is moving up at the speed of 2 m/s in a shaft. At the instant the elevator is at 20 m from the top, a ball is dropped from the top of the shaft. The ball rebounds elastically from the elevator roof. To what height can it rise relative to the top of the shaft ? (**Ans.** 8·34 m)

19. An elevator is provided with a weighing machine (making use of principle of spring balance) at its floor. A man weighing 60 kg stands on the platform of the machine and finds that the reading shoots to 62 kg when the elevator starts and then comes down to 60 kg again. How do you explain this ?

(**Ans.** Elevator starts with acceleration ·33 ms^{-2} upward and then moves with uniform speed upward)

20. A projectile of mass 50 kg is projected vertically upward with a velocity of 100 m/s. After 5 seconds it explodes into two pieces, one piece of 20 kg moving vertically upward with velocity 150 m/s. Calculate the velocity of other piece and also algebraic sum of the momenta of the system 3 seconds after.

(**Ans.** 15 m/s downwards; 1080 kg ms^{-1})

21. A ball strikes an identical ball with velocity 50 m/s and after collision which is perfectly elastic it is found to move with velocity 30 m/s. Show that the two balls move at right angles to each other after collision.

22. A cord is tied at one end to a spring balance of weight 3 kg supporting a mass 5 kg. The cord passes ovei a smooth fixed pulley carrying a load of 10 kg at the other end. What will be the reading of spring balance when the system moves ? How will the reading vary if the load of 10 kg be replaced by a load of 6 kg ?)

[*Hint* : Consider free-body diagrams of 5 kg, 10 kg and the system (5 kg body +3 kg spring).] (**Ans.** 5·55 kg, 4·3 kg)

23. A ball moving with a speed of 9 ms^{-1} strikes an identical ball such that after collision the direction of each ball makes 30° on either side of the original line of motion. Find the speed of the two balls after collision. Is collision elastic ? (**Ans.** $3\sqrt{3}$ ms^{-1}; No)

24. A plumb bob hanging from the ceiling of railroad car acts as an accelerometer. Derive the general expression relating to the horizontal acceleration a of the car with the angle θ made by the bob with the vertical. Find θ when $a=1{\cdot}5$ ms^{-2} and a when $\theta=10°$.

[*Hint* : Apply D'Alembert's principle.] (**Ans.** $a=g\tan\theta$; 8°42′; 1·73 ms^{-2}]

25. Show that in case of elastic collisions between a particle of mass m_1 with a particle of mass m_2 initially at rest, (a) the maximum angle through' which m_1 can be deflected is given by $1-\cos^2\theta_m=\frac{m_2^2}{m_1^2}$ when $m_1>m_2$,

(b) $\theta_1+\theta_2=\frac{\pi}{2}$ when $m_1=m_2$.

26. A 6000 kg rocket is set for firing. If the exhaust speed is 1000 ms^{-1}, how much gas must be ejected per second to supply the thrust needed, (a) to overcome the weight of the rocket, (b) to give the rocket an initial acceleration of 19·6 ms^{-2} ? (**Ans.** 58·8 kgs^{-1}; 176·4 kgs^{-1})

27. A machine gun is mounted on a flat railroad car. The gun is firing bullets at the rate of 10 bullets per second each of mass 10 gm. The bullets come out with velocity 500 ms^{-1} relative to the car. Calculate the acceleration of the car at the instant when its mass is 200 kg. Also calculate the force at that instant.

(**Ans.** ·25 ms^{-2}; 50 N)

28. A train of mass M is travelling with a uniform velocity on a level line. The last carriage whose mass is m gets detached from the train. The driver of the train discovers it only after travelling a distance l and then he shuts off steam. Show that when both parts come to rest the distance between them is $\left(\frac{Ml}{M-m}\right)$. Assume resistance to motion to be uniform and proportional to mass and the pull of the engine to be a constant.

29. A cannon and a supply cannon balls are inside a sealed railroad car. The cannon fires to the right, the car recoiling to the left. The cannon balls remain in

the car after hitting the far end. Show that no matter how the cannon balls are fired, the railroad car cannot travel more than its own length L, assuming it to start from rest.

[*Hint* : Apply the principle of conservation of momentum.]

30. A jet air plane is travelling at 200 ms^{-1}. The engine of the plane takes in 250 m^3 of air having mass 32 kg each each second. The air is used to burn 3 kg of fuel each second. The energy is used to compress the products of combustion to eject them at the rear of the plane at 500 ms^{-1} relative to the plane. Find the power of the jet plane in H. P. (1 H. P. = 746 watt).

[*Hint* : Resultant thrust = forward thrust − backward thrust and power = force × velocity] (**Ans.** 2975 H.P.)

31. Sand drops from a stationary hopper at the rate of 5 kg s^{-1} on to a conveyor belt moving with a constant speed of 2 ms^{-1}. What is the force required to keep the belt moving and what is the power delivered by the motor moving the belt ?

[*Hint* : Tangential force exerted on the belt $= u_{rel}\,\dfrac{dM}{dt}$ where u_{rel} is the tangential relative velocity of the sand.] (**Ans.** 10 newton; 20 watt)

32. A right angled triangular block of mass M rests on a horizontal table. The inclination of the block is 30° with the table. A cubical block of mass m is placed on the incline of the block. What horizontal force must be applied on M so that m may remain stationary relative to the block ?

[*Hint* : Apply D'Alembert's principle and consider free-body diagrams of m and the system $(m+M)$.] $\left(\textbf{Ans. } (M+m)\dfrac{g}{\sqrt{3}}\right)$

33. A rocket has initial mass of 100 kg, 90% of which is fuel. This fuel is turned into a gas and ejected from the rocket at the rate of 1 kg/sec. The velocity of fuel ejection is 500 ms^{-1} relative to the rocket. Neglecting the force of gravity on the rocket, find the acceleration of the rocket immediately after all the fuel is burnt.

[*Hint* : $F = u_{rel}\,\dfrac{dM}{dt} = Ma$ or $Ma = 500 \times 1 =$ a constant.] (**Ans.** 50 ms^{-2})

34. A wedge of mass M rests with one face in contact with a horizontal smooth surface. A particle of mass m is placed on the smooth face of it, inclined at an angle α to the horizontal and released. Find the velocity of the wedge and that of the particle relative to the wedge when the particle has slipped a distance x down the face of the wedge. Also calculate the acceleration of the wedge and that of the particle relative to the wedge.

[*Hint* : Apply principle of conservation of momentum along the horizontal and energy conservation principle. See Ex. 8]

$$\left(\textbf{Ans. } \sqrt{\frac{2m^2 gx \sin\alpha\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}},\ \sqrt{\frac{2\,(M+m)\,gx \sin\alpha}{(M+m\sin^2\alpha)}},\ \frac{(M+m)g\sin\alpha}{M+m\sin^2\alpha},\ \frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}\right)$$

35. Two identical bogies 1 and 2 with one man in each move without friction with constant velocities v_1 and v_2 along the parallel rails toward each other. When the bogies get opposite each other, the men exchange their positions by jumping in the direction perpendicular to the rails. As a consequence, bogie 1 stops and bogie 2 keeps on moving in the same direction with velocity v. Find the initial velocities of the bogies if the mass of each of them be M (without man) and the mass of each man be m.

[*Hint* : Consider import and export of tangential momentum to the bogies and apply momentum conservation principle to the system.]

$$\left(\textbf{Ans. } v_1=\frac{mv}{M-m},\ v_2=\frac{Mv}{M-m}\right)$$

36. Two identical bogies move one after the other without friction with constant velocity v_0. A man of mass m rides the rear bogie. At a certain moment the man jumps into the front bogie 1 with velocity u relative to his bogie numbered 2. If the mass of each bogie (without man) is equal to M, find the velocities with which the bogies will move after that.

[*Hint* : $u=u'-v'$ where u' is the actual velocity of the man and v' is the actual velocity of the bogie after the man jumps off.]

$$\left(\textbf{Ans. } v_1=v_0+\frac{mu}{M+m},\ v_2=v_0-\frac{mMu}{(M+m)^2}\right)$$

37. Assuming that the car in problem 12 is initially at rest and holds n men, each of mass m, prove that if the men run with relative velocity v_{rel} and jump in succession they can impart to the car a greater velocity than if they all run and jump simultaneously.

[*Hint* : When the men run together, the effect is the same as for one man of mass nm.

$$\therefore \qquad \Delta v=\frac{nm}{M+nm}v_{rel}.$$

When they run and jump in succession,

$$\Delta v=\sum_{i=1}^{i=n}\frac{mv_{rel}}{M+im}=mv_{rel}\left(\frac{1}{M+m}+\frac{1}{M+2m}+\frac{1}{M+3m}+\cdots\cdots+\frac{1}{M+nm}\right)\Bigg]$$

38. Five men lined up at one end of a floating raft, initially at rest, run in succession with velocity $v_{rel}=3\ \text{ms}^{-1}$ relative to the raft and dive off at the other far end. Neglecting the resistance of water to the horizontal motion of the raft, find its velocity after the last man dives. Each man weighs 77 kg and raft weighs 453 kg.

(**Ans.** $1{\cdot}733\ \text{ms}^{-1}$)

39. Two masses P and Q are connected by the arrangement as shown in Fig. 5-5. Find the downward acceleration of the mass Q.

[*Hint* : Considering the fact that the total length of the string is the same, it is found that when Q goes down by x, P goes up by $x/2$. Hence the acceleration of Q is double the acceleration of P. Now consider free-body diagrams of P and Q.]

$$\left(\text{Ans. } 2g\cdot\frac{2Q-P}{4Q+P}\right)$$

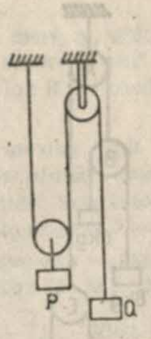


Fig. 5-5

40. Neglecting friction and the mass of the pulleys, find the acceleration of the mass Q assuming that $P=Q$ (Fig. 5-6).

[*Hint* : When Q goes down by x, P goes up by $x/2$. Hence acceleration of P is half the acceleration of Q. Consider free-body diagrams of P, Q and the movable pulley and apply Newton's second law to their motions.]

$$\left(\text{Ans. } \frac{2g}{5}\right)$$

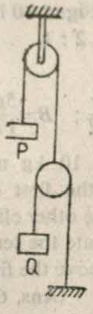


Fig. 5-6

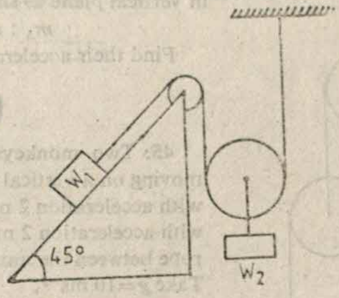


Fig. 5-7

41. Find the tension in the string during motion of the system if $W_1=200$ kg, $W_2=100$ kg, μ (between W_1 and plane)$=\cdot2$ and assume the pulleys to be without mass. (Fig. 5-7) (Ans. 63 3 kgf)

42. Calculate the accelerations of the pulleys B and C and the tension in the string passing over the pulley A of the figure Fig. 5-8.

$$\left(\text{Ans. } \frac{g}{11};\ 6\cdot23\text{ kgf}\right)$$

43. Calculate the acceleration of the block B of the figure Fig. 5-9, assuming surfaces and pulleys to be smooth. $\left(\text{Ans. } \frac{3F}{17m}\right)$

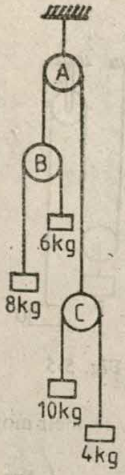


Fig. 5-8

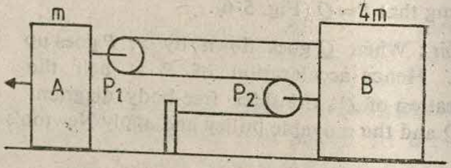


Fig. 5-9

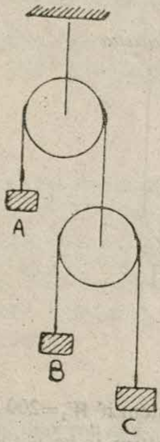


Fig. 5-10

44. A system of masses and pulleys is arranged in vertical plane as shown in the Fig. 5-10 here.

$$m_A : m_B : m_C = 3 : 2 : 1.$$

Find their accelerations.

$$\left(\textbf{Ans. } A=\frac{g}{17};\ B=\frac{5g}{17};\ C=\frac{7g}{17}\right)$$

45. Two monkeys of masses 10 kg and 8 kg are moving on a vertical light rope, the first climbing up with acceleration 2 ms^{-2} while the other climbing down with acceleration 2 ms^{-2}. Calculate the tension of the rope between the monkeys and above the first monkey. Take $g=10\ ms^{-2}$.

(**Ans.** 64 N; 184 N)

46. A rope is stretched between two stationary boats on the surface of a lake. A man in the first boat pulls the rope with a constant force of 100 N. The distance between the boats is 100 m in the beginning. The mass of the first boat with the man is 250 kg, while the mass of the other is 500 kg. Find : (i) the velocity of approach of the two boats 15 s after the start, (ii) the time taken for the two boats to meet each other.

[*Hint* : Consider relative acceleration and apply kinematic equations.]

(**Ans.** 3 ms^{-1}; 18·3 s)

47. A thin uniform chain is hanging vertically and its bottom end is touching a table. The mass of the chain is m and its length is l. At the moment $t=0$, the chain is released. Calculate the force that the chain exerts on the table after t seconds.

(**Ans.** $\frac{3}{2} \cdot \frac{m}{l} g^2t^2$)

Inelastic Collisions.

48. A sphere of mass m_1 collides with a sphere of mass m_2 which is at rest. Prove that whatever be the initial direction of the first sphere, it will move off at right angles to the line of centres at the time of collision if the coefficient of restitution is m_1/m_2.

49. Two equal smooth spheres of radius a are moving with equal speeds u in opposite direction along parallel lines whose distance apart is a. The coefficient of restitution between them is $\frac{1}{3}$. Find their velocities after impact. (**Ans.** $u\sqrt{3}$, perpendicular to original directions)

50. A simple pendulum of length L is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude becomes less than 60° ?
[IIT 1987] (**Ans.** 3)

51. A small metal ball falls vertically and strikes a smooth plane inclined at an angle θ to the horizontal ($\theta < 45°$). The ball rebounds horizontally. Show that a fraction ($1-e$ of the kinetic energy is lost during the impact and that if m is the mass and u is the speed of the ball before impact, the impulse on the plane is $mu \sec\theta$. Here e is the coefficient of restituiton.

□□□

CHAPTER 6

MOMENT OF A FORCE : TORQUE : CENTRE OF MASS AND CENTRE OF GRAVITY : EQUILIBRIUM OF RIGID BODIES : MACHINES

(a) The Moment of a Force—*The turning effect of a force on a body* about a point is called its moment and the amount of moment is given by

Moment = force × perpendicular distance of the point from the line of action of the force called the arm of the force $= (F \times d)$ Nm.

The moment of a force in an axial vector. It is along the direction of translation of a screw perpendicular to the plane containing the force and the point and rotating from the position vector $\vec{r}$ to the force $\vec{F}$ through the smaller angle. That is $\vec{\tau}$ (moment of a force) $= \vec{r} \times \vec{F}$ where $\vec{r}$ is the position vector of the point with respect to any arbitrary frame of reference.

(b) Torque or Couple—*A pair of two equal unlike parallel forces is called Couple or Torque.*

Torque (τ) = force × perpendicular distance between forces called arm of couple or torque $= F \times d$ Nm.

$$\tau = F.d \text{ Nm.}$$

(c) Centre of mass—The centre of mass of a body is that point in it *where its whole mass may be supposed to be concentrated and the body may be considered as a particle so far the action of a system of parallel forces acting on the elementary particles in proportion to the mass of the particles is concerned.*

If M is the mass of the body and $m_1, m_2, m_3 \ldots$ are the elementary masses at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots$ then distance of centre of mass is given by

$$M \vec{r} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \ldots$$

The x and y co-ordinates of the centre of mass of a system of particles is given by

$$\bar{x}=\frac{m_1x_1+m_2x_2+m_3x_3+\ldots}{m_1+m_2+m_3+\ldots}=\frac{\Sigma m_ix_i}{\Sigma m_i}$$

where m_i is the mass of ith particle which is at a distance x_i from the y-axis and that of a body

or $$\bar{x}=\frac{\int dm\, x}{\int dm}$$

and $$\bar{y}=\frac{\int dm\, y}{\int dm} \text{ or } \frac{\Sigma m_iy_i}{\Sigma m_i}.$$

Motion of centre of mass—The centre of mass is given by the formula, (vectorially)

$$M\vec{r}=m_1\vec{r}_1+m_2\vec{r}_2+\ldots$$

$$\therefore\quad M\frac{d\vec{r}}{dt}=m_1\frac{d\vec{r}_1}{dt}+m_2\frac{d\vec{r}_2}{dt}+\ldots$$

or $M\vec{v}=m_1\vec{v}_1+m_2\vec{v}_2+\ldots$ where $\vec{v}$ is the velocity of centre of mass and $\vec{v}_1, \vec{v}_2\ldots$ are velocities of elementary particles of the body.

Again, $$M\frac{d\vec{v}}{dt}=m_1\frac{d\vec{v}_1}{dt}+m_2\frac{d\vec{v}_2}{dt}+\ldots$$

or $M\vec{a}=m_1\vec{a}_1+m_2\vec{a}_2+\ldots$ where $\vec{a}$ is the acceleration of centre of mass and $\vec{a}_1, \vec{a}_2, \vec{a}_3,\ldots$ acceleration of the elementary particles of the body

or $\vec{F}=\vec{f}_1+\vec{f}_2+\vec{f}_3+\ldots$ where $\vec{F}$ = force effective on the body at the centre of mass and $f_1, f_2, f_3\ldots$ are the forces on the elementary masses. These forces on the elementary particles include both external and internal forces.

$$\therefore\quad M\vec{a}=\vec{F}_{external}+\vec{F}_{internal}.$$

But $\vec{F}_{internal}=0$ because action and reaction are equal and opposite. $\therefore$ $M\vec{a}=\vec{F}_{external}$. This equation is the equation of motion of the centre of mass. Further if no external force acts on the body then $M\vec{a}=0$ or $\vec{a}=0$. This means change of velocity $=0$. *If the body is initially at rest, then centre of mass must remain stationary.*

(d) Centre of gravity—Centre of gravity is that point where *the whole mass may be supposed to be concentrated so far the gravity* (i.e. force of attraction due to the earth) *on the particles is concerned.* For a small body the centre of mass and centre of gravity are identical points.

(e) Equilibrium of rigid bodies—A body is said to be in equilibrium when it neither moves in a line nor does it rotate. Condition for equilibrium,

$$\Sigma \vec{f}=0 \quad \text{and} \quad \Sigma \vec{\tau}=0.$$

i.e. (i) Sum of resolved parts of all forces in any direction must be zero. (ii) The algebraic sum of moments of all forces about any point must be zero.

Stability of equilibrium—Equilibrium is stable when the system returns to its original position after being slightly displaced. Condition for *stability* is that *the vertical line through the centre of gravity must pass through the base of contact of the body or system of bodies.* More fundamental condition is that *the potential energy of the body or system of bodies must be a minimum.*

(f) Machines—Machine is a device by which force applied at some point of it (called the load) is overcome by means of another force applied at some other point of it with alteration in direction or magnitude or both (called the effort).

$$\text{Mechanical advantage of a machine}=\frac{W\text{ (load)}}{F\text{ (effort)}}.$$

$$\text{Efficiency of a machine}=\frac{\text{Useful work}}{\text{Total work}}$$

$$\text{i.e., } \frac{\text{Work done on load (out-put)}}{\text{Work done by effort (in-put)}}.$$

$$\text{Velocity ratio}=\frac{\text{Velocity of effort}}{\text{Velocity of load}}$$

$$\text{or} \quad \frac{\text{Displacement of effort in any time}}{\text{Displacement of load in same time}}.$$

In any machine,

Mechanical advantage = Efficiency × Velocity ratio.

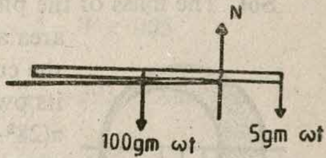
Lever, Axle and Wheel, Capstan, Jack, etc. are machines working on the fact that "moment of load about any point or line = moment of effort about that point or line". A jack is a 'nut and screw' system. The load is placed on the top of the screw. The screw is turned by a long rod. The effort is applied at the far end of the rod. When the rod is turned through one complete rotation, the load goes up a distance equal to the pitch of the screw. A capstan is a heavy barrel capable of rotating about a fixed vertical axis. The load works tangential to the barrel and the effort is applied at the far end of a rod by which the barrel is turned.

EXAMPLES

Ex. 1. *A metre ruler, weighing* 100 *gm rests on a table with a part projecting over the edge. Find the length of the part projecting out if a* 5 *gm body hung at the end just tilts the ruler.*

(Pat. 1970 A)

Sol. There are three forces acting on the ruler, (i) its weight ·1 g downward at its cg, (ii) normal reaction (N) of the table, (iii) ·005 g at the extreme end. When the ruler is about to turn, the normal reaction is only at the edge.



Taking moment about the edge of the table we have for rotational equilibrium of the ruler when it is about to turn

$$\cdot 005\, g \times x \searrow + \cdot 1\, g\, (\cdot 5 - x) \nwarrow = 0$$

or $$\cdot 005 g\, x - \cdot 1 g\, (\cdot 5 - x) = 0 \text{ or } x = \cdot 476 \text{ m. } \textbf{Ans.}$$

Ex. 2. *Find the centre of mass of a homogeneous semicircular plate of radius a.*

Sol. Take the straight edge of the plate as x-axis and the centre O as origin. The symmetry of the figure about y-axis requires that cg be somewhere on the y-axis. Thus x_{cm} (distance of cg along x-axis) = 0.

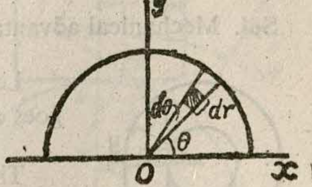


Fig. 6.1

Consider a point of polar coordinates (r, θ). An elementary area about the point is $r d\theta\, dr$ and the elementary mass at the point is $r d\theta\, dr \rho$ where ρ is the mass per unit area.

The height of the centre of gravity of the block above the datum plane through the centre of the sphere in the displaced position

$$= r\cos\Delta\psi + r\Delta\psi\sin\Delta\psi + h\cos\Delta\psi.$$

In the equilibrium position the height of the cg above the same datum plane $= r + h$.

For stability of equilibrium we have

$$r\cos\Delta\psi + r\Delta\psi\sin\Delta\psi + h\cos\Delta\psi > r + h.$$

Since $\Delta\psi$ is small, $\sin\Delta\psi = \Delta\psi$

$$\text{and } \cos\Delta\psi = 1 - \frac{\Delta\psi^2}{2};$$

$$\therefore\quad r\left(1 - \frac{\Delta\psi^2}{2}\right) + r\Delta\psi . \Delta\psi + h\left(1 - \frac{\Delta\psi^2}{2}\right) \geqslant r + h$$

$$\text{or}\quad r - \tfrac{1}{2} r\Delta\psi^2 + r\Delta\psi^2 + h - \frac{h\Delta\psi^2}{2} \geqslant r + h$$

$$r \geqslant h.$$

EXERCISES

1. Find the centre of gravity of a thin wire bent to semicircle of radius r.

(**Ans.** at a distance $\frac{2r}{\pi}$ from the centre)

2. Two like parallel forces P and Q act at the ends of a straight ruler of length l. What is the distance by which the resultant of the forces shift along the ruler if P is doubled ?

$\left(\textbf{Ans.}\quad \frac{PQl}{(P+Q)(2P+Q)}\right)$

3. A circular section of radius r is cut out of a uniform disc of radius R, the centre of the hole being $\frac{R}{2}$ from the centre of the original disc. Locate the centre of gravity of the remainder.

(**Ans** at a distance $\frac{Rr^2}{2(R^2-r^2)}$ from the centre of the disc)

4. A cylindrical vessel of radius 1 m and height 2 m is half filled with water. Find the centre of mass of the system consisting of cylinder and water. The cylinder has mass 10 kg per square metre of it and density of water 1000 kg m^{-3}.

(**Ans.** ·514 m above the base)

5. A flexible chain of weight W hangs between two fixed points A and B at the same level. The inclination of the chain with the horizontal at the two points

of support is θ. Calculate the tension of the chain at end point and also at the lowest point.

[*Hint* : Consider equilibrium of each half of the chain.]

$$\left(\textbf{Ans.}\ \text{(i)}\ \frac{W}{2\sin\theta},\ \text{(ii)}\ \frac{W}{2}\cot\theta\right)$$

6. A thin horizontal bar *AB* of negligible mass and length l is hinged to a vertical wall at *A* and supported at *B* by thin wire *BC* which is tide to a point *C* vertically above in the same wall. *BC* makes angle θ with the horizontal. A weight *W* is moved along the bar. Show how the tensile force *T* in the thin wire changes with the distance x of the weight from the wall.

$$\left(\textbf{Ans.}\ T=\frac{Wx}{l\sin\theta}\right)$$

7. Two particles *P* and *Q* are initially at rest 1 metre apart. *P* has a mass of 1 kg and *Q* a mass of 3 kg. *P* and *Q* attract each other with a constant force. At what distance from *P*'s original position do the particles collide ?

(**Ans.** ·75 m)

8. A table has a heavy circular top of radius 1 m and mass 20 kg. It has four light legs of length 1 m fixed symmetrically on its circumference. What is the maximum weight which may be placed anywhere on this table without toppling ?

(**Ans.** 48·3 kg)

9. A man weighing 60 kg lifts a weight of 40 kg to the roof of a building which is 9 m high. Calculate the useful work done by him. Considering the man as a machine calculate his efficiency. ($g=9{\cdot}8\ \text{ms}^{-2}$)

(**Ans.** 3528 joule; 40%)

10. What is the mechanical advantage of an inclined plane as a machine when inclination of the plane is 30° with the horizontal, if the force acts (i) horizontally, (ii) along the plane ? What force along the plane is needed to raise a scooter of weight 150 kg ? (**Ans.** $\sqrt{3}$; 2; 75 kg)

11. The fulcrum of a lever of length 1 m is at its middle point. If the fulcrum is moved by ·25 m towards the load, how are efficiency and velocity ratio affected ? (**Ans.** efficiency remains same, velocity ratio is increased 3 times)

12. In a wheel and axle machine the radii of the wheel and axle are ·5 m and ·05 m respectively. A weight of 100 kg is raised when it is hanging by a rope from the axle by applying a force of 12 kg on the circumference of the wheel. Find the mechanical advantage, velocity ratio and efficiency of this machine.

(**Ans.** 8·3; 10; 83·3%)

13. A jack of pitch 4 mm is worked by a handle of length 50 cm. There is friction between the screw and the nut of it. To overcome the frictional force an extra effort by ·004 times the load has to be applied. Calculate the least force required to raise and lower a load of 500 kg. (Take $g=10\ \text{ms}^{-2}$)

(**Ans.** 26·36 newton; 13·6 newton)

14. A pulley fixed to the ceiling carries a thread with bodies of masses m_1 and m_2 attached at its ends. The masses of the pulley and thread are negligible, friction is absent. Find the acceleration of the centre of mass (a_{cm}) of this system.

$$\left[\textbf{Ans.}\ a_{cm}=\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 g\right]$$

15. A particle of mass m is placed upon a smooth face, inclined at an angle α to the horizontal, of a prism, also of mass m, which is resting upon a smooth horizontal table. Show that when the particle has moved a distance x down the face of the prism, the prism itself will have moved a distance of $\frac{1}{2} x \cos\alpha$.

16. Suppose you have to pile up uniform bricks to erect a leaning wall by giving offset to each brick by a constant fraction, $\frac{1}{n}$, of a brick length l above the one beneath. How many bricks can be piled up before the wall will collapse? How high wall can you erect if the size of each brick is 7·5 cm × 12·5 cm × 25 cm and offset is only $\frac{1}{10}$ of the length of each brick? What is the total offset you get?

[*Hint*: The vertical line through C. G. of the wall must pass through the base. The moment it will go beyond the base brick, the wall will collapse.]

(**Ans.** $(n+1)$ bricks including the lowest one; 82·5 cm high wall can be erected: offset = 25 cm)

17. What is the maximum offset that one can obtain by piling up for identical bricks each of length l?

[*Hint*: C. G. of any two from top must pass through the edge of the third beneath. The uppermost brick may overhang by $\frac{l}{2}$.] $\left(\textbf{Ans. } \frac{11}{12} l\right)$

18. A man weighing 60 kg is standing at the centre of flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weighs 200 kg. How far is he from the shore at the end of this time?

[*Hint*: As the system is free from external force, centre of mass of the system consisting of man and boat must not move.]

(**Ans.** 13·8 m)

19. A block of mass M with a semi-circular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest from the top point A (Fig. 6-5) The cylinder slips on the semi-circular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track? How fast is the block moving when the cylinder reaches the bottom of the track?

(**I. I. T.** 1983)

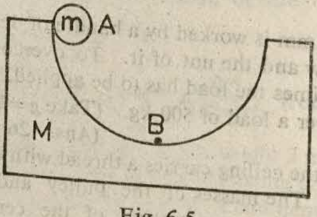


Fig. 6-5

$\left(\textbf{Ans. } \frac{m(R-r)}{M+m} \text{ to the left; } V = \sqrt{\frac{2m^2 g(R-r)}{M(M+m)}}\right)$

20. A 50 kg plank of length 6 m rests on the ground at one end and on frictionless roller on the top of a wall of height $h=3$ m. The plank projects a little beyond the wall. The plank remains in equilibrium for any value $\theta \geq 70°$, where θ is the angle made by the plank with the horizontal. Find the coefficient of friction between the plank and the ground.

[*Hint* : Apply the equilibrium condition that $\Sigma \vec{\tau}=0$ and $\Sigma \vec{f}=0$.] (**Ans.** ·33)

21. In the step-ladder (show in the Fig. 6-6) 8 m long and hinged at *C*, *BD* is a tie rod 2·5 m long half way up. A man of 86 kg climbs 6 m along the ladder. Find the tension in the tie rod and the forces exerted on the ladder by the floor assuming the floor frictionless and ladder weightless.

[*Hint* : Consider the 'free body' diagram of the sides of the ladder.]

(**Ans.** $R_A=54\cdot0$ kgf, $R_E=32\cdot2$ kgf, $T=21\cdot2$ kg)

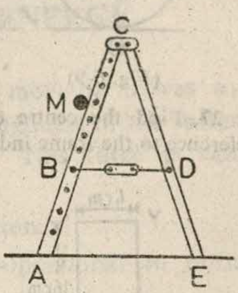


Fig. 6.6

22. A pulley arrangement is shown in the Fig. 6.7. *A* has a diameter of 30 cm and the cylinder *B* has diameter of 20 cm. The working handle has an arm of 50 cm. The direction of winding of the rope is opposite to that of *B*. Calculate the mechanical advantage of the machine.

[*Hint* : To find the displacement of the load when the handle is rotated through one full rotation, consider the fact that the length of the rope remains same.]

(**Ans.** 20)

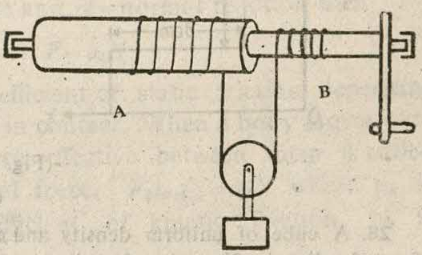


Fig. 6-7

23. Show that a non-homogeneous cylinder on a horizontal plane has a position of stable equilibrium when its centre of gravity is vertically below the axis *O* of the cylinder.

24. A dumb-bell consists of two weights W_1 and W_2 ($W_2>W_1$) and is hinged at point distant l_1 and l_2 from W_1 and W_2 respectively. Establish the criterion of stable equilibrium for the vertical position of the dumb-bell.

(**Ans.** $W_2 l_2 > W_1 l_1$)

25. A hemispherical cup of radius r and having its centre of gravity at C distant c from its centre rests on the top of a spherical surface of radius R as shown in the Fig. 6-8. Assuming that there is sufficient friction to prevent slipping, establish the criterion of stability of the cup in the position shown.

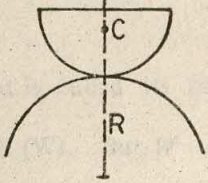
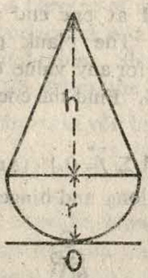


Fig. 6-8

$$\left[\textbf{Ans. } R > r\left(\frac{r}{c}-1\right)\right]$$

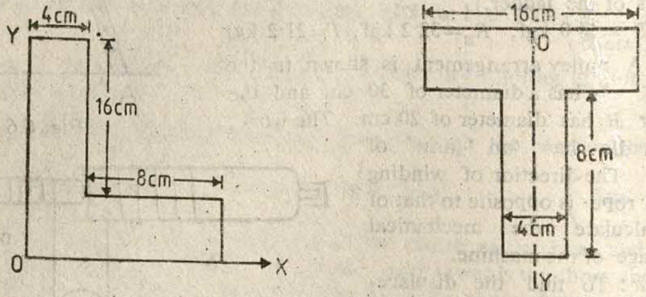


(Fig. 6-9)

26. A solid of uniform density is made by of a hemisphere and a right circular cone which have a common base of radius r. Determine the largest value of h of the cone consisten with stability of the body in the vertical position.

(**Ans.** $h_{max}=\sqrt{3}\,r$)

27. Find the centre of mass of the following L and T shaped planks with reference to the frame indicated in the diagram (Fig. 6-10).



(Fig. 6-10)

(**Ans** : 3·7 cm ; 7·7 cm and 0 and 4 cm)

28. A cube of uniform density and edge a is balanced on a cylindrical surface of radius r. Show that the criterion for stability of equilibrium of the cube, assuming the friction is sufficient to prevent slipping, is $r>a/2$.

29. Assuming that a car whose cg is at a distance b from the rear-wheel and at a distance c from the front wheel and at a height h above the pavement, has sufficient power and there is sufficient friction, find the maximum acceleration that it would be able to develop without tipping over backward. $\left(\textbf{Ans. } \frac{h}{h}\right)$

□□□

CHAPTER 7

FRICTION, WORK, POWER AND ENERGY : WORK-ENERGY THEOREM : PRINCIPLE OF CONSERVATION OF ENERGY

1. Friction—(a) Whenever a body tends to move or moves over another body, there arises a force due to roughness of the surfaces in contact which tends to oppose the motion. This force is called frictional force.

Laws of friction—(i) Friction opposes motion.

(ii) The force of limiting friction is proportional to normal reaction.

If F = limiting frictional force and N = normal reaction then

$$F \propto N \text{ or } F = \mu_s N$$

where μ_s is a constant called coefficient of static friction depending on the nature of the surfaces in contact. When a body moves over another body the frictional force effective between them is called dynamic (or kinetic) frictional force. $F_{kinetic} = \mu_k N$ where μ_k is another constant called coefficient of kinetic friction. μ_k is always less then μ_s.

2. Work, power and energy—(i) **Work** is said to be done when there is displacement of the point of application and the amount of work done is,

Work done = $Fs \cos\theta$ joule (J) where F = force, s = displacement and θ = angle between force and displacement.

$$\text{or } W = \vec{F} \cdot \vec{s}.$$

(ii) **Power**—Rate of doing work by an agent is called its power.

Power $= \frac{W}{t}$ joule per second (Js^{-1}) or watt (W). But $W = F \cdot s$.

$$\therefore \quad P = \frac{Fs}{t} = F \times v \text{ watt } \left(\because \quad v = \frac{s}{t}\right).$$

(iii) **Energy**—Capacity of doing work is called energy. In mechanics capacity of doing work may be due to motion, position (e.g. water at high level) and configuration (e.g. a compressed spring).

Energy due to motion is called kinetic energy and energy due to position or configuration is called potential energy.

The gravitational potential energy of a body at a height $h = (U_0 + mgh)$ *joule* (J) where U_0 is the gravitational energy of the body in the datum (reference) plane.

The potential energy of a stretched or compressed spring $= \frac{1}{2}kx^2$ where k is the 'force constant' of the spring and x = elongation or compression of the spring.

The kinetic energy of a body of mass 'm' and velocity $v = \frac{1}{2}mv^2$.

Conservative and non-conservative forces.

If the work done by a force in moving a body from one point to another depends only on the position of the points and *not on the path*, the force is said to be conservative. The gravitational force, the spring force, the electric and magnetic forces are examples of conservative force. If the work done depends on the path, it is said to be non-conservative force. Frictional force is the only non-conservative force with which we have to deal.

3. Work-energy theorem : The work done *by* the external forces acting *on a body* is *always equal to the change in kinetic energy of the body.*

4. Principle of conservation of energy : Energy can be transferred from one kind to another, but it cannot be created nor destroyed; *the total energy is constant.*

EXAMPLES

Ex. 1. *A piece of ice slides down a plane inclined at 45° to the horizontal in a time which is twice the time it takes to slide down the same plane when it is frictionless. Calculate the coefficient of friction between ice and the inclined plane.* (Bih. Univ. 1967)

Sol. Considering free body diagram of the piece of ice and its motion along and perpendicular to the plane,

$$mg \sin \theta - f_{friction} = ma$$

where a is the acceleration down the plane:

$$mg \cos \theta = N \text{ (normal reaction).}$$

But $\quad f_{friction} = \mu\, N = \mu\, mg \cos\theta$

$\therefore \quad mg \sin\theta - \mu\, mg \cos\theta = ma$

or $\quad a = g \sin\theta - \mu g \cos\theta = g \left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right) = \frac{g}{\sqrt{2}}\left(1 - \mu\right).$

When the plane is frictionless, $\mu = 0$; then

$$a = g \sin\theta = g \sin 45^\circ = \frac{g}{\sqrt{2}}.$$

Let t_1 be the time taken by the ice piece to slide down the frictional plane and t_2 down the frictionless plane.

Then s (length of the plane) $= \frac{1}{2}.\ \frac{g}{\sqrt{2}}(1-\mu).t_1^2$

$$\text{and } s = \frac{1}{2}.\ \frac{g}{\sqrt{2}} .\ t_2^2$$

$$\therefore \quad \frac{1}{2}.\ \frac{g}{\sqrt{2}}(1-\mu)t_1^2 = \frac{1}{2}.\ \frac{g}{\sqrt{2}}\ t_2^2.$$

$$\begin{aligned} &\text{But} && t_1 = 2t_2 \\ &\therefore && (1-\mu)\, 4t_2^2 = t_2^2 \\ &\text{or} && 1 - \mu = \tfrac{1}{4} \\ &\text{or} && \mu = 1 - \tfrac{1}{4} = \tfrac{3}{4} = \cdot 75. \quad \textbf{Ans.} \end{aligned}$$

Ex. 2. *A body of mass 5 kg is being pulled up along a rough inclined plane with an acceleration 5 cm s^{-2} by means of a weightless string running parallel to the plane. If the inclination of the plane be 45° to the horizontal and the coefficient of friction between the body and the plane be ·3, find the pull in the string.* $(g = 9{\cdot}8 \text{ ms}^{-2})$

Sol. Considering the free-body diagram of the body and its motion along and perpendicular to the plane,

$$P - \mu\, N - mg \sin 45^\circ = m \times \cdot 05$$

$$\text{and } N = mg \cos 45^\circ$$

$$\begin{aligned} \therefore\ P &= \mu\, N + mg \sin 45^\circ + \cdot 05\, m \\ &= \cdot 3 \times mg \cos 45^\circ + mg \sin 45^\circ + \cdot 05 m \\ &= \left(\cdot 3 \times 5 \times 9{\cdot}8 \times \frac{1}{\sqrt{2}} + \frac{5 \times 9{\cdot}8}{\sqrt{2}} + \cdot 05 \times 5\right) \\ &= 10{\cdot}4 + 34{\cdot}6 + \cdot 25 \end{aligned}$$

or $\quad P = 45{\cdot}25$ newtons. **Ans.**

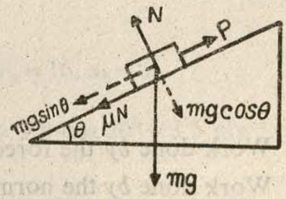


Fig. 7-1

Ex. 3. *A* 10 *kg body is pushed up a frictionless* 45° *inclined plane* 4 *m long by a horizontal force.* (*a*) *If the speed at the bottom is* 4 ms^{-1} *and at the top* 8 ms^{-1}, *how much work is done by the force ?* (*b*) *Suppose the plane is not frictionless and that* $\mu_k = \cdot 15$. *What work will the same force do ? How far up the plane does the body go ?*

Sol. The three real forces acting on the body are (i) the applied force *F*, (ii) the gravitational pull *mg*, and (iii) the normal reaction *N* of the plane.

Work done *by* the gravitational pull $= 10 \times 9{\cdot}8 \times (-4 \sin 45°)$

$$= -\frac{98 \times 4}{\sqrt{2}} = -277 \text{ joules}$$

Work done *by* the normal reaction $=0$.

Let *W* be the work done *by* the force. Then by the 'work-energy' theorem,

$$-277 + W = \tfrac{1}{2} \times 10 \times 8^2 - \tfrac{1}{2} \times 10 \times 4^2$$

or $\quad W = 277 + 320 - 80 = 517$ joules. **Ans.**

Also work done by the force $= F \times 4 \cos 45°$

$$= \frac{F \times 4}{\sqrt{2}} = 2\sqrt{2}\, F$$

$$\therefore \quad 2\sqrt{2}\, F = 517 \quad \text{or} \quad F = 182{\cdot}8 \text{ newtons.}$$

If the plane is not frictionless then there are four real forces acting on the body. Let it go up by *S*. Then

Work done *by* the gravitational pull $= 10\, g \times (-S \sin 45°)$

$$= -\frac{10\, gS}{\sqrt{2}} = -69{\cdot}3S.$$

Work done by the frictional force $= F_{friction} \times (-S)$

$$= -\mu\, NS = -\mu\, mg \cos\theta S$$

$$= -{\cdot}15 \times 10 \times 9{\cdot}8 \times \frac{1}{\sqrt{2}} S$$

$$= -10{\cdot}4\, S.$$

Work done *by* the force $= 182{\cdot}8 \times S \cos 45° = 129{\cdot}3S.$

Work done *by* the normal reaction $=0$.

By the 'work-energy' theorem,

$$-69{\cdot}3\, S - 10{\cdot}4\, S + 129{\cdot}3\, S = \tfrac{1}{2} \times 10 \times 4^2$$

or $\quad 49{\cdot}6\, S = 80 \quad$ or $\quad S = 1{\cdot}6$ m

$\therefore$ Work done by the force $= 129{\cdot}3 \times 1{\cdot}6 = 207$ joule. **Ans.**

Ex. 4. *A 8 kg block and a 16 kg block connected together by a string slide down a 30° inclined plane. The coefficient of friction between 8 kg block and the plane is ·1 and that between 16 kg block and the plane is ·2. Find (a) acceleration of the block, (b) the tension in the string, assuming that the 8 kg block leads, (c) describe the motion if the blocks are reversed.*

Sol. Let us first consider the problem in a general way. The real forces acting on the bodies are shown in their free-body diagrams (Fig. 7-2).

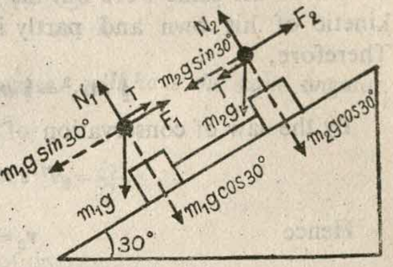


Fig. 7-2

Considering motion of the first body along and perpendicular to the plane,

$$m_1g\sin\theta - T - F_1 = m_1a$$

where F_1 is the frictional force;

$$N_1 - m_1g\cos\theta = 0.$$

$$\therefore\quad m_1g\sin\theta - T - \mu_1m_1g\cos\theta = m_1a \qquad \text{.. (i).}$$

Considering motion of the second body along and perpendicular to the plane,

$$m_2g\sin\theta + T - F_2 = m_2a$$

and $$N_2 - m_2g\cos\theta = 0$$

$$\therefore\quad m_2g\sin\theta + T - \mu_2\, m_2g\cos\theta = m_2a \qquad \text{... (ii).}$$

Adding (i) and (ii), we have

$$(m_1+m_2)\, g\sin\theta - (\mu_1\, m_1 + \mu_2\, m_2)\, g\cos\theta = (m_1+m_2)\, a$$

or $$a = g\sin\theta - \left(\frac{\mu_1\, m_1 + \mu_2\, m_2}{m_1+m_2}\right) g\cos\theta$$

and $$T = \frac{m_1\, m_2\, (\mu_2 - \mu_1)}{m_1+m_2}\, g\cos\theta.$$

In the first case $\mu_1 = \cdot 1$, $m_1 = 8$ and $m_2 = 16$, $\mu_2 = \cdot 2$.

$$\therefore\quad a = 9{\cdot}8\sin 30° - \left(\frac{\cdot 1\times 8 + \cdot 2\times 16}{6+16}\right)\times 9{\cdot}8\cos 30° = 3{\cdot}5\ \text{ms}^{-2}$$

$$T = \frac{8\times 16(\cdot 2 - \cdot 1)\times 9{\cdot}8\cos 30°}{8+16} = 4{\cdot}5\ \text{newton.}$$

Ex. 5. *A man can throw a stone horizontally with velocity $v_1 = 5$ m/s. What velocity v_2 can he generate in the same stone if he* *throws it with the same force while standing on skates on smooth ice? The mass of the stone $m=1$ kg, the mass of the man $M=60$ kg. Will the man produce the same power in each case?*

Sol. In the first case the man performs work

$$W=\tfrac{1}{2}\,mv_1^2.$$

If, in the second case, he throws the same stone with the same force he will do the same work but his work will be converted partly into kinetic of his own and partly into the kinetic energy of the stone. Therefore,

$$\tfrac{1}{2}\,mv_1^2=\tfrac{1}{2}\,mv_2^2+\tfrac{1}{2}\,MV^2.$$

By the law of conservation of momentum,

$$mv_2=MV.$$

Hence

$$v_2=v_1\sqrt{\frac{M}{m+M}}$$

$$=5\sqrt{\frac{60}{60+1}}=4{\cdot}96\ \text{ms}^{-1}.\quad \textbf{Ans.}$$

The velocity of the man after he has thrown the stone,

$$V=v_1\sqrt{\frac{m^2}{M(M+m)}}.$$

$\therefore$ The velocity of the stone relative to the man is

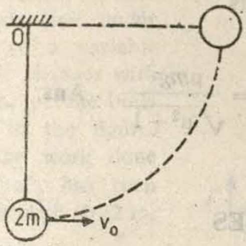
$$v_{rel}=v_2+V=v_1\sqrt{\frac{M+m}{M}}>v_1.$$

Since, power = force × velocity, the man must develop a greater power because the velocity of the stone relative to man is now greater.

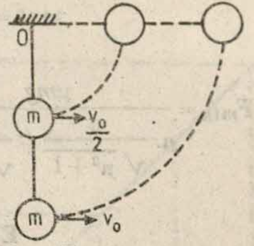
Ex. 6. *A mass 2 m is fixed at one end of a light rod of length l. What horizontal velocity should be imparted to the lower end so that it may just take up the horizontal position? If the mass is now rearranged with m at the lower end and the other m at the middle, what velocity is required?*

Sol. Considering conservation of energy between the vertical position and the horizontal position, we have

$$-(2m)gl+\tfrac{1}{2}\,(2m)v_0^2=0\quad\text{or}\quad v_0=\sqrt{2gl}.\quad \textbf{Ans.}$$



Case I



Case II

In the second case the *two masses will have the same angular velocity but different linear velocities.*

$$\therefore \quad \frac{v_0}{l}=\frac{v_2}{\frac{l}{2}} \quad \text{or} \quad v_2=\frac{v_0}{2}.$$

By the principle of conservation of energy,

$$mgl+mg\frac{l}{2}=\frac{1}{2}\,mv_0^2+\frac{1}{2}\,m\left(\frac{v_0}{2}\right)^2$$

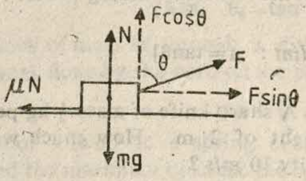
$$\text{or} \qquad \frac{3}{2}\,mgl=\frac{5}{8}\,mv_0^2 \quad \text{or} \quad v_0=\sqrt{\frac{12}{5}gl}. \quad \textbf{Ans.}$$

Ex. 7. *A particle of mass m rests on a horizontal floor with which it has a coefficient of friction* μ. *It is desired to make the body move by applying the minimum possible force. Find this minimum force and the direction in which has to be applied.* (IIT 1987)

Sol. Let F be any force applied in any direction (at θ) with the vertical to move the body. Resolving the forces along and perpendicular to the horizontal plane we have

$$\mu N=F\sin\theta$$

$$\text{and} \quad N+F\cos\theta=mg.$$



$$\text{Hence} \quad F=\frac{\mu mg}{\mu\cos\theta+\sin\theta}.$$

Obviously, F is minimum when $(\mu\cos\theta+\sin\theta)$ is maximum, that is,

$$\frac{d}{d\theta}\,(\mu\cos\theta+\sin\theta)=0$$

$$\text{or} \qquad -\mu\sin\theta+\cos\theta=0$$

or $$\tan\theta = \frac{1}{\mu}. \quad \textbf{Ans.}$$

$$\therefore \quad F_{min} = \frac{\mu mg}{\mu . \dfrac{\mu}{\sqrt{\mu^2+1}} + \dfrac{1}{\sqrt{\mu^2+1}}} = \frac{\mu mg}{\sqrt{\mu^2+1}}. \quad \textbf{Ans.}$$

EXERCISES

(A)

1. A block weighing 5 kg is pressed against a vertical wall by a normal force of 150 newtons, the coefficient of friction being ·5. Find the force required to cause the block to move (a) horizontally, (b) vertically upward, (c) vertically downwards.

(**Ans.** (a) 75 newtons, (b) 124 newtons, (c) 26 newtons)

2. A body of 4 kg rests in limiting equilibrium on a rough plane when slope is 30°. The slope is subsequently increased to 60°. Calculate (i) aoefficient of limiting friction between the body and the surface, (ii) acceleration down the plane if coefficient of kinetic friction is ·4, (iii) force required to prevent it from sliding down the plane, (iv) force required just to move it up the plane.

($g = 9{\cdot}8$ m/s^2) (**Ans.** (i) ·5774, (ii) 6·53 m/s^2, (iii) 22·63 newtons or 2·31 kg, (iv) 45·26 newtons or 4·62 kg)

3. The length of a simple pendulum is ·4 m The bob of pendulum is drawn aside till the string is horizontal. It is released from this horizontal position. How fast will it be when it reaches the lowest point in its swing ? $g = 9{\cdot}8$ m/s^{-2}

[*Hint* : Loss in potential energy = gain in kinetic energy.] (**Ans.** 2·8 ms^{-1})

4. A block slides down an inclined plane of slope θ with constant velocity. It is then projected up the plane with an initial speed u. How far up the incline will it move before coming to rest ? Will it slide down again ?

[*Hint* : $\mu = \tan\theta$] $\left(\textbf{Ans.} \ \dfrac{u^2}{4g\sin\theta}\text{; No}\right)$

5. A sharp knife of mass ½ kg penetrates 5 cm into wood when it falls from a height of 2 m. How much will it penetrate when thrown horizontally with velocity 10 m/s ?

(**Ans.** 12·44 cm)

6. A bullet of mass 20 gm travelling horizontally at 100 ms^{-1} embeds itself in the centre of the block of wood of mass 1 kg and suspended by a light vertical string of 1 m in length. Calculate the maximum inclination of the string to the vertical.

[*Hint* : Consider conservation of momentum and conservation of energy.]

(**Ans.** 37°)

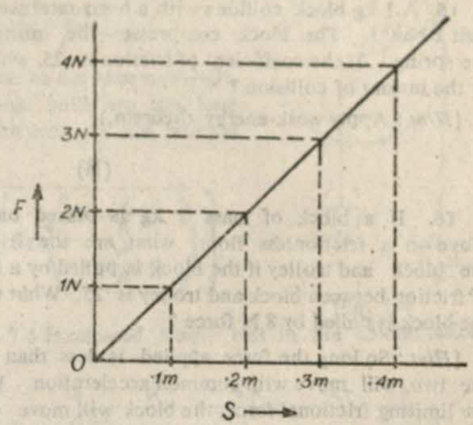


Fig. 7-3

7. A body moves in a straight line under the action of a variable force which changes with displacement of the body as shown in the figure. What is the work done when the body has been displaced through (i) ·2 m, (ii) ·4 m ?

(**Ans.** (i) ·2 joule, (ii) ·8 joule)

8. A motor cyclist is moving up an incline 1 in 100 at the speed of 40 kilometres per hour. What is the horse power of the engine if the motor cycle and its rider together weigh 200 kg, coefficient of friction between tyre and road is ·1 and air resistance to the system at this speed is ·5 newton per kg of the system ? (746 watt = 1 H. P.) (**Ans.** 4·7 H. P.)

9. A truck can move up a road having a grade of 1 m rise every 50 m with speed of 24 kilometres per hour. How fast will the same truck move down the hill with the same horse power ? The air resistance to the motion of the truck is 1/25th of its weight. (**Ans.** 72 kph)

10. A railroad flat car is loaded with crates having coefficient of friction ·25 with the car. If the car is moving at 30 kph, in what short distance can the car be stopped without letting the crates slide ? (**Ans.** 14·16 m)

11. The handle of a floor mop makes an angle θ with the vertical. If μ_k and μ_s be coefficients of kinetic and static friction between floor and mop, show that if θ is smaller than certain value θ_0, the mop cannot be made to slide over across the floor, no matter how great a force is directed along the handle towards the centre What is the angle θ_0 ? (**Ans.** $\theta_0 = \tan^{-1} \mu_s$)

12. A cord is used to lower vertically a block of mass M through a distance d at a constant acceleration $g/4$. Find the work done by the cord on the block.

(**Ans.** $\frac{3}{4}$ Mgd)

13. A 2 kg block is dropped from a height of ·4 m on to a spring of force constant $k = 1960$ newtons per metre. Find the maximum distance the spring will be compressed. (Take $g = 9{\cdot}8$ ms^{-2})

[*Hint* : Consider conservation of energy.] (**Ans.** ·1 m)

14. A light spring is hung vertically from fixed support and a heavy mass 'm' is attached to its lower end. The mass is then slowly lowered to its equilibrium position. This stretches the spring by an amount d. If the same body is permitted to fall instead, through what distance does it stretch the spring ?

[*Hint* : Consider conservation of energy.] (**Ans.** $2d$)

15. A 1 kg block collides with a horizontal weightless spring of force constant 2 Nm^{-1}. The block compresses the spring 4 m from the rest position of the spring. If the coefficient of friction is ·25, what was the speed of the block at the instant of collision ?

[*Hint* : Apply work-energy theorem.] (**Ans.** 7·2 ms^{-1})

(B)

16. If a block of mass 2 kg is placed on a trolley of mass 20 kg free to move on a frictionless floor, what are the frictional force, accelerations of the block and trolley if the block is pulled by a force of 3 N ? The coefficient of friction between block and trolley is ·25. What will be values of all the three if the block is pulled by 8 N force ?

[*Hint* : So long the force applied is less than the limiting frictional force the two will move wtih common acceleration. When the force applied exceeds the limiting frictional force, the block will move on the trolley with different acceleration.] (**Ans.** 2·73 N ; ·136 ms^{-2} ; ·136 ms^{-2}; 4·9 N; 1·55 ms^{-2}; ·245 ms^{-2})

17. A rectangular block of mass M and height $2h$ and width $2c$ rests on a flat car which moves horizontally with constant acceleration 'a'. Determine the value of 'a' at which slipping of the block on the car will be prevented if the coefficient of friction is μ and the value of acceleration at which tippings of the block will be prevented, assuming sufficient friction to prevent slipping.

[*Hint* : Apply D'Alemberts principle and conditions for rotational and translational equilibrium.]

$$\left(\textbf{Ans.}\quad \mu g\,;\quad \frac{cg}{h}\right)$$

18. An arrow weighing ·0145 kg is shot from a 15·876 kg draw bow at full draw $d = \cdot 4$ m. Assuming a linear relation between draw and force, calculate the veloctiy v with which the arrow leaves the bow.

[*Hint* : Here $F = kd$. When $d = \cdot 4$ m, $F = 15{\cdot}876\,g$ newton.

$$\therefore\quad k = \frac{15{\cdot}876 \times 9{\cdot}8}{\cdot 4} = 388{\cdot}7\ Nm^{-1}$$

strain potential energy$=\frac{1}{2}kd^2 = \frac{1}{2}mv^2$ or $v = d\sqrt{k/m}$]

(**Ans.** 65·27 ms^{-1})

19. A flexible but inextensible chain of length l is placed on a smooth table with an initial length a hanging down the table. Calculate the velocity with which the chain will leave the table if released from rest.

[*Hint* : Taking the table as the datum and m as the mass per unit length of the chain, apply the principle of conservation of energy.]

$$\left(\textbf{Ans.}\quad v = \sqrt{\frac{g}{l}(l^2 - a^2)}\right)$$

20. Calculate the velocity v of the falling mass P after it has gone down by x in an axle and wheel system of radii r and R respectively (Fig. 7-4).

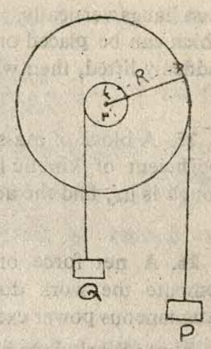


Fig. 7-4

[*Hint* : Let Q go up by y when P goes down by x.

Then $\frac{x}{y}=\frac{R}{r}$. $\therefore\ \dot{x}\left(\frac{dx}{dt}\right)=\frac{R}{r}\dot{y}$.

Apply 'work-energy' theorem to the *system* $(P+Q)$. Remember that the gravitational pulls are the lone forces external to the *system*, the tensions are internal forces of the system.]

$$\left[\textbf{Ans.}\ \sqrt{2gx\left(\frac{P-\frac{r}{R}Q}{P+\frac{r^2}{R^2}Q}\right)}\right]$$

21. If the system of the Fig. 7-5 is released from rest in the configuration shown by solid lines, find the maximum distance h that the weight will fall. (Neglect friction and masses of pulleys.)

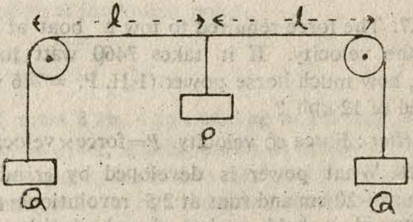


Fig. 7-5

[*Hint* : Apply 'work-energy' theorem to the compound system consisting of the three masses. To find instantaneous velocities of the masses apply the condition that the length of the string remains constant. Also remember that gravitational pulls are the lone forces external to *the system*, the tension of the string is the internal force of the system.]

$$\left(\textbf{Ans.}\ h=\frac{4PQl}{4Q^2-P^2}\right)$$

22. A body of mass m starts from rest down a plane of length l at an angle θ with the horizontal. (a) Find the body's speed at the bottom. (b) How far will it slide horizontally on a similar surface after reaching the bottom of the incline. The coefficient of friction between the body and the plane is μ.

[**Ans.** (a) $\sqrt{2gl(\sin\theta-\mu\cos\theta)}$; (b) $\frac{l}{\mu}(\sin\theta-\mu\cos\theta)$]

23. A 4 kg block is put on the top of a 5 kg block. In order to move the 4 kg block over the 5 kg block a force of 12 newton is required. Assuming a frictionless table, find (a) the maximum horizontal force which can be applied to the lower block so that the blocks will move together and (b) the resulting acceleration.

[*Hint* : $12=\mu.4g$ or $\mu=\cdot3$. The two will move together so long the frictional force between the two will not reach the limiting value. Consider separately the free-body diagrams and apply Newton's second law to their motion.]

(**Ans.** (a) 27 newton, (b) 3 ms^{-2})

24. A block A of 10 kg rests on a rough horizontal table and it is connected to another block of mass 5 kg by a string passing over a frictionless pulley. This

mass hangs vertically. (a) Determine the minimum weight of a third block C which can be placed on A to prevent it from sliding if $\mu_s = \cdot 2$. (b) The block C is suddenly lifted, then what is the acceleration of the block A ? ($\mu_k = \cdot 18$)

[**Ans.** (a) 15 kg, (b) 2 ms^{-2})

25. A block of mass m slides down an inclined right angled trough. If the coefficient of kinetic friction between the block and the material composing the trough is μ_k, find the acceleration of the block down the plane.

(**Ans.** $g(\sin\theta - \sqrt{2}\,\mu_k \cos\theta)$

26. A net force of 5 newton acts on a 15 kg body initially at rest. Compute the work done by the force in the first, second and third second and instantaneous power exerted by the force at the end of the third second.

[*Hint* : Work done by external force = change in kinetic energy.

Instantaneous power = force × instantaneous velocity.]

(**Ans.** ·83 joule; 2·5 joule; 4·17 joule; 5 watt)

27. The force required to tow a boat at constant velocity is proportional to the velocity. If it takes 7460 watt to tow a certain boat at a speed of 4 kph, how much horse power (1 H. P. = 746 watts) will be required to tow it at a speed of 12 kph ?

[*Hint* : Force ∝ velocity. P = force × velocity] (**Ans.** 90 H. P.)

28. What power is developed by grinding machine whose wheel has a radius of 20 cm and runs at 2·5 revolutions per second when the tool to be sharpened is held against the wheel with a force of 20 kgf ? The coefficient of friction between the tool and the wheel is ·3.

[*Hint* : Power $= Fv = F\omega r = \mu N\omega r$ where N is the normal reaction.]

(**Ans.** 184·7 watt)

29. Consider the system of weights $P_1 = 1$ kg, $P_2 = 2$ kg and $P_3 = 5$ kg and $P_4 = \cdot 5$ kg as shown in fig. 7-6. $\alpha = 30°$, $g = 9{\cdot}8\ ms^{-2}$ and the coefficient of friction between the weights and the planes is ·2. Find (a) the acceleration of the system, (b) tensions in the strings between P_1 and P_2 and that between P_2 and P_3 respectively, and (c) the force with which P_4 presses down on P_3.

[*Hint* : Make 'free body' diagram of each and apply Newton's law of motion after resolving the forces acting on a body along and perpendicular to the plane on which it lies.]

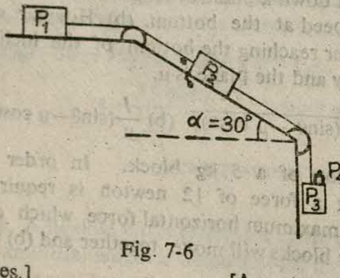


Fig. 7-6

[**Ans.** (a) 6·87 ms^{-2}; (b) 8·83 N; 16·12 N; (c) 1·465 N]

30. A plane surface, inclined at 37° to the horizontal is accelerated horizontally till a block of mass m originally at rest with respect to the plane, just starts to slip up the plane. The coefficient of static friction between surfaces in contact is $\mu = \frac{5}{9}$. Find the acceleration at which the plane is accelerated.

[*Hint* : Applying D'Alembert's principle bring the system to rest. Make free-body diagram and apply second law of motion.]

(**Ans.** $a=\dfrac{\cos\theta+\sin\theta}{\cos\theta-\mu\sin\theta}\,g=21{\cdot}7$ ms^{-2})

31. Suppose that the string of simple pendulum is elastic and k is its force constant. Show that the string will stretch by an amount $\Delta l=\dfrac{3mg}{k}$ when $\Delta l<<l$. Also show that the speed of the bob at the bottom is $v=\sqrt{2g\left(l-\dfrac{3mg}{2k}\right)}$. The amplitude of the pendulum is 90°.

[*Hint* : From energy consideration at the lowest point we have, (taking the horizontal plane through the point of suspension as the datum plane)

$$0=\tfrac{1}{2}mv^2+\tfrac{1}{2}k(\Delta l)^2-mg(l+\Delta l).$$

From consideration of circular motion at the lowest point,

$$k\Delta l-mg=\frac{mv^2}{l+\Delta l}\,.$$
]

32. Three blocks A, B and C of mass 8 kg, 4 kg and 3 kg respectively are placed on a horizontal floor one above the other, A being on the floor and B over A and C over B. C is prevented from moving by a massless rigid rod fixed to the nearest wall while A and B are connected by a light flexible horizontal string passing around a pulley fixed to the wall. Calculate the force required to move A away from the wall if the coefficient of sliding friction between any two surfaces is ·25.

[*Hint* : When two bodies are in motion, one exerts frictional force on the other in direction opposite to its motion and the other also exerts same force on the first in direction opposite to its own motion. Consider then motion of each body considering all the forces acting on each.] (**Ans.** 8 kg wt.)

33. The cable of a 4000 kg elevator (lift) snaps when the elevator is at rest at the second floor so that the bottom is at a distance 12m above a cushioning spring whose force constant is $k=10^4$ kg m^{-1}. A safety device clamps the guide rails so that a constant frictional force 1000 kg opposes the motion of the elevator. Find (a) the speed of the elevator just before it hits the spring, (b) the distance by which the spring is compressed after first hit, (c) the distance that the elevator will bounce back up the shaft, and (d) the total distance that the elevator will describe before coming to rest.

[*Hint* : Apply 'work-energy' theorem and remember that spring force and gravitational force are conservative and the frictional force is non-conservative.

(**Ans.** 13·3 ms^{-1}; 3 m; 6 m; 48·75ml]

34. Two masses 1·65 kg and 3·30 kg joined together by a massless rod paralle to an incline of 30°, on which both slide down along the plane with 1·65 kg trailing behind 3·30 kg. The coefficient of friction between the first mass and the plane is ·226 and that between the second mass and the plane is ·113. Find (a) the tension in the rod, (b) the common acceleration of the two masses.

(I. I. T. 1979)

[*Hint* : Make 'free-body' diagram of the two bodies and consider their motion along and perpendicular to the plane.] (**Ans.** 1·06 N; 3·62 ms^{-2})

35. A 40 kg body is pushed up a frictionless 30° incline 10 m long by a horizontal force. (a) If the speed at the bottom is 2 ms^{-1} and at the top is 10 ms^{-1}, how much work is done by the force ? (b) Suppose the plane is not frictionless ($\mu_k=\cdot 5$), then what work will this force do ? How far up the plane does the body go ? (**Ans.** 3880 joules; 349·0 joules; ·9m)

36. A particle of mass m is accelerated by a constant force for t seconds. Two observers observe it from two frames S and S', S is stationary and S' is moving with uniform velocity u with respect to S. Show that for each observer the work done is equal to the gain in kinetic energy but for observer in S' it is $\frac{1}{2}\,ma^2t^2$ and for the observer in S it is $\frac{1}{2}\,ma^2t^2+maut$ where a is the acceleration of the particle. The particle is initially at rest in S'.

[*Hint* : $x=x'+ut$, ∴ $v=v'+u$. According to the theory of relativity laws of Physics are the same in all frames.]

37. An escalator joins one floor with another 10 m above. The escalator is 20 m long and moves along its length at 1·0 ms^{-1}. (a) What power its motor must deliver if it is required to carry a maximum of 100 persons per minute of average mass 60 kg ? (b) If a man walks up the escalator in 10 s, how much work does the motor do on him ? (c) If this man turned around at the middle and walked down the escalator so as to stay at the same floor, would the motor do work on him ? [*Hint* : Apply 'work energy' theorem.]

(**Ans.** 9800 watt or 13·14 H. P.; 2940 joules; No)

38. A ball of mass m is projected horizontally with veloctiy v into the barrel of a spring gun of mass M initially at rest on a frictionless table. The mass m sticks in the barrel at the point of maximum compression of the spring. What fraction of the initial energy of the ball is stored in the spring ?

[*Hint* : Apply energy conservation principle to the *system* to obtain the maximum work done on the spring which is stored as potential energy of the spring. The spring will be porgressively compressed till the two acquire a common velocity.]

(**Ans.** $\frac{M}{(m+M)}$)

39. A block of mass 2 kg slides on a frictionless table with a speed of 10 ms^{-1}. Directly in front of it and moving in the same direction is a block of mass 5 kg moving at 3 ms^{-1}. A massless spring with force constant $k=1120$ Nm^{-1} is attached to the backside (the side facing 2 kg mass) of it. When the blocks collide, what is the maximum compression of the spring ?

[*Hint* : Apply 'work energy' theorem to the system of blocks and principle of conservation of momentum.] (**Ans.** ·25 m)

40. An ideal massless spring is compressed by 1 m by a force of 100 newton. The same spring is placed at the bottom of a frictionless inclined plane ($\theta=30°$). A 10 kg mass is released from the top of the incline and is brought to rest momentarily after compressing the spring by 2 m. Through what distance does the mass slide before coming to rest ?.

[*Hint* : Apply work-energy theorem.]

(**Ans.** 4·08 m)

41. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in Fig. 7-7. The masses move such that the string between P_1 and P_2 is parallel to the incline and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4·0 kg each and the coefficient of kinetic friction between the masses and the surfaces is ·25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity, find

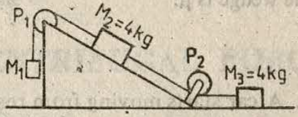


Fig. 7-7

(i) the mass of M_1,

(ii) the tension in the horizontal portion.

($g=9{\cdot}8\ \text{ms}^{-2}$, $\sin 37^\circ=\frac{3}{5}$) (I. I. T. 1981)

[Ans. (i) 4·2 kg, (ii) 9·8 N]

42. The handles of a table-drawer are equidistant from the sides of the drawer and are distant c from each other. Show that it will be impossible to pull the drawer out by pulling one handle if $l \leqslant \mu c$ where l is the length of the drawer from back to front and μ is the coefficient of friction.

[*Hint* : Consider a force applied at inclination θ with the knob of the handle. Apply conditions for translational and rotational equilibrium. Hence show that it is possible to be drawn when

$$\tan\theta \geqslant \frac{l-\mu c}{\mu^2 c+2\mu^2 x+2\mu l}\cdot$$

Since tanθ is positive, $l \geqslant \mu c$.]

43. A fixed pulley carries a weightless thread with masses m_2 and m_1 at its ends ($m_2 > m_1$). There is friction between the thread and the pulley. If it is such that the thread starts slipping when the ratio of m_2 to m_1 is n_0, find :

(a) the friction coefficient, (b) the acceleration of the masses when $\frac{m_2}{m_1}=n>n_0$.

$$\left(\textbf{Ans.}\ \ \mu=\frac{\log n_0}{\pi};\ a=\frac{g(n-n_0)}{(n+n_0)}\right)$$

[*Hint* : Consider an element of the thread of width $d\theta$ at an angular distance θ from the horizontal diameter of the pulley. The forces at the ends are T and $T+dT$. If dN is the normal reaction of the pulley on the element, then df (frictional force on the element) $=\mu dN$. Resolving the tensions at the ends along and perpendicular to the radius vector of the element, we have for equilibrium of the element $dT=df$ and $dN=Td\theta$. Integrate from T_1 to T_2. $T_1=m_1g$ and $T_2=m_2g$. These are obtained by considering equilibrium of m_1 and m_2.]

44. A block of mass m is placed on the rough incline of a wedge of mass M and inclination α to the horizontal. Calculate the maximum and minimum acceleration that can be imparted to the wedge so that the block may remain stationary relative to the wedge. The coefficient of friction between the block and the wedge is μ.

$$\left(\text{Ans. } \frac{\mu\cos\alpha+\sin\alpha}{\cos\alpha+\mu\sin\alpha}g;\quad \frac{\sin\alpha-\mu\cos\alpha}{\mu\sin\alpha+\cos\alpha}g\right)$$

45. A car starts moving from rest along the circumference of a horizontal circle of radius $R=40$ m with a constant tangential acceleration $a_t=0{\cdot}62$ ms^{-2}. The coefficient of sliding friction is $\mu={\cdot}2$. What distance will the car describe without sliding ?

[*Hint* : The car will start sliding when the frictional force attains its upper bound.]

$$\left(\text{Ans. } s=\tfrac{1}{2}R\sqrt{\left(\frac{\mu g}{a_t}\right)^2-1}=60\text{ m}\right)$$

□□□

CHAPTER 8

CENTRIPETAL AND CENTRIFUGAL FORCES

(Pseudo Force)

1. Centripetal force—According to Newton's first law of motion a body moves in a straight line unless a force acts on it. So to keep a body move in a curved path a force is essentially needed towards the centre of the path. The force so needed is called the centripetal force.

If a body of mass 'm' is going round a circular path of radius 'r' with uniform speed 'v' then

$$\text{Centripetal force} = \frac{mv^2}{r} \text{ newton (N) or } m\omega^2 r \text{ newton (N)}$$

where ω is the angular speed of rotation of the body. In circular motion, v is related to ω by

$$v = \omega r.$$

The force directed away from the centre is called centrifugal force.

2. Motion of a bicyclist in a curved path—While taking turn round a curved path a cyclist automatically leans inwards for balance because when he leans inwards a force arises out of the reaction of the ground towards the centre of the path. This provides necessary centripetal force which is so essentially needed to move in a curved path.

If v = speed of cyclist, r = radius of the path then inclination of the cyclist with the vertical required for 'balance' is given by

$$\tan\theta = \frac{v^2}{rg}.$$

3. Banking of tracks—To keep automobiles move in safety in a curved path, the road itself is inclined a little towards the centre of the path so that there arises a force towards the centre of the path out of the reaction of the ground and that provides necessary centripetal force. Thus no lateral pressure will arise between outer wheel and road and safety of drive round the curved path is ensured.

The angle by which the road has to be inclined with the **horizontal** is given by

$$\tan\theta=\frac{v^2}{r.g}.$$

Exactly for the same reason a railway track is also inclined towards the centre of the curved path for 'safety'.

4. Circular motion with variable velocity—If a particle moves in a circular path with variable speed v, then it will have both tangential acceleration (a_t) and radial or centripetal acceleration (a_r). They are given by

$$a_t=\frac{dv}{dt} \text{ and } a_r=\frac{v^2}{r}$$

where r is the radius of the circle.

The resultant acceleration (a) is given by

$$a=\sqrt{a_t^2+a_r^2}.$$

The resultant force is given by

$$F=ma=m\sqrt{a_t^2+a_r^2}.$$

EXAMPLES

Ex. 1. *A stone weighing 500 gm is tied to a string 50 cm long and whirled in a horizontal plane top of a frictionless table with a speed of 150 cm per second. Determine the tension of the string. Also find out the maximum speed with which the stone can be whirled if the string can bear at most a load of 20 kg.*

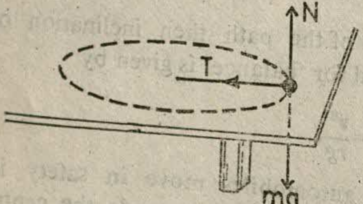


Fig. 8-1

Sol. Here, centripetal force = tension of the string $=\frac{mv^2}{r}$.

N, normal reaction of the table $=mg$.

Now, $m=500$ gm $=\cdot5$ kg; $v=150$ cm per second $=1\cdot5$ ms^{-1}; $r=50$ cm $=\cdot5$ m.

$\therefore$ Tension $=\frac{\cdot5(1\cdot5)^2}{\cdot5}=2\cdot25$ N. **Ans.**

Maximum speed is attained when the tension of the string becomes equal to the breaking forces of the string.

$$\therefore \quad 20 \times 9{\cdot}8 = \frac{{\cdot}5 v^2_{max}}{{\cdot}5}.$$

$$v_{max} = \sqrt{20 \times 9{\cdot}8} = 14 \text{ ms}^{-1}. \quad \textbf{Ans.}$$

Ex. 2. *A train is moving at the rate of* 60 *km per hour round a curve of radius* 500 *m and the distance between the rails is* 170 *cm. By how much the outer rail be raised above the inner one so that there may be no lateral thrust on the outer rail ?* $(g = 9{\cdot}8 \text{ ms}^{-2})$

Sol. The angle by which the track is inclined to avoid lateral thrust is given by

$$\tan\theta = \frac{v^2}{r.g}.$$

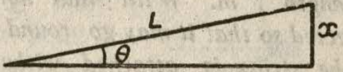


Fig. 8-2

Since θ is small,

$$\tan\theta = \theta = \sin\theta = x/L$$

where x = required distance by which outer rail is to be raised and L = distance between rails.

$$\therefore \quad x/L = \frac{v^2}{r.g}, \text{ or } x = \frac{v^2 L}{r.g}.$$

Here,

$$v = 60 \text{ kph} = \frac{60 \times 1000}{60 \times 60} = \frac{100}{6} \text{ m s}^{-1};$$

$$r = 500 \text{ m}; \; L = 170 \text{ cm} = 1{\cdot}7 \text{ m}.$$

$$\therefore \quad x = \frac{(100/6)^2 . 1{\cdot}7}{500 \times 9{\cdot}8} = {\cdot}0986 \text{ m}. \quad \textbf{Ans.}$$

Ex. 3. *The roadway bridge over a canal is in the form of an arc of a circle of radius* 20 *m. What is the maximum speed with which a car can cross the bridge without leaving the ground at the highest point ?*

Sol. The real forces acting on the car at any point are (i) its weight vertically downward, and (ii) the normal reaction N of the road. At the highest point the normal reaction is vertically upward. Hence at the highest point the force towards the centre is $mg - N$. If v is the speed of the car, then

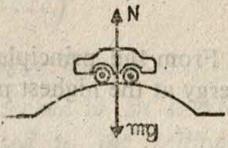


Fig. 8-3

$$mg - N = \frac{mv^2}{r} \text{ or } N = mg - \frac{mv^2}{r}.$$

The normal reation N can never be negative, i.e. $N \geqslant 0$.

$$\therefore \quad mg - \frac{mv^2}{r} \geqslant 0 \quad \text{or} \quad g \geqslant \frac{v^2}{r} \quad \text{or} \quad v \leqslant \sqrt{gr}.$$

$$\therefore \quad v_{max} = \sqrt{gr}.$$

Here, $r = 20$ m; $g = 9{\cdot}8$ ms^{-2}.

$$\therefore \quad v = \sqrt{20 \times 9{\cdot}8} = 14 \text{ ms}^{-1}. \quad \textbf{Ans.}$$

Ex. 4. *A small body of mass $\frac{1}{2}$ kg is tied to one end of a string of length 1 m. With what horizontal velocity the body must be projected so that it may go round the peg to which the other end of the string is attached without slackening of the string at the highest point ?*

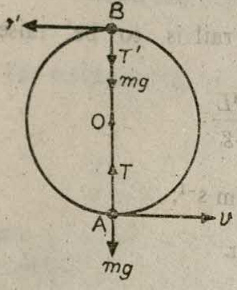


Fig. 8-4

Sol. At the lowest point A,

centripetal force $= T - mg = \dfrac{mv^2}{r}$

and kinetic energy $= \frac{1}{2}mv^2 =$ total energy.

At the highest point B,

centripetal force $= T' + mg = \dfrac{mv'^2}{r}$

and total energy $= \underset{\text{(k.e.)}}{\frac{1}{2}mv'^2} + \underset{\text{(p.e.)}}{mg(2r)}$.

In order that the string may not just slacken at the highest point we have $T' = 0$.

$$\therefore \quad mg = \frac{mv'^2}{r} \quad \text{or} \quad mv'^2 = mgr.$$

From the principle of conservation of energy we must have total energy at the highest point = total energy at the lowest point.

$$\therefore \quad \tfrac{1}{2}mv'^2 + 2mgr = \tfrac{1}{2}mv^2$$

or $$\tfrac{1}{2}mgr + 2mgr = \tfrac{1}{2}mv^2$$

or $$v = \sqrt{5gr} = \sqrt{5 \times 9{\cdot}8 \times 1} \quad (\because \; r = 1 \text{ m})$$

$$= \sqrt{49{\cdot}0}$$

or $$v = 7 \text{ ms}^{-1}. \quad \textbf{Ans.}$$

Ex. 5. *Show that the maximum tension in the string of a simple pendulum, when the amplitude* θ_0 *is small, is* $mg\,(1+\theta_0^2)$ *At what position of the pendulum is the tension a maximum ?*

Sol. From consideration of circular motion of the pendulum at B, we have

$T - mg\cos\theta = \dfrac{mv^2}{r}$, where r is the length of the string.

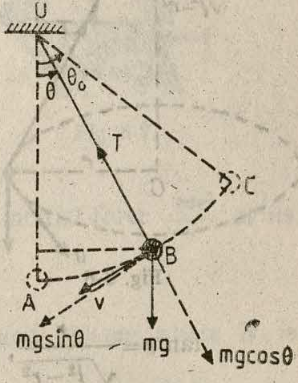


Fig. 8-5

From consideration of conservation of energy, total energy at any point B = total energy at the highest point C (when $\theta = \theta_0$)

$$\tfrac{1}{2}mv^2 + mgr(1-\cos\theta) = \text{a constant}$$
$$= mgr(1-\cos\theta_0)$$

or $\quad mv^2 = 2mgr(\cos\theta - \cos\theta_0)$

or $\quad \dfrac{mv^2}{r} = 2mg(\cos\theta - \cos\theta_0)$;

$$\therefore \quad T = mg\cos\theta + 2mg(\cos\theta - \cos\theta_0)$$
$$= 3mg\cos\theta - 2mg\cos\theta_0.$$

Obviously T is maximum when $\cos\theta$ is maximum.

But maximum value of $\cos\theta$ is 1.

Hence T is maximum when $\cos\theta = 1$

or $\quad \theta = 0°$

and $\quad T_m = 3\,mg - 2\,mg\cos\theta_0 = mg\left[3 - 2\left(1 - \dfrac{\theta_0^2}{2}\right)\right]$

$\left(\text{expansion of } \cos\theta \text{ is } \cos\theta = 1 - \dfrac{\theta^2}{2!} + \dfrac{\theta^4}{4!} \cdots\right)$

$\therefore \quad T_m = mg[3 - 2 + \theta_0^2] = mg(1+\theta_0^2)$. Proved.

Ex. 6. *A sphere of mass* 200 *gm is attached to an inextensible string of length* 1·5 *m whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius* 50 *cm. Calculate the time period of one revolution and tension of the string.*

Sol. Suppose T = tension of the string and θ is the angle made by the string with the vertical and v is the speed of rotation of the sphere.

$$=\frac{mv^2}{(R+h)}$$

where R = radius of the earth and h = height of the satellite.

$$\therefore \quad v^2=g'(R+h)$$

$$\text{or} \quad v=\sqrt{g'(R+h)}$$

$$=\sqrt{7{\cdot}5\times(6400+300)\times 1000}=\sqrt{7{\cdot}5\times 67\times 10^5}$$

$$=7{\cdot}09\times 10^3 \text{ ms}^{-1}. \quad \textbf{Ans.}$$

Ex. 9. *A particle of mass m moves in a circle of radius R in such a way that its speed (v) varies with distance (s) as $v=a\sqrt{s}$ where a is a constant. Calculate the acceleration and force on the particle.*

Sol. We have here $\quad v=a\sqrt{s}$

or $\quad v^2=a^2s.$

Differentiating, $\quad 2v\dfrac{dv}{dt}=a^2\dfrac{ds}{dt}.$

But $\quad v=\dfrac{ds}{dt}.$

$$\therefore \quad 2v\frac{dv}{dt}=a^2v \quad \text{or} \quad \frac{dv}{dt}=\tfrac{1}{2}a^2.$$

$$\therefore \quad a_t \text{ (tangential acceleration)}=\frac{dv}{dt}=\tfrac{1}{2}a^2;$$

$$a_r \text{ (radial acceleration)}=\frac{v^2}{R}=\frac{a^2s}{R}.$$

$$\therefore \quad \text{Total acceleration}=\sqrt{a_t^2+a_r^2}=\sqrt{\frac{a^4}{4}+\frac{a^4s^2}{R^2}}$$

$$=\tfrac{1}{2}a^2\sqrt{1+\frac{4s^2}{R^2}}. \quad \textbf{Ans.}$$

$$F=\tfrac{1}{2}a^2m\sqrt{1+\frac{4s^2}{R^2}}. \quad \textbf{Ans.}$$

EXERCISES

(A)

1. A bucket containing water is tied to one end of a rope 2 m long and rotating in a circle about the other end in a vertical plane. Find the minimum number of rotations per minute in order that water may not spill. (g=9·8 ms^{-2})
[*Hint* : $mg=m\omega^2r$.]

(Ans. 21·1)

2. A motor cyclist goes round a circular race course at 150 kmph. How far from the vertical must he lean inward to keep his balance if the track is 1·5 km long ? (**Ans.** 36°42′)

3. In circus a motorcyclist shows a trick of moving in a vertical plane without falling. Explain the physics of this trick. What is the minimum speed of the motorcyclist if the radius of the iron cage in which he shows his trick is 8 m ? (**Ans.** 8·85 ms^{-1})

4. A gramophone disc is set revolving in a horizontal plane at the rate $33\frac{1}{3}$ revolutions per minute. It is found that a small coin placed on the disc will remain there if its centre is not more than 5 cm from the axis of rotation. Calculate the coefficient of friction between the coin and the disc.

Hint : Centripetal force $= m\omega^2 r =$ Frictional force $= \mu R = \mu mg$.] (**Ans.** ·06)

5. The C. G. of a loaded truck is at a height of 3 m above the ground and the distance between its wheels is 3 m. What is the maximum speed with which it can go round a curved road of radius 200 m without being turtled ? (**Ans.** 31·3 ms^{-1})

6. What is the maximum speed with which an automobile can round an unbanked curve of radius 200 m without skidding if the coefficient of friction between tyre and road is ·3 ? ($g = 9{\cdot}8\ ms^{-2}$)

[*Hint* : Centripetal force $= mv^2/r = \mu R = mg$.] (**Ans** 24·25 ms^{-1})

7. Calculate the apparent weight of a body of 10 kg at a place of latitude 60°. The radius of the earth $= 6{\cdot}4 \times 10^6$ m.

[*Hint* : Apparent weight $= mg - mv^2 \cos^2\lambda/r$ where λ is the latitude of the place

$$= mg - m\omega^2 r \cos^2\lambda \text{ newton}$$

$$= (m - m\omega^2 r \cos^2\lambda/g) \text{ kg.}]$$

(**Ans.** 9·9914 kg)

8. A small body is tied to an end of an inextensible string of negligible mass and length r, the other end being clamped to a rigid suppport. What is the minimum velocity with which the body must be projected horizontally so that it can go completely round a vertical circle without slackening of the string at the highest point ?

[*Hint* : In circular motion in a vertical plane speed and tension change from point to point because acceleration due to gravity also acts on it. The only thing that remains constant is its total energy. The string will not just slacken when tension is zero.] (**Ans.** $\sqrt{5gr}$)

9. An artificial satellite is revolving in a circular orbit at a height of 600 km. from the earth's surface If its time of revolution be 100 minutes, calculate the acceleration due to gravity acting on it towards the earth. Radius of earth $= 6{\cdot}4 \times 10^6$ m. (**Ans.** 7·68 ms^{-2})

10. A mass m on a frictionless table is attached to hanging mass M by a cord of length l passing through a hole in the table. Find the speed with which m must spin for M to stay at the highest point. If this speed is doubled, what will be the force with which M will press the table ?

$$\left(\textbf{Ans. } v = \sqrt{\frac{Mgl}{m}}\ ;\ 3\,Mg\right)$$

11. A road is banked correctly for speed 40 kmph. What is the maximum speed with which a car can negotiate the bend without skidding if coefficient of friction between road and tyre is ·3 and radius of the bend is 1 km ?

[*Hint* : $v_m^2=v^2+\mu gr$.] (**Ans.** 55·35 ms^{-1})

12. The artificial satellite Aryabhatta was reported to revolve round the earth in 96·2 minutes at an average height 600 km. With what velocity was it projected ? What is the acceleration due to gravity on the satellite ?

(Radius of the earth=6400 km.) (**Ans.** $7{\cdot}62\times10^3$ ms^{-1} ; 8·29 ms^{-2})

13. The bob of a simple pendulum of length 1 m hangs at rest. It is suddenly projected horizontally with velocity 10 ms^{-1}. Calculate the velocity of bob and tension of string when it is horizontal. The mass of bob is $\frac{1}{2}$ kg.

[*Hint* : See worked out example 4.] (**Ans.** 8·97 ms^{-1} ; 40·2 N)

14. A small brass bob is revolving in a horizontal circle with constant speed 2 ms^{-1} at the end of a string of length 1 m. Find the time required for one revolution of the body. What angle does the string make with the vertical ?

(**Ans.** 1·814 s, (ii) 35°)

15. A roadway bridge over a rivulet is in the form of quarter of a circle of radius 60 m. What is the maximum speed with which a car must pass over the bridge without leaving the ground at any point of it ? (**Ans.** 20·39 ms^{-1})

16. A string of length 1 m is fixed at one end and carries a mass of 1 kg at the other end. The string makes $2/\pi$ revolutions per second around a vertical axis passing through the fixed end. Calculate (i) angle of inclination of the string with the vertical, (ii) the tension in the string, and (iii) the linear velocity of the mass. [**Ans.** (i) 53°8′, (ii) 16 newton, (iii) 3·2 ms^{-1}]

17. A small block of mass *m* slides along a frictionless loop-the-loop (loop inside loop) track. What should be the ratio of the radius of the outer loop to radius of the inner loop so that the block may not lose contact at the highest point of the inner loop ?

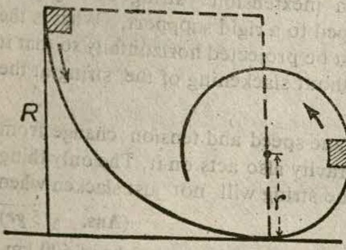


Fig. 8-8

(**Ans.** $R:r=5:2$)

18. Calculate the height of the 'Apple' (India's first stationary satellite) above the surface of the earth. (Radius of the earth=6400 km and $g=9{\cdot}8$ ms^{-2})

(**Ans.** $35{\cdot}9\times10^6$ m)

(B)

19. A stone weighing 1 kg is tied to a string 1 m long and whirled on the horizontal plane top of a rough plane with a speed of 2 ms^{-1}. Calculate the tension of the string if the coefficient of friction between the table and the stone is ·3.

(**Ans.** 1·06 newton)

20. Calculate the frequency of revolution of electron in a hydrogen atom assuming that the radius of the first Bohr orbit is 5×10^{-11} m. Mass of electron$=9{\cdot}0\times10^{-31}$ kg; charge of electron$=1{\cdot}6\times10^{-19}$ coulomb; $\frac{1}{4\pi\epsilon_0}=9\times10^{9}$ SI units. (**Ans.** $7{\cdot}2\times10^{15}$ rev/sec)

21. A brass bob is suspended by an inextensible weightless string of length l from a peg fixed on a vertical wall. A nail is fixed in the wall at a distance d vertically below the point of suspension of string. The bob is then drawn to a position where the string becomes horizontal and it is released from there. Show that d must be at least $\cdot6l$ if the bob is to swing completely around in a circle centred on the nail.

22. A point mass m starts to slide from rest from the top of a smooth solid sphere of radius r. Calculate the angle at which the mass flies off the sphere. If there is friction between the mass and sphere, does the mass fly off at a greater or lesser angle than this angle ? (**Ans.** $48^\circ\ 12'$; greater angle)

23. A very small cube of mass m is placed on the inside of a conical funnel of semivertical angle $(\pi/2-\theta)$. The funnel is then set in rotation. If the coefficient of static friction between the cube and the funnel is μ and the centre of the cube is at a distance r from the axis of rotation, what are the largest and smallest angular velocity of funnel with which it can be rotated so that the block will not move with respect to the funnel ?

[**Ans.** $\omega_{max}=\sqrt{\frac{g}{r}\left(\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\right)}$; $\omega_{min}=\sqrt{\frac{g}{r}\left(\frac{\sin\theta-\mu\cos\theta}{\cos\theta+\mu\sin\theta}\right)}$]

24. A simple pendulum of length l is suspended in a car that is travelling with a constant speed v around a circle of radius r. If the pendulum undergoes small oscillations about its equilibrium position, what will its frequency of oscillation be ?

[**Ans.** $\frac{1}{2\pi}\sqrt{\frac{g}{l}\left(1+\frac{v^4}{r^2g^2}\right)^{\frac{1}{2}}}$]

25. A body is suspended on a spring balance in a ship sailing along the equator with a speed v. Show that the scale reading will be very close to $W_0\left(1\pm\frac{2\omega v}{g}\right)$, where ω is the angular speed of the earth and W_0 is the reading of spring balance when the ship is at rest. Explain the plus or minus sign.

[*Hint* : $W_0=mg-m\omega^2R$; $W=mg-m\,\frac{(\omega R+v)^2}{R}$ when the ship is moving from west to east.]

(**Ans.** Plus sign when ship is moving from east to west and minus sign when ship is moving from west to east, i.e. in the direction of rotation of the earth which rotates from west to east.)

26. A satellite revolving in a circular equatorial orbit of radius $R=2\times10^4$ km from west to east appears over a certain point at the equator every $t=11{\cdot}6$ hours. Calculate the mass of the earth. The gravitational constant $G=6{\cdot}67\times10^{-11}$ SI units.

[*Hint* : $\omega_{app} = \omega_{real} - \omega_{earth}$ or $\omega_{real} = 2\pi\left(\frac{1}{T} + \frac{1}{t}\right)$] (**Ans.** 6×10^{24} kg)

27. A 40 kg mass hanging at the end of a rope of length l, oscillates in a vertical plane with an angular amplitude of θ_0. What is the tension in the rope when it makes an angle θ, with the vertical ? If the breaking strength of the rope is 80 kg, what is the maximum angular amplitude with which the mass can oscillate without the rope breaking ?

[*Hint* : See worked out example 5.]

[**Ans.** $T = 392\,(3\cos\theta - 2\cos\theta_0)$ N; $\theta_0 = 60°$)

28. A large mass M and a small mass m hang at the two ends of a string passing through a smooth tube. The mass m moves around in a circular path in a horizontal plane. The length of the string from the mass m to the top of the tube is l and θ is the angle this length makes with the vertical. What should be the frequency of rotation of the mass m so that M remains stationary ?

[*Hint* : See Ex. 6.] (**Ans.** $f = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$)

29. In problem 17 if the mass m allowed to fall from a height $2R$, find the reactions of the track on the sliding mass at the lowest point, at the highest point, at the mid point. Take $R : r = 5 : 2$. (**Ans.** 11 mg, 5 mg, 8 mg)

30. In the figure of problem 17 if the block starts from rest when $R = 5r$, what is the reaction of the inner track on the block at the highest point ? From what height should it fall so that it may exert force equal to its weight on the track at the highest point ? (**Ans.** 5 mg, $3r$)

31. Due to rotation of the earth a plumb bob may not hang along the earth's gravitational pull but slightly deviates from this direction. Calculate the deviation (a) at 45° latitiude, (b) at the poles, (s) at the equator.

[**Ans.** (a) 0°7′, (b) 0°, (c) 0°]

32. A mass m is moving inside a vertical circular track of radius R. There is no friction. Find the minimum speed (v_{min}) at which m will round the circle without losing contact with the track. Suppose $v = {\cdot}775\ v_{min}$. The mass will move up to certain point P where it will lose contact with the track. Find the angular height of the track. (**Ans.** $\sqrt{5gR}$; $\sin^{-1}\frac{1}{3}$)

33. If a simple pendulum of mass m is released from a horizontal position, find the tension in the string as function of the angle θ with the horizontal.

(**Ans.** $T = 3\,mg\sin\theta$)

34. A simple pendulum consists of a heavy mass supported by a light thin rod. It is held inverted in the position of unstable equilibrium and then released. Find the angle defining the position of its downward fall at which the axial force in the rod changes from compression to tension. (**Ans.** 48°11′)

35. In problem 15 if a motor cyclist passes over the bridge at 18 ms^{-1}, at which point will the reaction become equal to zero ? (**Ans.** 56°34′)

36. Calculate the ratio of the reaction of a convex bridge at the highest point to the reaction of a concave bridge at its lowest point when a car passes at 30 kmph and radius of either bridge is 20 m. (**Ans.** ·48 ; 1)

37. A heavy particle of mass 2 kg is tied to two strings, each of length 25 cm and they are tied to two points 30 cm apart on the same vertical line. The particle is whirled in a horizontal plane with speed 1·8 ms^{-1}. Calculate the ratio of the tensions of the two strings. (**Ans.** 9·3 : 1)

38. A body is suspended by a string of length 1 m and is projected horizontally with velocity 4 ms^{-1}. Calculate its tangential and radial accelerations when the string rises by 60° from its initial position.

(Ans. 8·5 ms^{-2}, 6·2 ms^{-2})

39. A mass of ·675 kg on frictionless table is attached to a string which passes through a hole in the table at the centre. If the radius of the circle is 5 m when the speed is 10 ms^{-1}, find the tension of the string. It is found that on drawing down the string by ·2 m, the tension is increased 4·63 times. Calculate the work done by the string on the revolving mass during the reduction of the radius.

[*Hint* : Apply 'work-energy' theorem to find work done by the force.]

(**Ans.** 135 N; 60 joules)

40. A particle is placed inside a hemispherical bowl which rotates about its vertical axis with constant angular speed ω. It is just prevented from sliding down when the line joining it to the centre of the hemisphere is 45° with the vertical. The radius of the bowl is $10\sqrt{2}$ m. The coefficient of friction between the particle and the bowl is ·5 and $g=10$ ms^{-2}. Find the magnitude of ω.

$$\left(\textbf{Ans. } \frac{1}{\sqrt{3}}\right)$$

41. A rigid rod of length l but of negligible mass has a heavy mass 'm' attached to its one end and it is hinged freely at the upper end. It is inverted and then released. What is its speed at the lowest point and what is the tension in the suspension at that instant ? When the same is released from horizontal position, at what angle from the vertical will the tension in the suspension equal the weight ?

[*Hint* : Consider conservation of energy and dynamics of circular motion.]

(**Ans.** $2\sqrt{gl}$; $5\,mg$, $\theta=71°$)

42. A coppor rod 1 m long is rotated in a horizontal plane surface about a vertical axis passing through its middle. What is the number of revolutions per second at which this rod ruptures ?

Breaking stress of copper $=2{\cdot}9\times10^8$ Nm^{-2}; density of copper $=8900$ kgm^{-3}.

(**Ans.** 81·3)

34. A rod of length l rotates with uniform velocity ω about an axis perpendicular to its length and passing through its end. Calculate the force exerted by one-half on the other. (**Ans.** $\frac{3}{8}m\omega^3 l$)

[*Hint* : Consider the dynamics of circular motion of an element at a distance x and thickness dx. Assume that T is the tension at a distance x.]

44. A small body is placed on the top of a smooth sphere of radius R. Then the sphere is imparted a constant acceleration $a=ng$ in the horizontal direction and the body begins to slide down the sphere. Find : (a) the velocity of the body relative to the sphere at the moment of break-off, (b) the angle θ_0 between

the vertical and the radius vector drawn from the centre of the sphere to the break-off point. Calculate θ_0 for $n=1$, that is, for $a=g$.

(**Ans.** $v=\sqrt{\frac{2gR}{3}}$; $\cos\theta_0=\frac{2+n\sqrt{5+9n^2}}{3(1+n^2)}$, $\theta\approx 17^\circ$)

[*Hint* : Apply D'Alembert's principle and work-energy theorem taking into consideration work done by real forces and inertial force.]

45. A particle of mass m begins to slide down a fixed smooth sphere from the top of its vertical diameter. Calculate its tangential acceleration, radial acceleration, and total acceleration when it breaks off. $\left(\textbf{Ans.}\ \frac{\sqrt{5}}{3}g;\ \frac{2}{3}g;\ g\right)$

46. A particle of mass m is suspended by a string of length l from a fixed rigid support. A sufficient horizontal velocity $v_0=\sqrt{3gl}$ is imparted to it suddenly. Calculate the angle made by the string with the vertical when the acceleration of the particle is inclined to the string by 45°. (**Ans.** 90°)

47. A man stands on a running train (speed 60 kmph) with his feet 60 cm apart facing forward. If the train 'rounds a curve' of radius 450 m in a counterclockwise sense, what per cent of his weight rests on his left leg, if his centre of mass is 1·2 m above the floor ? (**Ans.** 37·4%)

48. A stone is thrown horizontally with velocity $v_0=10\ \text{ms}^{-1}$. Find the radius of curvature of the path in $t=3$ s after the motion began. Neglect air resistance. (**Ans.** $R=306$ m)

□□□

CHAPTER 9

MOMENT OF INERTIA

1. Moment of Inertia—The moment of inertia of a body or particle is its inertia for rotational motion. The moment of inertia of a particle of mass m at a distance r from the axis of rotation is mr^2 and that of a body is Σmr^2 where summation extends over elementary particles constituting the body.

Thus, $\boxed{I \text{ (moment of inertia)} = \Sigma mr^2}$.

Unit of moment of inertia : In SI the unit of moment of inertia is kgm^2.

2. Radius of Gyration—Radius of gyration of a body is the distance of the point from the axis of rotation where the whole mass of the body may be supposed to be concentrated so far its moment of inertia is concerned. It is given by $I = MK^2$ when I = moment of inertia of the body, M = mass of the body and K = radius of gyration. Since it is a distance, its unit is metre (m).

3. The theorem of Perpendicular Axes—If OX and OY are two perpendicular axes in the plane of a lamina and OZ is an axis normal to the plane then

$$\boxed{I_z = I_x + I_y}$$

where I_x, I_y and I_z are respectively moment of inertia of the lamina about OX, OY and OZ respectively.

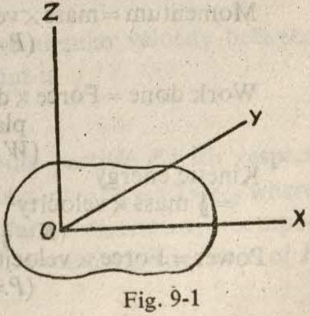


Fig. 9-1

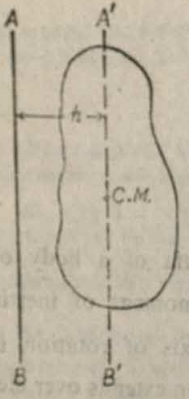


Fig. 9-2

4. Theorem of Parallel Axes—The moment of inertia of any body about any axis *AB* is equal to the moment of inertia of the body about an axis *A′B′* passing through its centre of mass and parallel to the given axis plus the product of mass of the body and square of the distance between the axes, i.e.,

$$\boxed{I=I_0+Mh^2}.$$

5. Analogy between Translatory motion and Rotational motion—

All the laws and formulae of translatory motion can be extended to rotational motion simply by replacing 'mass' by 'moment of inertia' and linear displacement by angular displacement. The full chart of analogy between these two types of motion is given below :

Linear motion	Angular motion
Linear displacement (x)	Angular displacement (θ)
Linear velocity $\left(v=\dfrac{dx}{dt}\right)$	Angular velocity $\left(\omega=\dfrac{d\theta}{dt}\right)$
Linear acceleration $\left(f=\dfrac{dv}{dt}=\dfrac{d^2x}{dt^2}\right)$	Angular acceleration $\left(\alpha=\dfrac{d\omega}{dt}=\dfrac{d^2\theta}{dt^2}\right)$
Force = mass × acceleration ($F=mf$)	Torque = M. I. × angular acceleration ($\tau=I\alpha$)
Momentum = mass × velocity ($P=mv$)	Angular momentum = M. I. × angular velocity ($L=I\omega$)
Work done = Force × displacement ($W=Fx$)	Work done = Torque × angular displacement ($W=\tau\theta$)
Kinetic energy = ½ mass × velocity² ($K=\frac{1}{2}mv^2$)	Kinetic energy = ½M.I. × angular velocity² ($K=\frac{1}{2}I\omega^2$)
Power = Force × velocity ($P=Fv$)	Power = Torque × angular velocity ($P=\tau\omega$)

6. Expressions for M.I. of bodies of known geometrical shape about axis passing through centre of mass :

Body	Axes	Moment of inertia
1. *Thin rod*	About an axis passing through its C.M. and perpendicular to its length	$\frac{1}{12} Ml^2$
2. *A rod*	,,	$M\left(\frac{l^2}{12}+\frac{r^2}{4}\right)$
3. *Rectangular lamina*	About an axis passing through its C.M. and perpendicular to its plane	$M\left(\frac{l^2+b^2}{12}\right)$
4. *A ring*	,,	Mr^2
5. *A disc*	,,	$\frac{1}{2} Mr^2$
6. *Spherical shell*	About a diameter	$\frac{2}{3} Mr^2$
7. *Solid sphere*	,,	$\frac{2}{5} Mr^2$

7. Principle of conservation of angular momentum—

Under the mutual action and reaction of the components of a system, free from external torque, the algebraic sum of the angular momenta of the components of the system remains constant. For a system consisting of two bodies,

$$I_1\omega_1+I_2\omega_2=I_1\omega_1'+I_2\omega_2'.$$

8. Kinematic equations for rotational motion—

$$\omega=\omega_0+\alpha t$$

$$\theta=\omega_0 t+\tfrac{1}{2}\alpha t^2$$

$$\omega^2-\omega_0^2=2\alpha\theta.$$

9. Relative angular velocity—The relative angular velocity between two particles rotating about a common point is

$$\omega_{rel}=\omega_2-\omega_1.$$

The relative angular velocity of a rotating particle P with respect to another rotating particle Q at any instant $=(v_2-v_1)\times r_{12}$ where $v_2=$ component of velocity of P perpendicular to r_{12} and $v_1=$ component of velocity of Q along the same direction and $r_{12}=$ distance of P from Q.

EXAMPLES

Ex. 1. *Calculate the radius of gyration of a thin rod and mass* 1 *kg and length* 100 *cm about an axis passing through its centre of gravity and perpendicular to its length.*

Sol. For a thin rod m.i. about an axis passing through its C.G. and perpendicular to its length is,

$$I = \tfrac{1}{12}\, Ml^2.$$

For radius of gyration we have to equate I to MK^2.

$$\therefore \quad \tfrac{1}{12}\, Ml^2 = MK^2$$

or

$$K = \frac{l}{\sqrt{12}}.$$

Here $l = 100$ cm $= 1$ m.

$$\therefore \qquad K = \frac{1\text{ m}}{\sqrt{12}} = \frac{\sqrt{12}}{12} = \frac{3{\cdot}464}{12} = {\cdot}289\text{ m.} \quad \textbf{Ans.}$$

Ex. 2. *A flywheel of mass* 500 *kg and diameter* 1 *m is set in rotation. In* 5 *seconds it starts rotating at the rate of* 20 *revolutions per second. Calculate the torque applied.*

Sol. From theoretical point of view, a flywheel may be considered as a heavy disc capable of rotating about an axis passing through its C.G. and perpendicular to its plane.

$\therefore$ Moment of inertia of flyweel $= \tfrac{1}{2}Mr^2 = \tfrac{1}{2}.500.(\tfrac{1}{2})^2 = \tfrac{500}{8}$ kgm^2.

Now from formula analogous to $v = u + ft$, we have

$$2\pi 20 = \alpha \times {\cdot}5 \quad \text{or} \quad \alpha = 8\pi \text{ radian sec}^{-2}.$$

$\therefore$ Torque = moment of inertia × angular acceleration

$$= \tfrac{500}{8} \times 8\pi = 1571 \text{ Nm.} \quad \textbf{Ans.}$$

Ex. 3. *A solid sphere of mass* 1 *kg and radius* 10 *cm rolls without slipping with uniform velocity of* 100 *cm per second along a straight line on a horizontal table. Calculate its total energy.*

Sol. Rotational kinetic energy $= \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}.\left(\frac{2}{5}Mr^2\right)\left(\frac{v}{r}\right)^2$

$$= \tfrac{1}{5}Mv^2. \qquad (\because \quad v = \omega r)$$

Linear kinetic energy $= \tfrac{1}{2}Mv^2$.

$\therefore$ Total knetic energy $= \tfrac{1}{5}Mv^2 + \tfrac{1}{2}Mv^2 = \tfrac{7}{10}Mv^2$.

$\therefore$ Potential energy $= Mgh = Mgr$ (because C. G. is at a height equal to the radius of the sphere).

$\therefore$ Total energy $= \frac{7}{10}Mv^2 + Mgr$

$= \frac{7}{10} \times 1 \times 1^2 + 1 \times 9{\cdot}8(10 \times 10^{-2})$

$= \frac{7}{10} + {\cdot}98 = 1{\cdot}68$ joule. **Ans.**

Ex. 4. *A horizontal disc rotating freely about a vertical axis makes* 100 *r.p.m. A small piece of wax* 10 *gm falls vertically on the disc and adheres to it at a distance* 9 *cm from the axis. If the number of revolutions per minute (r.p.m.) is thereby reduced to* 90, *calculate the moment of inertia of the disc.*

Sol. Angular momentum of the disc $= I\omega$

$= I(2\pi n) \quad (\because \ \omega = 2\pi n)$

$= I.2\pi.\frac{100}{60}$

$= \frac{10\pi I}{3}.$

Moment of inertia of wax (considered as a point)

$= mr^2$

$= (10 \times 10^{-3}) \times (9 \times 10^{-2})^2 = 81 \times 10^{-6}$ kgm^2.

Angular momentum of the disc and wax $= (I + 81 \times 10^{-6}).\omega'$

$= (I + 81 \times 10^{-6}) \times 2\pi.\frac{90}{60}$

$= 3\pi(I + 81 \times 10^{-6}).$

From the principle of conservation of angular momentum, we have

$$\frac{10\pi I}{3} = 3\pi(I + 81 \times 10^{-6})$$

or $$10I = 9I + 729 \times 10^{-6}$$

or $$I = 729 \times 10^{-6} \text{ kgm}^2. \quad \textbf{Ans.}$$

Ex. 5. *How long does a disc take to roll down an inclined plane of length* 10 *m without slipping ? Inclination of the plane with the horizontal* $= 30^\circ$ *and acceleration due to gravity* $= 9{\cdot}8$ ms^{-2}.

Sol. When it is said that *a body rolls without slipping*, it means that its centre of mass moves forward and covers a linear distance while it itself goes on rotating about a horizontal axis passing through its centre of mass and the instantaneous resultant velocity of its point of contact with the ground is zero, that is, $v = \omega r$ where v is the linear velocity of its centre of mass and ω is its angular velocity.

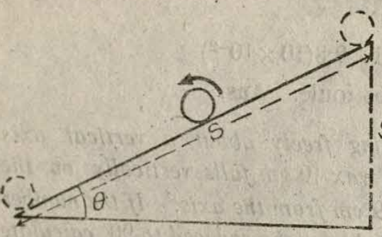


Fig. 9.3

Let the disc cover a distance S down the plane. Then it falls through a vertical distance $= S\sin\theta$.

$\therefore$ Loss in potential energy of the body $= MgS\sin\theta$.

Gain in kinetic energy of the body $= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

$$= \tfrac{1}{2}(MK^2)\omega^2 + \tfrac{1}{2}Mv^2,$$

where K = radius of gyration of the body

$$= \tfrac{1}{2}MK^2\left(\frac{v}{r}\right)^2 + \tfrac{1}{2}Mr^2 \qquad \left(\because \quad \omega = \frac{v}{r}\right)$$

$$= \tfrac{1}{2}Mv^2\left(\frac{K^2}{r^2} + 1\right).$$

From consideration of conservation of energy,

$$\tfrac{1}{2}Mv^2\left(\frac{K^2}{r^2} + 1\right) = MgS\sin\theta$$

or
$$v^2 = \frac{2gS\sin\theta}{\left(\frac{K^2}{r^2} + 1\right)}.$$

Let f be the effective acceleration down the plane. Then

$$v^2 = 2fS$$

$$\therefore \quad f = \frac{g\sin\theta}{\frac{K^2}{r^2} + 1}.$$

This gives the effective uniform acceleration down the plane. Here, I (m.i. of the disc about its axis) $= \frac{1}{2}Mr^2 = MK^2$.

$$\therefore \quad K^2 = \frac{r^2}{2}.$$

$$\therefore \quad \frac{K^2}{r^2} + 1 = \tfrac{1}{2} + 1 = \tfrac{3}{2}.$$

$$\therefore \quad f = \frac{g\sin 30^\circ}{\frac{3}{2}} = \tfrac{2}{3}g\tfrac{1}{2} = \frac{9{\cdot}8}{3} = 3{\cdot}27 \text{ ms}^{-2}.$$

Applying $S = ut + \frac{1}{2} ft^2$, we have

$$10 = \tfrac{1}{2} \times 3{\cdot}27 t^2, \quad \text{or } t = \sqrt{\frac{20}{3{\cdot}27}} = 2{\cdot}47 \text{ s.} \quad \textbf{Ans.}$$

Ex. 6. *A man stands on a frictionless platform which is rotating with a speed of* 1 *revolution per sec., his arms outstretched and he holds a weight in each hand. With his hands in this position the total rotational inertia of the man, the weights and the platform is* 6 *kgm*^{2}. *If by drawing in the weights the man decreases the rotational inertia by* 2 *kgm*^{2}, *calculate the resulting speed of the platform and increase in kinetic energy. How do you account for the increase of kinetic energy* ?

Sol. The system consisting of the man, weights and the platform is free from external torque and hence their angular momentum is conserved.

$\therefore \quad I_1 \omega_1 = I_2 \omega_2.$

$I_1 = 6$ kgm^2 and $I_1 - I_2 = 2$ kgm^2 or $I_2 = 4$ kgm^2.

$\omega_1 = 1$ rev./sec.

$\therefore \quad 6 \times 1 = 4 \times \omega_2$, or $\omega_2 = 1{\cdot}5$ rev./sec.

$$\text{Increase in kinetic energy} = \tfrac{1}{2} I_2 \omega_2^2 - \tfrac{1}{2} I_1 \omega_1^2$$
$$= \tfrac{1}{2} \times 4 \times (\tfrac{3}{2})^2 - \tfrac{1}{2} \times 6 \times 1^2$$
$$= 4{\cdot}5 - 3 = 1{\cdot}5 \text{ joule.} \quad \textbf{Ans.}$$

This increase in energy is supplied by the man, who does work when he pulls the parts of his body together.

Ex. 7. (*a*) *Show that so far the energy of a rolling body without slipping is concerned it is equivalent to pure rotation with the same angular speed about an axis through the point of contact of the rolling body.*

(*b*) *Show that angular momentum of a body of mass M rolling without slipping about any point is equal to its angular momentum about its centre plus the moment of momentum of the particle of the same mass M concentarted at its centre and moving with speed of the centre of mass of the body about the same point.*

Sol. (a) Suppose that a disc of radius r is rolling on a surface with angular velocity ω about an axis through its centre of mass while the centre of mass is advancing with speed v.

Then, K (kinetic energy of the rolling body) = linear kinetic energy + rotational energy $= \frac{1}{2} M v^2 + \frac{1}{2} I_0 \omega^2$ where I_0 = moment of inertia of the body about an axis through its centre of mass

$$= \tfrac{1}{2} M \omega^2 r^2 + \tfrac{1}{2} I_0 \omega^2 \qquad (\because \; v = \omega r).$$

Suppose now that the body is not rolling but it is simply rotating about an axis through the point of contact with the same angular velocity.

Then K (kinetic energy of the body) $=\frac{1}{2}I\omega^2$ where $I=$ moment of inertia about this new axis.

But

$$I=I_0+Mr^2, \text{ by theorem of parallel axes.}$$

$$\therefore \quad K=\tfrac{1}{2}(I_0+Mr^2)\omega^2=\tfrac{1}{2}I_0\omega^2+\tfrac{1}{2}Mr^2\omega^2. \quad \text{Proved.}$$

(b) Let us consider any point P at a distance x below the centre of the body, say, a disc. Suppose m is an elementary mass above the centre at a distance r. Then its speed (linear) $=\omega r$ and its angular momentum about $P=m\omega r\times(r+x)$ ($\because$ angular momentum is defined as moment of momentum).

$\therefore \quad L_P$ (angular momentum of the body about P)

$$=\Sigma m\omega r(r+x)=\Sigma m\omega r^2+\Sigma m\omega rx$$

$$=\omega\Sigma mr^2+Mv_{cm}x$$

$$\left(\because \quad v_{cm}=\frac{\Sigma mv}{M}=\frac{\Sigma m\omega r}{M}\right)$$

or

$$L_P=I\omega+Mv_{cm}x. \quad \text{Proved.}$$

Ex. 8. *A ball is thrown down a lawn in such a way that it slides with a speed v_0 initially without rolling. It gradually picks up rotational motion. Prove that it will roll without sliding, that is, its motion will be pure rolling when its speed falls to $\frac{5}{7}v_0$.*

Sol.

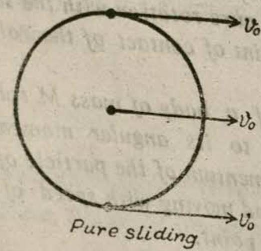


Fig. 9-4

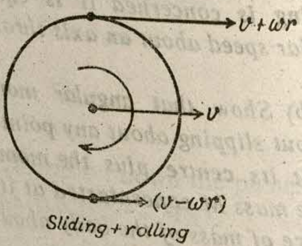


Fig. 9-5

When motion is pure sliding, all the points of the body has the same linear speed, namely v_0 which is the speed of its centre of mass.

Then, $\quad K$ (kinetic energy of the body) $=\frac{1}{2}Mv_0^2$.

When motion is partly sliding and partly rolling, linear speed of the lowest point is $v-\omega r$ and that of the highest point is $v+\omega r$ and speed of centre of mass is v.

Motion is reduced to pure rolling, when the linear speed of the lowest point is zero.

∴ The sphere will roll without sliding when

$$v - \omega r = 0$$

or

$$v = \omega r.$$

F (frictional force on the disc) $= \mu N = \mu mg = ma$ or $a = \mu g$ where a is the linear retardation produced by friction.

Let the linear velocity be reduced from v_0 to v in t seconds.

Then $$v = v_0 - at \quad \text{or} \quad t = \frac{v_0 - v}{a} = \frac{v_0 - v}{\mu g}.$$

Considering rotational motion of the sphere we have

$$\text{Torque} = \mu\, mg\, r = (\tfrac{2}{5}\, mr^2)\,\alpha$$

where α is the angular acceleration produced

or $$\alpha = \frac{5\mu g}{2r}.$$

Now from $\omega = \omega_0 + \alpha t$ we have $\omega = 0 + \dfrac{5\mu g}{2r}\, t = \dfrac{5\mu g}{2r} \cdot \dfrac{v_0 - v}{\mu g}$

or $$\omega = \tfrac{5}{2}\frac{v_0 - v}{r}, \quad \text{or} \quad \omega r = \tfrac{5}{2}(v_0 - v).$$

But $$\omega r = v.$$

∴ $$v = \tfrac{5}{2}\,(v_0 - v) \quad \text{or} \quad v = \tfrac{5}{7}\, v_0.$$

Ex. 9. *Show that a cylinder will slip on an inclined plane if the coefficient of static friction between plane and cylinder is less than $\frac{1}{3}$ tanθ where θ is the angle of inclination with the horizontal.*

Sol. Let us solve the problem of Ex. 5 from consideration of dynamics of the body. Considering translational motion, we have $R - mg \cos\theta = 0$ and $mg \sin\theta - F = mf$. where F is the frictional force and R is the normal reaction.

Considering rotational motion about the centre of mass, we have $F.r = I_0\alpha$ where α is the angular acceleration. But $\alpha = f/r$ just as $\omega = v/r$ when rolling without slipping.

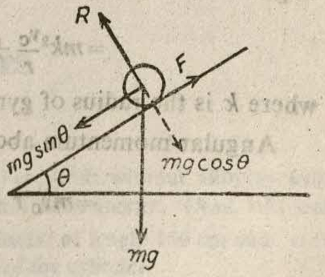


Fig. 9-6

$$\therefore \quad Fr = \tfrac{1}{2}mr^2 \times f/r \quad \text{or} \quad F = \tfrac{1}{2}mf.$$

Putting the value of F, we have

$$f = \tfrac{2}{3}g \sin\theta.$$

$$\therefore \quad F = \tfrac{1}{3}mg \sin\theta.$$

The body will slip if $F > F_{lim}$. But $F_{lim} = \mu R = \mu mg\cos\theta$.

$\therefore$ The body will slip if $\tfrac{1}{3}mg\sin\theta > \mu mg \cos\theta$ or $\mu < \tfrac{1}{3}\tan\theta$.

Proved.

Ex. 10. *Show that if a body rolls without slipping along a horizontal plane with velocity* v_C *and reaches a plane inclined at* θ *with the horizontal, the velocity with which it starts up the inclined plane is* $v_C\left(1 - \dfrac{1-\cos\theta}{\frac{k^2}{r^2}+1}\right)$. *Suppose that there is sufficient friction to prevent slipping.*

Sol. Suppose that just before impact A is the instantaneous point of contact and B is the point where it touches the plane immediately after A. When the body strikes the plane, it (the plane) exerts a very large force F on the body. If we take B as a 'moment centre', the moment of this external force is zero and so we may apply conservation of angular momentum about B.

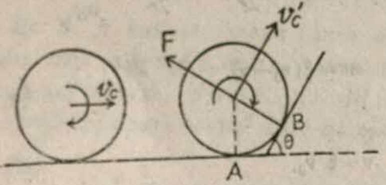


Fig. 9-10

Angular momentum of the body before impact about $B = I\omega + mv_C\, r\cos\theta$ (remembering that moment of momentum is also angular momentum)

$$= mk^2\frac{v_C}{r} + mv_C r\cos\theta \quad (\because \quad v_C = \omega r)$$

where k is the radius of gyration.

Angular momentum about B after impact $= mv_C'\, r + I\omega'$

$$= mv_C' r + mk^2.\frac{v_C'}{r} = mv_C' r\left(\frac{k^2}{r^2}+1\right).$$

$$\therefore \quad mk^2\frac{v_C}{r} + mv_C r\cos\theta = mv_C' r\left(\frac{k^2}{r^2}+1\right)$$

or $$v_C' = \frac{v_C\left(\frac{k^2}{r^2}+\cos\theta\right)}{\frac{k^2}{r^2}+1} = v_C\left(1-\frac{1-\cos\theta}{\frac{k^2}{r^2}+1}\right).$$ **Proved.**

Ex. 11. *Two particles P and Q are rotating in the anticlockwise direction about a common centre O in circles of radii 2 m and 5 m with linear speeds 63 m/s and 21 m/s respectively. Calculate the relative angular velocity with reference to the common centre O. What is maximum angular velocity of P relative to Q in the clockwise direction and that in the anticlockwise direction ?*

Sol. The angular velocity of P about $O=\frac{63}{2}=31\cdot5$ rad/s

The angular velocity of Q about $O=\frac{21}{5}=4\cdot2$ rad/s.

$\therefore$ Relative angular velocity about $O=31\cdot5-4\cdot2$
$=27\cdot3$ rad/s. **Ans.**

When the two particles come on the same radial line and are on the same side of the common centre O, the maximum relative velocity of P at right angles to the line PQ is $63-21=42$ m/s and its moment about Q is clockwise.

$\therefore$ Maximum relative angular velocity in the clockwise direction

$$=\frac{42}{5-2}=14 \text{ rad/s.}$$ **Ans.**

When the two particles come on the same radial line and are on the opposite side of the common centre O, then the maximum relative velocity is $63+21=84$ m/s and its moment about Q is anticlockwise.

$\therefore$ Maximum relative angular velocity of P in the anticlockwise

direction relative to $Q=\frac{84}{5+2}=12$ rad/s. **Ans.**

EXERCISES

(A)

1. A sphere of mass 10 kg and diameter 10 cm rolls without slipping with a velocity of 50 cm per second. Calculate its total kinetic energy. (**Ans.** 1·75 joule)

2. Calculate the moment of inertia of a cylinder of length 150 cm and radius 5 cm and density 8×10^3 kgm^{-3} about the axis of the cylinder.

(**Ans.** ·1178 kgm^2)

3. A uniform rod rotates in a horizontal plane about an axis perpendicular to its length and passing through one extreme end of it. Calculate moment of inertia and kinetic energy of the rod if its length is 3 m, its mass 6 kg and angular velocity 20 radians per second. (**Ans.** 18 kgm^2; 3600 joule)

4. A sphere starting from rest rolls for 5·3 second without slipping along a smooth plane 1 m in length of which the upper end is raised by 1 cm above the lower end. Find the acceleration due to gravity. (**Ans.** 9·97 ms^{-2})

5. A couple 10 Nm is applied to a flywheel of mas 10 kg and radius 50 cm. What is the resulting acceleration ? For how many seconds the couple must be applied and then removed to produce 100 rpm (revolutions per minute in the wheel ? (**Ans.** 8 rads^{-2}; 1·3 s)

6. If the earth suddenly contracts to half of its present radius without losing any mass, how many hours will there be in a day ?

[*Hint* : Apply the principle of conservation of angular momentum.]

(**Ans.** 6 hours)

7. A sphere, a disc and a ring of the same mass and radius are allowed to roll down an inclined plane from the same height without slipping. Prove that the sphere reaches down first, the disc the next and the ring the last.

8. A flywheel of mass 5 kg and radius 10 cm rolls down a plane inclined at 30° to the horizontal. It rolls from rest through 100 cm in 10 s. Neglecting friction find (a) the kinetic energy of the flywheel at the end of 10 s, (b) the moment of inertia of the flywheel about its axis. [**Ans.** (a) 24·5 J; (b) 12·2 kgm^2]

9. Two point masses m_1 and m_2 are joined by a weightless rod of length r. Calculate the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the rod.

[*Hint* : Centre of mass is the point which divides r in the inverse ratio of the masses i.e. $\frac{r_1}{r_2}=\frac{m_2}{m_1}$ and $r_1+r_2=r$.] $\left(\textbf{Ans.}\ \frac{m_1m_2}{m_1+m_2}r^2\right)$

10. A thin uniform rod AB of mass m and length l is hinged at one end to the level floor and stands vertically. If it is allowed to fall, with what angular velocity will it strike the floor ? $\left(\textbf{Ans.}\ \sqrt{\frac{3g}{l}}\right)$

11. A hoop of radius 3 m weighs 150 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 15 cm/sec. How much work has to be done to stop it ?

[*Hint* : A hoop is a ring.] (**Ans.** 3·375 J)

12. A sphere rolls up an inclined plane of inclination angle 30°. At the bottom of the inclined plane the centre of mass of the sphere has a translational speed of 5 ms^{-1}. (a) How far does the sphere travel up the plane ? (b) How long does it take to return to the bottom ? (**Ans.** 3·57 m; 2·8 s)

13. A student sits on a stool that is free to rotate about a vertical axis. He holds his arms extended horizontally with 4 kg weight in each hand. The stool is set in rotation with angular speed of ·5 revolution per second. Calculate the

angular speed of the student when he pulls his hands to his sides, assuming that armlength of student is 90 cm and his rotational inertia is 7 kgm^2.

[*Hint* : Use conservation principle.] **(Ans.** ·96 revolution per second**)**

14. A solid sphere of 1 kg reaches the bottom of an inclined plane after rolling without slipping over a horizontal plane with velocity of 20 ms^{-1}. The inclination of the plane with the horizontal is 30°. What is the velocity of the centre of mass of the sphere immediately after it starts moving up the plane ? Up to what distance along the plane will it rise ? Assume sufficient friction to prevent slipping. **(Ans.** 18 ms^{-1}; 46 m**)**

(B)

15. A thin uniform rod of mass '*m*' and length *l*, pivoted freely at its base, is allowed to fall from a vertical position. Calculate its angular velocity and acceleration when it has turned through 60°.

[*Hint* : Find loss in potential energy and equate it to gain in kinetic energy.)

$$\left(\textbf{Ans. } \sqrt{\frac{3g}{2l}};\ \frac{3\sqrt{3g}}{4l}\right)$$

16. What must be the relation between length and radius of a cylinder of given mass and density so that its moment of inertia about an axis through its centre of mass and perpendicular to its length may be minium ?

(Ans. $\sqrt{3} : \sqrt{2}$**)**

17. A circular disc is of mass *M* and radius *r*. From it a circular piece is cut out with radius of disc as its diameter. Find moment of inertia of the remainder about the axis of the disc. **(Ans.** 13/32 Mr^2**)**

18. A string is wrapped around a cylinder of mass *M* and radius *r*. The string is pulled vertically upward to prevent the centre of mass from falling as the cylinder unwinds the s tring. (a) What is the tension in the string ? (b) How much work has been done on the cylinder once it has reached an angular speed ω ? (c) What is the length of string unwound in the time it acquires angular speed ω ?

[*Hint* : From consideration of linear motion of cylinaer, $T-Mg=0$. Work done = Gain in kinetic energy.]

$$\left(\textbf{Ans. } Mg;\ \frac{1}{4}Mr^2\omega^2;\ \frac{1}{4}\ \frac{r^2\omega^2}{g}\right)$$

19. A small sphere of mass *m* and radius *r* rolls without slipping on the inside of a large hemisphere of radius *R* whose axis of symmetry is vertical. It starts at the top from rest. What is the kinetic energy of the small sphere at the bottom ? What fraction is rotational and what translational ? What normal force does the small sphere exert on the hemisphere at the bottom ?

[*Hint* : Consider circular motion of centre of mass at the bottom to calculate normal reaction. For others consider conservation of energy.]

[Ans. $mg\,(R-r)$; $\frac{2}{7}$; $\frac{5}{7}$; $\frac{17}{7}\,mg$**]**

20. A billiard ball is struck by a cue when it starts moving with velocity v_0 without rolling. How far will the ball move before it ceases to slide on the table if μ is the coefficient of friction between ball and the table ?

[*Hint* : See Ex. 8.) $\left(\text{Ans. } \frac{12}{49}\,\frac{v_0^2}{\mu g}\right)$

21 Explain why the cushion of a billiard table is made to receive the impact of the billiard ball at a height of $0{\cdot}7\,d$ where d is the diameter of the ball.

[*Hint* : A billiard ball is a sphere rolling without slipping on the table. A orlling body without slipping is equivalent to a purely rotating body rotating about the line of contact. Therefore, there is gradation of speed from point of contact to the top, it is being zero at the lowest point and ωd at the top. The resultant of all such velocities is $\frac{1}{2}\,\omega d^2$ acting at a point $\frac{2}{3}d$ or $0{\cdot}7d$ above the pt. of contact.]

22. A right circular cylinder of radius r and mass m is suspended by a cord that is wound around its surface. If it is allowed to fall, prove that its centre of gravity will follow a vertical rectilinear path and find the acceleration along this path. Determine also that tensile force in the cord. (**Ans.** $\frac{2}{3}\,g$; $\frac{1}{3}\,mg$)

23. The mass of the earth is increasing at the rate 1 part in 5×10^{19} per day by the accretion of meteors falling normally upon the earth's surface. Find the corresponding rate of change of the period of rotation of the earth supposing earth to be a sphere of uniform density.

[*Hint* : Consider the deposit as a spherical shell and apply principle of conservation of angular momentum.] (**Ans.** 8×10^{-19} hour per day)

24. A metre stick of length l and mass M lies on a frictionless table. The stick is free to move in any way on the table. A hockey ball of mass m moving perpendicularly to the stick with speed v collides elastically with the stick at one end. What must be the mass of the ball so that it remains at rest immediately after the collision ?

[*Hint* : 'Elastic collision' means that kinetic energy is conserved in the process. Consider conservation of kinetic energy, conservation of angular momentum and linear momentum, i.e.,

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}Mv'^2+\tfrac{1}{2}I\omega^2,\ mv\frac{l}{2}=I\omega,\ mv=Mv'.$$

Eliminate v'^2 and ω^2 from the first with the help of the other two. Use the expression $I=1/12\ Ml^2$.] (**Ans.** $M/4$)

25. A long light thread is wound partly around a circular cylinder of radius r and mass m and partly on a similar cylinder but free to turn about a fixed axis. The movable one is allowed to fall from rest. Find the velocity of the falling cylinder as a function of the height h through which it has fallen. Does the cylinder fall without slipping ?

(**Ans.** $v=\sqrt{8/5\,gh}$); No, it falls with slipping)

26. A timber of mass m rests on two rollers, each of mass $m/2$ and radius r, and is pulled along a horizontal plane by a force P. Assuming there is no slipping and treating the rollers as right circular cylinders, find the acceleration of the timber. $\left(\text{Ans. } \frac{8}{11}\cdot\frac{P}{m}\right)$

27. Show that in order to get a billiard ball to roll without sliding from the start the cue must hit the ball not at the centre (that is, a height above the table equal to the ball's radius R) but exactly at a height $\frac{2}{5}R$ above the centre.

[*Hint* : Consider linear motion and angular motion of the ball. Apply the condition for rolling without sliding. $F=ma$ and $\tau=F\times h=\frac{2}{5}mR^2\times\alpha$. Condition rolling without silding is $a=\alpha R$.]

28. Two masses 500 gm and 460 gm are suspended from the ends of a light string passing over a frictionless heavy pulley of radius 5 cm. When released from rest the heavier mass is observed to fall 75 cm in 5 s. What is the rotational inertia of the pulley ? (*Hint* : When the pulley is heavy the tensions on the two sides are different.) (Ans. $1{\cdot}39\times10^{-2}$ kgm^2)

29. A solid cylinder of weight 20 kg and radius 7·5 cm is placed on an inclined plane having inclination 30° to the horizontal. A light thin tape is wound around the cylinder and is taken tangentially from the upper surface parallel to the plane and finally attached to a 5 kg body after passing over a light smooth pulley. Find the tension in the tape and linear acceleration of the cylinder up the plane, assuming no slip. (Ans. 49 N; 0)

30. A point A is located on the rim of a wheel of radius R which rolls without slipping along a horizontal surface with velocity V. Find the total distance traversed by the point A between successive moments at which it touches the surface. (Ans. 8 R)

31. A plank of mass m_1 with a sphere of mass m_2 on it rests on a smooth horizontal surface. A constant horizontal force F is applied to the plank. With what acceleration will the plank and the sphere move provided there is sliding between the plank and the sphere ?

$$\left(\text{Ans}\quad \frac{7F}{7m_1+2m_2};\ \frac{2F}{7m_1+2m_2}\right)$$

* * *

CHAPTER 10

SIMPLE HARMONIC MOTION

1. Definition and equation of simple harmonic motion (S.H.M.) : The simple harmonic motion is defined as the motion in which acceleration is always directed towards a fixed point in the path of motion and is proportional to the displacement from that fixed point.

Suppose x = displacement of the body at any time t from that point to which acceleration is always directed.

Then $\frac{dx}{dt} = v$ (velocity of the body at time t) and $\frac{d^2x}{dt^2}$ = acceleration of it at that very time t.

Motion is simple harmonic when $\frac{d^2x}{dt^2} \propto -x$

or $\frac{d^2x}{dt^2} = -\mu x$ where μ is a constant. This is equation of S.H.M. in the differential form. The solution of this equation is (see author's Introductory Physics Part I, Ch. 7)

$$x = a\sin(\omega t + \alpha) \text{ where } \omega = \frac{2\pi}{T} = \sqrt{\mu}.$$

Here a is the peak value of x and is called the amplitude of simple harmonic motion and ω is called the cyclic frequency.

The constant α determines the initial 'state of motion' of the particle. The 'state of motion' of a particle is called phase. Thus α determines initial phase of the particle. This is called 'epoch'. The value of epoch depends on the instant from which we reckon time. If we reckon time when the particle is at its mean position then $x = 0$ at $t = 0$.

$$\therefore \quad 0 = a\sin(\omega.0 + \alpha) \quad \text{or} \quad \sin\alpha = 0 \quad \text{or} \quad \alpha = 0.$$

Thus epoch is zero if we reckon time from the instant when the particle passes through its mean position. In this case displacement at any instant is given by $x = a\sin\omega t$.

2. Velocity of S. H. M. : We have, $x = a\sin(\omega t + \alpha)$

and $$v = \frac{dx}{dt}.$$

$\therefore$ $v = a\omega\cos(\omega t+\alpha) = v_0\cos(\omega t+\alpha)$ *where* $v_0 = a\omega$, *maximum velocity of particle in* S. H. M.

When the particle is timed with mean position as explained above,

$$v = a\omega\cos\omega t$$

or

$$v = a\omega\sqrt{1-\frac{x^2}{a^2}}$$

or

$$v = \omega\sqrt{a^2-x^2}.$$

3. Acceleration of S. H. M. : We have, $f = \dfrac{dv}{dt}$

and

$$v = a\omega\cos(\omega t+\alpha)$$

$$\therefore \quad f = a\omega.\{-\omega\sin(\omega t+\alpha)\} = -a\omega^2\sin(\omega t+\alpha).$$

When the particle is timed with mean position,

$f = -a\omega^2\sin\omega t = -f_0\sin\omega t$ where $f_0 = a\omega^2$, maximum value of acceleration of S. H. M. But $x = a\sin(\omega t+\alpha)$.

$$\therefore \quad f = -\omega^2 x.$$

4. Energy of S. H. M. : (i) Kinetic energy of S. H. M. at time t

$$= K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}ma^2\omega^2\cos^2(\omega t+\alpha) = \tfrac{1}{2}m\omega^2(a^2-x^2).$$

(ii) Potential energy at time t = work done in displacing the particle through x.

Now, force on the particle when displacement is z

$$= \text{mass} \times \text{acceleration}$$

$$= m \times (-\omega^2 z)$$

$$= -m\omega^2 z;$$

elementary work done in displacing through $dz = m\omega^2 z dz$.

$\therefore$ Work done in displacing through $x = \displaystyle\int_0^x m\omega^2 z dz$

$$= m\omega^2\left[\frac{z^2}{2}\right]_0^x = \tfrac{1}{2}m\omega^2x^2.$$

$\therefore$ U, potential energy at time $t = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2a^2\sin^2(\omega t+\alpha)$.

$\therefore$ Total energy at time t

$$= \tfrac{1}{2}m\omega^2a^2\cos^2(\omega t+\alpha) + \tfrac{1}{2}m\omega^2a^2\sin^2(\omega t+\alpha)$$

$$= \tfrac{1}{2}ma^2\omega^2.$$

Thus, energy of a particle in S.H.M. $= \frac{1}{2}ma^2\omega^2$ joule (J).

EXAMPLES

Ex. 1. *A particle executes simple harmonic motion of period* 16 *s. Two seconds later after it passes through the centre of oscillation its velocity is found to be* 2 *ms*$^{-1}$. *Find the amplitude.*

Sol. Obviously the particle is timed with the mean position.

Hence we may take, $x=a\sin\omega t$ and $v=a\omega\cos\omega t$.

Here, $t=2$, $v=2$ ms^{-1} and $T=16$ s;

$$\therefore \quad 2=a.\frac{2\pi}{16}\cos\frac{2\pi}{16}.2$$

$$\text{or}\quad 2=\frac{\pi a}{8}\cos\frac{\pi}{4} \quad\text{or}\quad a=\frac{16\sqrt{2}}{\pi}=\frac{16\times 1{\cdot}414}{3{\cdot}142}=7{\cdot}2 \text{ m. } \textbf{Ans.}$$

Ex. 2. *A body executing* S. H. M. *has an amplitude of* 10 *cm and its time period is* 1·5 *s. Calculate the time taken by the body to travel a distance* $5\sqrt{3}$ *cm from its rest position.*

Sol. We have, $x=a\sin\omega t$.

$$\therefore \quad 5\sqrt{3}=10\sin\frac{2\pi}{1{\cdot}5}t$$

$$\text{or}\quad \sin\frac{2\pi t}{1{\cdot}5}=\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}.$$

(*Note* : It is to be noted here that it will be absolutely wrong to write $\frac{\sqrt{3}}{2}$ as sin 60° because angle must be in radian.]

$$\therefore \quad \frac{2\pi t}{1{\cdot}5}=\frac{\pi}{3} \quad\text{or}\quad t=\frac{1{\cdot}5}{6}={\cdot}25 \text{ s. } \textbf{Ans.}$$

Ex 3. *A spring of force constant k and negligible mass is attached to a rigid mass m at one end and clamped to a strong support at the other end. The mass is displaced a little and then released. Show that the body executes simple harmonic motion and calculate its time period. If the spring is cut into three equal pieces and the same body is supported by one such piece in a similar way, how many times time period will be increased or decreased ?*

Sol. Method I. 'Force constant' of a spring is the force called into play per unit change of its length.

Let us suppose that x is the displacement of the body at any time t.

Then force called into play $= kx$. This force is opposite to displacement x. $\therefore$ Force $= -kx$ in the +ve direction.

We have, Force = mass × acceleration

$$\therefore \quad -kx = m \times \frac{d^2x}{dt^2}$$

$$\text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x.$$

$\therefore \quad \frac{d^2x}{dt^2} \propto -x.$ Hence motion is simple harmonic. In simple harmonic motion we have

$$f \text{ (acceleration)} = -\omega^2 \times x \text{ (displacement)}.$$

$$\therefore \quad \text{Here,} \quad \omega^2 = \frac{k}{m}$$

$$\text{or} \quad \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \quad \left(\because \ \omega = \frac{2\pi}{T}\right)$$

$$\text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}; \text{ or } f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

'Force constant' is **inversely proportional** to the length of the spring. Hence, force constant (k') of each piece will be three times the force constant of original spring.

$$\therefore \quad k' = 3k$$

$$\text{and} \quad T' = 2\pi\sqrt{\frac{m}{3k}}$$

$$\therefore \quad T' = \frac{T}{\sqrt{3}}.$$

Hence time period is reduced $\sqrt{3}$ times. **Ans.**

Method II. (*From energy consideration*) : The total energy of the system (spring + body) is

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2.$$

$$\text{Differentiating,} \quad 0 = \frac{1}{2}m2v\frac{dv}{dt} + \frac{1}{2}k2x\frac{dx}{dt}$$

($\because$ E is a constant)

or $$mv\frac{dv}{dt}=kxv \qquad \left(\because \quad v=\frac{dx}{dt}\right)$$

or $$\frac{dv}{dt}=-\frac{k}{m}x$$

or $$\frac{d^2x}{dt^2}=-\frac{k}{m}x \qquad \left(\because \quad \frac{dv}{dt}=\frac{d^2x}{dt^2}\right).$$

Hence motion is simple harmonic and

$$\omega^2=\frac{k}{m}, \quad \text{or} \quad T=2\pi\sqrt{\frac{m}{k}}.$$

Ex. 4. *A uniform U-tube contains a very little mobile liquid to a height of* 50 cm *in each limb. If the liquid in one limb is depressed and then released, find the period of oscillation of the liquid in the tube.*

Sol. Let the depression of the liquid level be x at time t.

Then difference of levels in the two limbs $=2x$.

Pressure due to this difference of levels $=2x\rho g$.

$\therefore$ Thrust on the liquid = area × pressure $=A\times 2x\rho g=2Ax\rho g$.

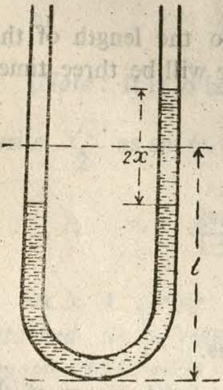


Fig. 10-1

This force acts on the liquid in the direction opposite to displacement.

$$\therefore \quad -2Ax\rho g=(A\times 2l\rho)\frac{d^2x}{dt^2}.$$

$$\therefore \quad \frac{d^2x}{dt^2}=-\frac{g}{l}x$$

or $$\frac{d^2x}{dt^2}\propto -x.$$

Hence motion is S. H. M.

In simple harmonic motion we have

$$f=-\omega^2x.$$

Here, $$\omega^2=g/l.$$

$$\therefore \quad T=2\pi\sqrt{\frac{l}{g}}=2\pi\sqrt{\frac{\cdot 5}{9\cdot 8}}=1\cdot 42 \text{ sec.} \quad \textbf{Ans.}$$

EXERCISES

(A)

1. A particle moves with simple harmonic motion; if the velocities at distances of 4 cm and 5 cm from the centre are 13 cm per second and 5 cm per second respectively, find the period and amplitude.

[*Hint* : Use the formula $v=\omega\sqrt{a^2-x^2}$.] (**Ans.** 1·57 sec, 5·15 cm)

2. A point describes simple harmonic motion in a line 4 cm long. The velocity when passing through the centre of the line is 12 cm per second. Find the period.

[*Hint* : Use the formula $v=\omega\sqrt{a^2-x^2}$.] (**Ans.** 1·047 sec)

3. A particle is moving with simple harmonic motion in a st. line. When the distances of the particle, from the equilibrium positions are x_1 and x_2, the corresponding velocities are u_1 and u_2. Find the period of the motion.

$$\left[\textbf{Ans. } 2\pi\left(\frac{x_2^2-x_1^2}{u_1^2-u_2^2}\right)^{\frac{1}{2}}\right]$$

4. The maximum velocity of a body undergoing S.H.M. is ·04 ms^{-1} and its acceleration at ·02 m from the mean position is ·06 ms^{-2}. Find its amplitude and period of vibration.

[*Hint*. : Use $v_m=a\omega$ and $f=\omega^2x$ numerically.] (**Ans.** $2{\cdot}31\times10^{-2}$ m; 3·63 s)

5. A test tube of weight 5·6 gm and external diameter 2·5 cm is floated vertically in water by placing 10·4 gm of mercury at the bottom of the tube. The tube is depressed by a small amount and then released. Find the time of oscillation. (**Ans.** ·36 s)

6. A metal cylinder of length 15 cm and density 8 gm per c.c. floats upright in mercury of density 13·6 gm per c.c. It is set into vertical oscillations; find the period of oscillation. (**Ans.** ·6 s)

7. A body of mass 10 gm executes S.H.M. with amplitude 2×10^{-2} m and time period 2 s. Calculate the energy of the particle. (**Ans.** $1{\cdot}974\times10^{-5}$ J)

8. A spring balance scale reading from 0 to 15 kg is 10 cm long. A body suspended from the spring is found to oscillate vertically with a frequency of 2 oscillations per second. How much does the body weigh ? (**Ans.** 9·3 kg)

9. When displacement is one-half the amplitude, what fraction of the total energy is kinetic and what fraction is potential ? At what displacement is the energy half kinetic and half potential ?

$$\left(\textbf{Ans. } \tfrac{3}{4};\ \tfrac{1}{4};\ \frac{a}{\sqrt{2}}\right)$$

(B)

10. A particle moves with equation $f+4x=0$, where x is its instantaneous displacement and f instantaneous acceleration. The maximum value of x is 20×10^{-2} m. How much time the particle will take to move from $x=\cdot02$ m to $x=\cdot08$ m? (**Ans.** ·157 s)

11. Assuming atoms of a solid of atomic weight 24 as harmonic oscillators of frequency 3×10^{15} Hz, calculate amplitude of vibration of the atoms if the energy of a kg mole of the solid is $1 \cdot 7 \times 10^4$ joule. (**Ans.** 2×10^{-15} m)

12. The balance wheel of a watch vibrates with an angular amplitude π radians and a period of ·5 s. Find (a) the maximum angular speed of the wheel, (b) the angular speed of the wheel when its displacement is $\frac{\pi}{2}$ radians and (c) the angular acceleration of the wheel when its displacement is $\frac{\pi}{4}$ radians.

[*Hint* : Use formulae of linear S.H.M. for this angular S.H.M. of the wheel.]

(**Ans.** 39·5 radian/sec; 34·2 radian/sec; 124 radian/sec^2)

13. A block is on a piston which is moving vertically with simple harmonic motion of period 1 s. At what amplitude of motion will the block and the piston separate ? If the piston has an amplitude of ·05 m, what is the maximum frequency for which the block and piston will be in contact continuously ?

[*Hint* : The block and piston will separate when acceleration of piston= acceleration due to gravity.] (**Ans.** ·25 m; 2·23)

14. A 3·6 kg block is suspended from a spring of force constant 540 Nm^{-1}. A bullet weighing 45 gm is fired into the block from below with vleocity of 180 ms^{-1} and comes to rest in the block. (a) Find the amplitude of the resulting simple harmonic motion. (b) What fraction of the original kinetic energy of the bullet is stored in the simple harmonic motion ?

[*Hint* : Apply conservation of momentum and conservation of energy to find the upper extreme position. The amplitude of an S.H.M. is the distance between the equilibrium position and one of the extreme positions.]

(**Ans.** ·183 m; ·012)

15. Two springs of force constants k_1 and k_2 are joined together and the combination is attached to a mass 'm' resting on a frictionless table at one end and the other end is clamped to a fixed support. Show that the frequency of oscillation of mass is $f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}$.

[*Hint* : Let the mass be displaced by x and the other end of the second spring by x'. Then $mf = -k_1(x - x')$. Considering the first spring and the mass as a single system we have $mf = -k_2 x'$.]

16. Two springs of force constants k_1 and k_2 are attached to the two opposite sides of a cube of mass 'm' resting on a frictionless table and clamped to two supports at their other end. Show that if the cube is displaced a little it will execute simple harmonic motion of frequency given by $f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$.

17. A point of mass m is suspended at the end of a massless wire of length l and cross-section A. If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line.

[*Hint* : If x is the displacement of mass m from equilibrium position then

$Y=\frac{\text{stress}}{\text{strain}}=\frac{F/A}{x/l}$ or $F=\frac{YAx}{l}$ and this force acts opposite to x. Now proceed as in example 3.]

$\left(\text{Ans. } \frac{1}{2\pi}\sqrt{\frac{YA}{ml}}\right)$

18. Two mass m_1 and m_2 are suspended together by a massless spring of force constant k. When the masses are in equilibrium m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation.

(I. I. T. 1981) $\left(\textbf{Ans. } \sqrt{\frac{k}{m_2}};\ \frac{m_1 g}{k}\right)$

19. An ideal gas is enclosed in a horizontal cylindrical container having a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is p_0 and when the piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from its equilibrium position. Assuming that the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and find the frequency of oscillation.

[*Hint* : Use the relation pV^{γ}=a constant for an ideal gas.]

$\left(\textbf{Ans. } f=\frac{1}{2x}\sqrt{\frac{\gamma p_0 A^2}{MV_0}}\right)$

20. If the mass of a spring 'm' is not negligible but small compared to the mass M suspended from it, show that the period of oscillation is given by $T=2\pi\sqrt{\left(M+\frac{m}{3}\right)\Big/k}$ where k is the force constant of the spring.

[*Hint* : If x is the instantaneous displacement of the lower end, then displacement of an element at a distance z from the fixed end is $\frac{xz}{l}$ and its velocity

$$=\frac{z}{l}\cdot\frac{dx}{dt}.$$

$$\text{The kinetic energy of the spring}=\int_{z=0}^{z=l}\frac{1}{2}\left(\frac{m}{l}\,dz\right)\left(\frac{z}{l}\,\frac{dx}{dt}\right)^2=\frac{1}{6}m\left(\frac{dx}{dt}\right)^2.$$

$$\therefore\ \text{Total energy of the system}=\frac{1}{6}m\left(\frac{dx}{dt}\right)^2+\frac{1}{2}M\left(\frac{dx}{dt}\right)^2+\frac{1}{2}kx^6$$

$$=\text{a constant.}$$

Differentiate and find T.]

21. A cylinder is attached to a light horizontal spring so that it can roll without slipping along the horizontal surface. The mass of the cylinder is m and the force constant of the spring is k. Show that the cylinder executes S.H.M. with period $T=2\pi\sqrt{\frac{3m}{2k}}$.

[*Hint* : $\frac{1}{2}kx^2+\frac{1}{2}m\left(\frac{dx}{dt}\right)^2+\frac{1}{2}I\omega^2$=a constant. Differentiate and find T.]

22. A point is moving in S.H.M. about a fixed point O. Its distance from O at a certain time is 1 cm and 1 second later its distance from O is 5 cm. After yet another second its distance is again 5 cm. Find the time of a complete oscillation.

$$\left(\textbf{Ans.}\ T=\frac{2\pi}{\cos^{-1}\frac{3}{5}}\right)$$

23. A plank rests symmetrically on two cylinders '$2l$' distance apart and rotating with uniform speed in opposite directions. If the plank is displaced a little and then released, show that the motion of the plank is simple harmonic. Calculate the period of oscillations of the plank. The coefficient of friction between the plank and either cylinder is μ. (**Ans.** $T=2\pi\sqrt{l/\mu g}$)

24. A horizontal disc is oscillating in its own plane harmonically with amplitude 'a' and period T. A body placed on the disc is about to slip on it. What is the coefficient of friction between the disc and the body ? (**Ans.** $\mu=4\pi^2a/T^2g$)

25. A body of mass m falls from a height h on to the pan of mass M of a spring balance. The force constant of the spring is k. Having 'm' to the pan the compound body consisting of m and M start vibrating. Find the amplitude of oscillation and its time period.

$$\left(\textbf{Ans..}\ a=\frac{mg}{k}\sqrt{1+\frac{2kh}{(M+m)g}};\ T=2\pi\sqrt{\frac{M+m}{k}}\right)$$

26. A particle executes S.H.M. with period T about a point O. It passes through a point P with velocity V along OP. Show that the time that elapses when it again comes to P is given by

$$t=\frac{T}{\pi}\tan^{-1}\frac{T.V}{2\pi.OP}.$$

27. Two particles are in S.H.M. of same amplitude and frequency along the same line about the same point. If the maximum separation between them is $\sqrt{3}$ times their amplitude, what is the phase difference between them ? $\left(\textbf{Ans.}\ \frac{2\pi}{3}\right)$

□□□

CHAPTER 11

GRAVITATION

1. Newton's law of Gravitation : (i) Material bodies attract each other.

(ii) The force of attraction between two point masses is proportional to the product of the masses and inversely proportional to the square of the distance between them.

Thus, $F=G\,\dfrac{m_1m_2}{d^2}$, where G is a universal constant called Gravitational constant.

Value of G in SI $=6{\cdot}6\times10^{-11}\text{Nm}^2\text{kg}^{-2}$.

2. Relation of g and G : Consider a body of mass 'm' on the surface of the earth.

Force of attraction on the body by the earth $=\dfrac{GmM}{R^2}$,

where $M=$ mass of the earth and $R=$ radius of the earth.

If g is the acceleration due to gravity, then from the formula

force = mass × acceleration, we have $mg=\dfrac{GMm}{R^2}$

or $$g=\frac{GM}{R^2}.$$

3. Gravitational field and potential : (i) The force of attraction experienced by a unit mass at a point is called intensity of gravitational field at that point. Unit of field is newton per kilogramme (Nkg^{-1}).

(ii) The work done in bringing unit mass from infinity up to a point in a gravitational field by an external agent is called gravitational potential of the field at that point. Gravitational potential at a point is essentially negative because work is done by the force itself. Unit of gravitational potential is joule per kilogramme (Jkg^{-1}).

4. Relation of field (g) and potential (V) at a point : The relation between field (g) and potential (V) at a point is

$$g=-\frac{dV}{dx}.$$

Body	Position of point	Potential	Field
1. A point of mass (m)	Anywhere at a distance r from mass	$-\frac{Gm}{r}$	$-\frac{Gm}{r^2}$
2. A spherical shell of mass M and radius a	(i) Point lying outside at a distance 'r' from the centre of the shell	$-\frac{GM}{r}$	$-\frac{GM}{r^2}$
,,	(ii) Point lying inside the shell at a distance r from the centre of the shell	$-\frac{GM}{a}$	Zero
3. A solid sphere of mass M and radius a	(i) Point lying outside at a distance r from the centre	$-\frac{GM}{r}$	$-\frac{GM}{r^2}$
,,	(ii) Point lying inside the sphere at a distance r from its centre	$-\frac{GM}{2a^3}(3a^2-r^2)$	$-\frac{GM}{a^3}r$

5. Velocity of escape : If a body is imparted sufficient velocity so that it can do work against the force of attraction of the earth from its surface right up to infinity, it will never return to the earth. This velocity is called the velocity of escape.

The work done from the surface up to infinity per unit mass is defined as potential of the surface. Hence potential of the surface gives us the value of work to be done per unit mass in carrying it (that unit mass) from the surface up to infinity. Now, potential of the surface of a sphere (considering earth as a sphere) $= -\frac{GM}{R}$.

Hence work to be done per unit mass from the surface up to infinity $= \frac{GM}{R}$.

Let $m =$ mass of the body.

$\therefore$ Work to be done by the body to escape $= \frac{mGM}{R}$.

Let $v =$ velocity of escape,

then kinetic energy of the body $= \frac{1}{2}mv^2$.

For escape we have, $\frac{1}{2}mv^2 = m\frac{GM}{R}$

or $$v = \sqrt{\frac{2GM}{R}}.$$

Since $g = \frac{GM}{R^2}$, $\therefore$ $v = \sqrt{2gR}$.

6. Satellite: Satellite is a body projected horizontally at a certain height above the surface of a planet so that it may go round the planet constantly.

Suppose m = mass of the satellite,

v = velocity with which it is to be projected horizontally at a height 'h' above the surface of the planet,

R = radius of the planet and M = its mass.

Here the centripetal force necessary for circular motion of satellite is provided by gravitational pull i.e. $\frac{GmM}{(R+h)^2}$.

Therefore, from consideration of circular motion, we have

$$\frac{GmM}{(R+h)^2} = \frac{mv^2}{(R+h)}$$

or $$v = \sqrt{\frac{GM}{(R+h)}}.$$

This is called the *orbital velocity* of the satellite.

Since $$v = \omega r,$$

$\therefore$ here $$v = \omega(R+h)$$

or $$\omega = \frac{v}{R+h} \text{ or } T = 2\pi\frac{(R+h)}{v} \quad \left(\because \ \omega = \frac{2\pi}{T}\right)$$

or $$T = 2\pi(R+h)\sqrt{\frac{(R+h)}{GM}}.$$

Since $$g = \frac{GM}{R^2},$$

$\therefore$ $$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}.$$

EXAMPLES

Ex. 1. *Calculate the mass of the sun, given that the distance between the sun and the earth is* $1{\cdot}5\times10^{11}$ *m and* $G=6{\cdot}66\times10^{-11}$ *SI units and take a year = 365 days.*

Sol. Centripetal force = Gravitational pull $=\dfrac{GmM}{d^2}$ where M = mass of the sun, m = mass of the earth and d = distance between the earth and the sun.

But centripetal force $= m\omega^2 d$.

$$\therefore \quad m\omega^2 d=\frac{GmM}{d^2}$$

or
$$M=\frac{\omega^2 d^3}{G}=\frac{\left(\frac{2\pi}{T}\right)^2 d^3}{G}=\frac{4\pi^2 d^3}{T^2 G}.$$

Here T = time of revolution of earth round the sun

$$=365\times24\times3600 \text{ s}$$

$$\therefore \quad M=\frac{4\pi^2(1{\cdot}5\times10^{11})^3}{(365\times24\times3600)^2\times6{\cdot}66\times10^{-11}}=1{\cdot}97\times10^{30} \text{ kg. } \textbf{Ans.}$$

Ex. 2. *Calculate the speed of projection necessary to send a body right out of the field of the earth's gravitational attraction.*

$G=6{\cdot}63\times10^{-11}$ *newton metre*^{2} *kg*$^{-2}$
Mass of the earth $=5{\cdot}97\times10^{24}$ *kg*
Radius of the earth $=6{\cdot}37\times10^{6}$ *m.*

Sol. Work to be done by a body of unit mass to go right out of the gravitational field of the earth

= potential of the surface of the earth

$$=\frac{GM}{R}.$$

Let v be the required velocity.

Then kinetic energy acquired by the body $=\frac{1}{2}.1.v^2=\frac{1}{2}v^2$.

To enable the body to escape we must have

$$\frac{1}{2}v^2=\frac{GM}{R}$$

or
$$v=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2\times6{\cdot}63\times10^{-11}\times5{\cdot}97\times10^{24}}{6{\cdot}37\times10^{6}}}$$

or
$$v=1{\cdot}12\times10^{4} \text{ ms}^{-1}.$$

Ex. 3. *A body is projected vertically upwards from the surface of the earth with an initial velocity v. Show that it will go up to a height h given by* $h=\frac{v^2R}{2gR-v^2}$ *and hence deduce the expression for escape velocity.*

Sol. Let us consider a body of unit mass.

Kinetic energy of the body $=\frac{1}{2}v^2$.

Potential energy of the body at the surface of the earth $=\frac{GM}{R}$.

Potential energy of the body at a height $h=\frac{GM}{R+h}$.

$\therefore$ Difference in potential energy $=GM\left(\frac{1}{R}-\frac{1}{R+h}\right)=\frac{GMh}{R(R+h)}$.

Now, work done = difference in potential energy.

Work is done at the cost of kinetic energy imparted to it.

$$\therefore \qquad \frac{1}{2}v^2=\frac{GMh}{R(R+h)}.$$

$$\text{But} \qquad g=\frac{GM}{R^2}$$

$$\therefore \qquad \frac{1}{2}v^2=\frac{gR^2h}{R(R+h)}, \text{ or } v^2=\frac{2gRh}{(R+h)}$$

$$\text{or} \qquad v^2R+v^2h=2gRh, \text{ or } h=\frac{v^2R}{2gR-v^2}.$$

For h to be infinity, we have $2gR-v^2=0$

$$\text{or} \qquad v=\sqrt{2gR}.$$

Ex. 4. *Calculate acceleration due to gravity on the surface of the earth from the following data.*

$G=6{\cdot}6\times10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

Mean density of the earth $=5\times10^3\ \text{kgm}^{-3}$

Radius of the earth $=6{\cdot}4\times10^6$ m.

Sol. We have, $\qquad g=\frac{GM}{R^2}$.

Taking earth as a perfect sphere,

$$M=\frac{4\pi}{3}R^3\rho$$

$$\therefore \qquad g=\frac{G}{R^2}\cdot\frac{4\pi}{3}R^3\rho=\frac{4\pi}{3}GR\rho$$

$$\therefore \qquad g=\frac{4\pi\times 6{\cdot}6\times 10^{-11}\times 6{\cdot}4\times 10^{6}\times 5\times 10^{3}}{3}$$

$$=4\pi\times 2{\cdot}2\times 6{\cdot}4\times 5\times 10^{-2}=8{\cdot}85\ \text{ms}^{-2}.\ \textbf{Ans.}$$

Ex. 5. *With what horizontal velocity must a satellite be projected at* 150 km *above the surface of the earth so that it will have a circular orbit about the earth ? What will be the period of rotation of the satellite ? Radius of the earth*$=$6400 km *and acceleration due to gravity*$=9{\cdot}8$ ms^{-2}.

Sol. From consideration of circular motion, we have

$$\text{Centripetal force}=\text{Gravitational pull}=\frac{GMm}{(R+h)^2}=\frac{mv^2}{(R+h)}$$

$$\therefore \qquad v=\sqrt{\frac{GM}{R+h}}.\ \text{But } g=\frac{GM}{R^2}.$$

$$\therefore \qquad v=\sqrt{\frac{gR^2}{R+h}}=\sqrt{\frac{9{\cdot}8\times(6{\cdot}4\times 10^6)^2}{(6{\cdot}4\times 10^6+0{\cdot}15\times 10^6)}}$$

$$\text{or} \qquad v=\sqrt{\frac{9{\cdot}8\times 6{\cdot}4^2\times 10^{12}}{6{\cdot}55\times 10^6}}=\sqrt{\frac{9{\cdot}8\times 6{\cdot}4^2\times 16^6}{6{\cdot}55}}$$

$$=7{\cdot}83\times 10^3\ \text{ms}^{-1}.$$

$$\text{Now,} \qquad v=\omega r.$$

$$\therefore \qquad 7{\cdot}83\times 10^3=\frac{2\pi}{T}.\ (6{\cdot}4\times 10^6+0{\cdot}15\times 10^6)$$

$$\therefore \qquad T=\frac{2\pi\times 6{\cdot}55\times 10^6}{7{\cdot}83\times 10^3}=\frac{2\pi\times 6{\cdot}55\times 10^3}{7{\cdot}83}=5{\cdot}255\times 10^3\ \text{s}$$

$$=1\ \text{hour}\ 27\ \text{minutes}\ 35\ \text{s}. \qquad \textbf{Ans.}$$

Ex. 6. *Calculate the binding energy of the earth-sun system. Mass of the earth*$=6\times 10^{24}$ kg, *mass of the sun*$=2\times 10^{30}$ kg *and distance between the earth and the sun*$=1{\cdot}5\times 10^{11}$ m *and gravitational constant* $=6{\cdot}6\times 10^{-11}$ Nm^2 kg^{-2}.

Sol. The work done in bringing two or more than two bodies from infinite separation up to the present configuration is the potential energy of the system. We know that the potential at a point is the work done per unit mass in bringing it from infinity up to the point. Therefore, if there are two point masses m_1 and m_2 separated by a distance r_{12} then potential at the position of m_2 due to $m_1 = -\frac{Gm_1}{r_{12}}$ and potential energy of the two-body system $= -\frac{Gm_1m_2}{r_{12}}$.

Similarly, potential energy of a three-body system

$$= -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_3m_1}{r_{31}}.$$

$\therefore$ Potential energy of the earth-sun system

$$= -\frac{6 \cdot 6 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^{30}}{1 \cdot 5 \times 10^{11}}$$

$$= -5 \cdot 3 \times 10^{33} \text{ joule.}$$

The minus sign indicates that work is done by the gravitational force. It would take an equivalent amount of work by an outside agent to separate these bodies from rest. But the earth is not at rest, rather it is moving round the sun. Because the kinetic energy of the earth in its orbit is half the magnitude of the potential energy of the earth-sun system, only half of this potential energy is needed to break up the system. The energy so needed is called Binding energy of the system.

$\therefore$ Binding energy of the earth-sun system $= \frac{5 \cdot 3}{2} \times 10^{33}$

$$= 2 \cdot 65 \times 10^{33} \text{ joule. } \textbf{Ans.}$$

Ex. 7. *Calculate the pressure inside the earth, caused by gravitational compression at a distance r from its centre. Take M as the mass of the earth and R its radius.*

Sol. Consider the equilibrium of an element of width dr at a distance r and transverse area $\triangle S$.

Let p be the compressional pressure at a distance r. Then,

Outward force on the element $= dp \triangle S$;

Inward gravitational pull on it $= \dfrac{G(\triangle S dr\rho)\left(\frac{4\pi}{3}r^3\rho\right)}{r^2}$, where ρ is the density of the earth.

For equilibrium of the element, we have

$$dp\,\triangle S=-G(\triangle S.dr)\frac{4\pi}{3}r\rho^2$$

or
$$dp=-\frac{4\pi}{3}G\rho^2 r\,dr.$$

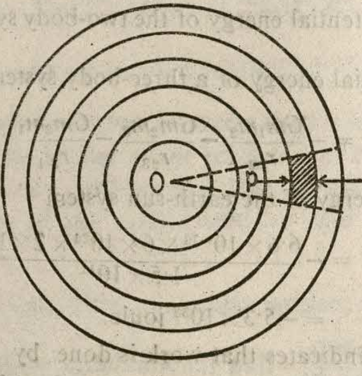


Fig. 11-1

Integrating, we have $p=-\frac{4\pi}{3}G\rho^2\,\frac{r^2}{2}+c.$

At $r=R$, $p=0$. $\therefore\quad 0=-\frac{2\pi}{3}G\rho^2R^2+c$ or $c=\frac{2\pi}{3}G\rho^2R^2$

$$\therefore\quad p=\frac{2\pi}{3}G\rho^2(R^2-r^2)=\frac{2\pi}{3}G\rho^2R^2\left(1-\frac{r^2}{R^2}\right).$$

We have, $M=\frac{4\pi}{3}R^3\rho$

$$\therefore\quad p=\frac{3GM^2}{8\pi R^4}\left(1-\frac{r^2}{R^2}\right).\quad \textbf{Ans.}$$

EXERCISES

(A)

1. Calculate the acceleration due to gravity at the earth's surface given that constant of gravitation $G=6{\cdot}7\times10^{-11}$ SI units, radius of the earth$=6{\cdot}4\times10^6$ m and mean density of earth$=5{\cdot}5\times10^3$ kgm^{-3}. (**Ans.** $9{\cdot}88$ ms^{-2})

2. With what velocity would a shell have to be projected horizontally so that it might describe a circular orbit round the earth ? Acceleration due to gravity $=9{\cdot}81$ ms^{-2} and radius of the earth$=6{\cdot}4\times10^6$ m. (**Ans.** $7{\cdot}9\times10^3$ ms^{-1})

3. With what velocity must a body be projected vertically upwards from the surface of the earth in order that it may never return to the earth ? ($G=6{\cdot}66\times10^{-11}$ SI units; mean density of the earth$=5{\cdot}525\times10^{3}$ kgm^{-3}; radius of the earth $=6{\cdot}38\times10^{6}$ m). (Ans. $1{\cdot}12\times10^{4}$ ms^{-1})

4. Compare the minimum velocity with which a particle would circulate round the earth near its surface with the minimum velocity with which the particle must be projected in order to move it completely beyond the earth's field. (Ans. $1:\sqrt{2}$)

5. Calculate the mean density of the earth given that constant of gravitation $G=6{\cdot}7\times10^{-11}$ SI units, radius of the earth$=6{\cdot}4\times10^{6}$ m and acceleration due to gravity$=9{\cdot}88$ ms^{-2}. (Ans. $5{\cdot}5\times10^{3}$ kgm^{-3})

6. How many hours will make a day if the earth were rotating at such a high speed that weight of a body on the equator were zero ? (radius of the earth $=6{\cdot}4\times10^{6}$ m; acceleration due to gravity$=9{\cdot}8$ ms^{-2})

[*Hint* : Weight at equator$=\dfrac{GmM}{R^2}-m\omega^2R=mg-m\omega^2R$] (Ans. 1·4 hour)

7. The mass of Mars is one-tenth of that of the earth and its diameter is only half of that of the earth. Compare the values of g on their surfaces. (Ans. 2 : 5)

8. Calculate the escape speed from the earth for an atmospheric particle 1000 km above the earth's surface. (radius of the earth$=6400$ km; and acceleration due to gravity$=9{\cdot}8$ ms^{-2}) (Ans. 10^{4} m/s)

9. An 800 kg mass and a 600 kg mass are separated by ·25 m. What is the gravitational field and potential at a point ·2 m from 800 kg and ·15 m from 600 kg ? (Ans. $2{\cdot}2\times10^{-6}$ Nkg^{-1} perpendicular to line joining centres; $-\ 5{\cdot}3\times10^{7}$ joule per kg)

10. A man can jump vertically 1·5 m on the surface of the earth. Calculate the radius of the planet of the same mean density as earth from whose gravitational field he would escape by jumping. (radius of the earth$=6{\cdot}41\times10^{6}$ metre)

[*Hint* : $v=\sqrt{2\times9{\cdot}8\times1{\cdot}5}$; $v_e=\sqrt{\dfrac{2GM}{R}}=\sqrt{\dfrac{2G}{R}\cdot\dfrac{4\pi}{3}R^3\rho}$; using $g=\dfrac{GM_e}{R_e^2}$ and $M_e=\dfrac{4\pi}{3}R_e^3\rho$ we have, $v_e=\sqrt{\dfrac{2gR^2}{R_e}}$.] (Ans. $3{\cdot}1\times10^{3}$ m)

11. A body weighs 20 kg on the surface of the earth. What will be its weight when it is at a height (i) equal to the radius of the earth, (ii) double the radius of the earth ? (Ans. 5 kg; 2·2 kg)

12. Two lead spheres of 20×10^{-2} m and 2×10^{-2} m in diameter are placed with their centres 1 m apart. Calculate the force of attraction between the spheres, given that the radius of the earth is $6{\cdot}37\times10^{6}$ m and its mean density is $5{\cdot}53\times10^{3}$ kgm^{-3}. Density of lead$=11{\cdot}5\times10^{3}$ kgm^{-3} and $g=9{\cdot}8$ ms^{-2}. (Ans. $1{\cdot}54\times10^{-10}$ newton)

13. A projectile is fired vertically from the earth's surface with an initial velocity of 10 km/sec. Neglecting atmospheric retardation, calculate how far above the surface of the earth would it go ? Radius of earth$=6400$ km.

[*Hint* : Kinetic energy acquired$=$Difference in potential energy of projectile when it is on the surface and at the point up to which it rises.] (Ans. $2{\cdot}5\times10^{4}$ km)

14. A short, straight and frictionless tunnel is bored through the centre of the earth and a body is released from the surface into. the tunnel. Show that the motion of the body in the tunnel will be simple harmonic and hence calculate the time taken by the body to travel from one end of the tunnel to the other. (radius of the earth$=6{\cdot}38\times10^6$ m and acceleration due to gravity at the surface $=9{\cdot}81$ ms^{-2}) (**Ans.** 42 minutes 14 seconds)

(B)

15. Two particles of mass m and M are initially at rest at infinite distance. Find their relative velocity of approach due to gravitational attraction when d is their separation at any instant.

[*Hint* : From consideration of conservation of energy,

$$\frac{GmM}{d}=\tfrac{1}{2}mv_1^2+\tfrac{1}{2}Mv_2^2.$$

For consideration of conservation of momentum,

$$mv_1-Mv_2=0.$$

Relative velocity of approach$=v_1+v_2$] $\left(\textbf{Ans. } \sqrt{\frac{2G(m+M)}{d}}\right)$

16. Three identical bodies of mass m are located at the vertices of an equilateral triangle with side l. At what speed must they move if they all revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle while still preserving the equilateral triangle ? $\left(\textbf{Ans. } \sqrt{\frac{Gm}{l}}\right)$

17. An iron ball of radius 1 m and density 8000 kgm^{-3} is placed in water. A bubble of radius 1 cm is at a distance 1·5 m from the centre of the ball. Will there be force of attraction or repulsion between them and by how much ? (Neglect the mass of air and take $G=6{\cdot}67\times10^{-11}$.)

[*Hint* : See Fig. 11-1. The element, if devoid of mass, will experience force of repulsion due to gravitational compressional pressure. Thus a bubble will experience force of repulsion, that is, a bubble will behave like 'a negative mass' equal to the mass of water displaced by it. $F=\frac{Gm_1m_2}{d^2}$ where m_1=mass of water displaced by the bubble and m_2=mass of the iron ball minus the mass of water displaced by it.] (**Ans.** $3{\cdot}7\times10^{-9}$ newton)

18. A particle is fired vertically from the surface of the earth with a velocity kv_e, where v_e is the escape velocity and $k<1$. Neglecting air resistance, calculate the height to which it will rise from the surface of the earth.

(**Ans.** $\frac{k^2}{1-k^2}$ times radius of the earth)

19. Calculate the pressure caused by gravitational compression at the centre of the earth. Express the result in terms of standard atmospheric pressure. Mass of the earth$=6{\cdot}0\times10^{24}$ kg; radius of the earth$=6{\cdot}4\times10^6$ m and g (at the surface of the earth)$=9{\cdot}8$ ms^{-2}. 1 standard atmospheric pressure$=10^5$ Nm^{-2}.

(**Ans.** $1{\cdot}7\times10^6$ atm.)

20. A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are equal to r_1 and r_2 respectively. Find the angular momentum L of this planet relative to the centre of the sun.

[*Hint* : Remember that at the maximum and minimum distances velocity is perpendicular to position vectors of the planet. Apply the principles of conservation of angular momentum and energy.] $\left(\text{Ans. } L=m\sqrt{\frac{2GMr_1r_2}{r_1+r_2}}\right)$

21. A pair of stars rotates about their centre of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centres are at distance d apart d being large compared to the size of either star. (a) Find their period of revolutions in terms of d, m and G. (b) Compare their kinetic energies and angular momenta about their common centre.

(Ans. $T=2\pi\sqrt{\frac{d^3}{3Gm}}$; 1 : 2; 1 : 2)

22. Two satellites A and B of equal masses, move in the equatorial plane of the earth, close to earth's surface. Satellite A moves in the same direction as that of the earth, while satellite B moves in the opposite direction. Find the ratio of the kinetic energy of B to that of A in the reference frame fixed to the earth. ($g=9{\cdot}8$ ms^{-2}, radius of the earth$=6{\cdot}37\times10^6$ m.) (Ans. 1·266 : 1)

* * *

CHAPTER 12

SIMPLE AND COMPOUND PENDULUM

1. **Simple Pendulum** : When a heavy particle is suspended by an *inextensible, weightless* and *perfectly flexible* string from an *unyielding support*, it is called a Simple Pendulum. In practice a body is suspended to set up a simple pendulum.

The distance from the centre of gravity of the body suspended up to the point of suspension is called the effective length of the simple pendulum. For small amplitude of vibration, the motion of simple pendulum is simple harmonic and its time period is given by

$t=2\pi\sqrt{\frac{l}{g}}$ where t = time period, l = effective length and g = acceleration due to gravity.

2. **Compound Pendulum** : Any rigid body capable of oscillating freely about an unyielding horizontal axis passing through the body is called a Compound Pendulum.

The time period of compound pendulum is given by

$t=2\pi\sqrt{\frac{k^2+l^2}{lg}}$ where l = distance of the centre gravity of the pendulum from the centre of suspension, k = radius of gyration of the pendulum about the axis passing through its centre of gravity and parallel to the axis of oscillation.

3. **Equivalent Simple Pendulum** : The simple pendulum which has the same time period as the given compound pendulum is called Equivalent Simple Pendulum. The length of equivalent simple pendulum is $L=(k^2/l+l)$.

4. **Interchangeability of centre of suspension and centre of oscillation** : A point on the line joining the centre of gravity with centre of suspension at a distance $k^2/l+l$ from centre of suspension and at a distance k^2/l from centre of gravity is called Centre of Oscillation.

The time period of a compound pendulum is the same about its centre of oscillation and centre of suspension. There are four points on a line through C. G. about which it has the same time period. Two points are on one side of C. G. at distances l and k^2/l and the other two on the other side of it at the same distances.

EXAMPLES

Simple Pendulum :

Ex. 1. *A pendulum which keeps correct time at a place where* $g = 9{\cdot}81\ ms^{-2}$ *is taken to a place where* $g = 9{\cdot}8\ ms^{-2}$. *Calculate by how much it will lose or gain in a day.*

Sol. A pendulum keeps correct time when its time period is exactly 2 sec.

$$\therefore \qquad 2 = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{9{\cdot}81}} \qquad \ldots \text{(i).}$$

When g decreases, t increases and hence it will lose at the second place. Let it lose by x seconds in a day. There are 86400 seconds in a day. A correct pendulum will make exactly 86400 beats (vibration) in a day. Since our pendulum loses x seconds in a day it will make $(86400 - x)$ beats in a day instead of 86400.

$\therefore$ Its time period at the second place $= 2 \times \dfrac{86400}{86400 - x}$.

$$\therefore \qquad 2 \times \frac{86400}{86400 - x} = 2\pi\sqrt{\frac{l}{9{\cdot}8}} \qquad \ldots \text{(ii).}$$

Dividing (i) by (ii), $\dfrac{86400 - x}{86400} = \sqrt{\dfrac{9{\cdot}8}{9{\cdot}81}}$

$$\text{or} \qquad \left(1 - \frac{x}{86400}\right) = \left(\frac{9{\cdot}81 - {\cdot}01}{9{\cdot}81}\right)^{\frac{1}{2}} = \left(1 - \frac{{\cdot}01}{9{\cdot}11}\right)^{\frac{1}{2}}$$

or $\quad 1 - \dfrac{x}{86400} = 1 - \dfrac{1}{2} \cdot \dfrac{{\cdot}01}{9{\cdot}81}$, neglecting other terms of the expansion

$$\text{or} \qquad x = \frac{86400 \times {\cdot}01}{2 \times 9{\cdot}81}$$

$$\text{or} \qquad x = 44\ \text{s.} \quad \textbf{Ans.}$$

Ex. 2. *A simple pendulum* $2\ m$ *long is arranged in an elevator. What will be its time period when* (a) *elevator is moving up with uniform velocity* 2 *m/s*, (b) *elevator is moving up with uniform acceleration* $2\ ms^{-2}$, (c) *elevator is going down with uniform acceleration* $2\ ms^{-2}$?

Sol. When the elevator is going up with acceleration f the acceleration relative to the ground is $(g+f)$.

$$\therefore \quad t=2\pi\sqrt{\frac{l}{g+f}}.$$

(a) When velocity is uniform, $f=0$.

$$\therefore \quad t=2\pi\sqrt{\frac{2}{9{\cdot}8}}=2{\cdot}84 \text{ s}.$$

(b) When elevator is going up with acceleration 2 ms^{-2},

$$f=2 \text{ ms}^{-2}.$$

$$\therefore \quad t=2\pi\sqrt{\frac{2}{9{\cdot}8+2}}=2\pi\sqrt{\frac{2}{11{\cdot}8}}=2{\cdot}59 \text{ s}.$$

(c) When elevator is going down with acceleration 2 ms^{-2},

$$f=-2 \text{ ms}^{-2}.$$

$$\therefore \quad t=2\pi\sqrt{\frac{2}{9{\cdot}8-2}}=2\pi\sqrt{\frac{2}{7{\cdot}8}}=3{\cdot}18 \text{ s. } \textbf{Ans.}$$

Ex. 3. *A simple pendulum swings with amplitude of* 5 *cm and time period* 2 *s. Calculate its maximum velocity, maximum acceleration and velocity when it is* 2 *cm away from its mean position.*

Sol. The motion of a simple pendulum is simple harmonic and hence to calculate its displacement, velocity and acceleration we may use formulae of S. H. M.

Maximum velocity of S. H. M. $=a\omega$.

$\therefore$ Maximum velocity of pendulum $= {\cdot}05\times\frac{2\pi}{2}={\cdot}16 \text{ ms}^{-1}$.

Maximum acceleration of S. H. M. $=a\omega^2={\cdot}05\left(\frac{2\pi}{2}\right)^2=4{\cdot}94 \text{ ms}^{-2}$.

Velocity of S. H. M. at a distance $x=\omega\sqrt{a^2-x^2}$.

$\therefore$ Velocity of pendulum at a distance 2 cm $=\frac{2\pi}{2}\sqrt{{\cdot}05^2-{\cdot}02^2}$

$$={\cdot}14 \text{ ms}^{-1}.$$

Ex. 4. *A simple pendulum loses* 50 *seconds in a day when taken to the bottom of a mine. Find the depth of the mine.*

(*Radius of the earth*$=$6400 *km*)

Sol. We have on the surface of the earth

$$2=2\pi\sqrt{\frac{l}{g}};\quad \text{or}\quad 1=\pi\sqrt{\frac{l}{g}}$$

and at the bottom of mine $2\times\dfrac{86400}{86400-50}=2\pi\sqrt{\dfrac{l}{g'}}$ where g' is the acceleration due to gravity at the bottom of mine.

Dividing the two, we have $\dfrac{86400-50}{86400}=\sqrt{\dfrac{g'}{g}}$.

Now we will calculate g and g' from consideration of gravitation.

$mg=\dfrac{GMm}{R^2}$ (G = gravitational constant, M = mass of the earth and R = radius of the earth)

or $mg=\dfrac{G}{R^2}\cdot\dfrac{4\pi}{3}R^3\rho=\dfrac{4\pi}{3}GR\rho$, where ρ = density of the earth

and similarly, $mg'=\dfrac{4\pi}{3}G(R-h)\rho$ ($\because$ only the inner mass is effective in producing attraction towards the centre).

Dividing, $\therefore\ \dfrac{g}{g'}=\dfrac{R}{R-h}$.

$$\therefore\ \frac{86400-50}{86400}=\sqrt{\frac{R-h}{R}}$$

or $1-\dfrac{50}{86400}=\left(1-\dfrac{h}{R}\right)^{\frac{1}{2}}=1-\dfrac{1}{2}\cdot\dfrac{h}{R}$, neglecting other terms in the expansion

or $h=\dfrac{2\times 50}{86400}\times R=\dfrac{100}{86400}\times 6400$ km

or $h=7{\cdot}4$ km. **Ans.**

Compound Pendulum :

Ex. 5. *The period of a disc of radius* 10 *cm executing small oscillations about a pivot at its rim is measured to be* ·784 *s. Find the value of* g, *the acceleration due to gravity.*

Sol. For a disc, I_0 (m.i. about its axis) $= \frac{1}{2} Mr^2 = Mk^2$.

$$\therefore \quad k^2 = r^2/2.$$

l = distance of C. G. from point of suspension (pivot) $= r$, radius of the disc;

$$\therefore \quad L = \frac{k^2}{l} + l = \frac{r^2/2}{r} + r = \frac{r}{2} + r = \frac{3}{2} r,$$

$$\therefore \quad t = 2\pi \sqrt{\frac{k^2/l + l}{g}} = 2\pi \sqrt{\frac{3r}{2g}}$$

or $$g = \frac{6\pi^2 r}{t^2} = \frac{6\pi^2 \times {\cdot}1}{{\cdot}784^2} = 9{\cdot}64 \text{ ms}^{-2}. \quad \textbf{Ans.}$$

Ex. 6. *A body of mass* ·2 *kg oscillates about a horizontal axis at a distance* 20 *cm from its ceantre of gravity. If the length of the equivalent simple pendulum be* ·35 *m, find its moment of inertia about the axis of suspension.*

Sol. L (length of equivalent simple pendulum) $= l + k^2/l$.

$$\therefore \quad {\cdot}35 = {\cdot}2 + k^2/{\cdot}2 \text{ or } k^2 = {\cdot}03 \text{ m}^2.$$

Now, $$I = I_0 + ml^2 = mk^2 + ml^2 = {\cdot}2({\cdot}03 + {\cdot}2^2)$$

or $$I = {\cdot}2 \times {\cdot}07 = {\cdot}014 \text{ kgm}^2. \quad \textbf{Ans.}$$

Ex. 7. *Show that the time period of a compound pendulum is minimum when it is suspended from a point at a distance equal to its radius of gyration.*

Sol. We have for the time period of a compound pendulum,

$$t = 2\pi \sqrt{\frac{k^2/l + l}{g}}.$$

Obviously t is minimum when $\frac{k^2}{l} + l$ is minimum.

Now, $$\frac{k^2}{l} + l = \left(\frac{k}{\sqrt{l}}\right)^2 + (\sqrt{l})^2 = \left(\frac{k}{\sqrt{l}} - \sqrt{l}\right)^2 + 2\frac{k}{\sqrt{l}} . \sqrt{l}$$

or $$\frac{k^2}{l} + l = \left(\frac{k}{\sqrt{l}} - \sqrt{l}\right)^2 + 2k$$

$\therefore \quad \frac{k^2}{l} + l$ is minimum when $\left(\frac{k}{\sqrt{l}} - \sqrt{l}\right)^2$ is minimum.

This is a positive quantity, its minimum value is zero.

Hence t is minimum when $\frac{k}{\sqrt{l}}-\sqrt{l}=0$ or $k=l$. **Proved.**

Ex. 8. *A circular hoop of radius 60 cm and weight 4 kg is suspended on a horizontal nail at its circumference.* (a) *What is its frequency of oscillation for small displacements from equilibrium ?* (b) *What is the length of equivalent simple pendulum ?*

Sol. We have for a hoop, $I_0=mr^2=mk^2$;

$\therefore\quad k^2=r^2$ and $l=r$ (given).

$$\therefore\quad t=2\pi\sqrt{\frac{k^2+l^2}{lg}}=2\pi\sqrt{\frac{r^2+r^2}{rg}}=2\pi\sqrt{\frac{2r}{g}}.$$

Comparing it with simple pendulum formula, $t=2\pi\sqrt{\frac{L}{g}}$.

We have, $L=2r=2\times{\cdot}6=1{\cdot}2$ m. **Ans.**

And $t=2\pi\sqrt{\frac{1{\cdot}2}{9{\cdot}8}}$.

$\therefore$ Frequency $=\frac{1}{t}=\frac{1}{2\pi}\sqrt{\frac{9{\cdot}8}{1{\cdot}2}}={\cdot}45\ \text{s}^{-1}$. **Ans.**

EXERCISES

Simple Pendulum :

(A)

1. Calculate the length of a second pendulum at a place where $g=9{\cdot}8$ ms^{-2}. (Ans. ·993 m)

[*Hint* : Second pendulum is that pendulum whose time period is exactly 2 seconds.]

2. A hollow light cylinder of length 10 cm is suspended by a string of length 95 cm. Calculate the time period of this simple pendulum (i) when the cylinder is completely filled with mercury, (ii) when half filled with mercury. ($g=9{\cdot}81$ ms^{-2}) [Ans. (i) 2·006 s, (ii) 2·031 s]

3. A faulty seconds pendulum loses 9 seconds per day. Find the required alteration in length so that it may keep correct time. (Ans. $\cdot02\times10^{-2}$ m)

4. A simple pendulum which beats a second on the surface of the earth where $g=9{\cdot}8$ ms^{-2} is taken to the top of a hill where it is found to lose 2 sec in an hour. What is the height of the hill ? (radius of the earth $=6{\cdot}4\times10^6$ m) (Ans. 3·55 km)

5. A simple pendulum beats seconds on the earth ($g=9{\cdot}8$ ms^{-2}) is taken on the moon where $g=3{\cdot}9$ ms^{-2}; find the number of seconds it will lose per day. (Ans. 31893 seconds)

6. The bob of a simple pendulum is of mass 50 gm and radius 1·5 cm. It is suspended by a string of length 98·5 cm. Calculate its maximum kinetic energy if it is drawn to one side by 5 cm and then released. (Ans. $6 \cdot 1 \times 10^{-4}$ joule)

7. A simple pendulum loses 50 seconds when taken to the top of a hill. Find the height of the hill. Radius of the earth=6400 km. (Ans. 3·7 km)

(B)

8. Two simple pendulums of lengths 1 m and 1·1 m respectively start swinging together with the same amplitude. Find the number of swings that will be executed by the longer pendulum before they will again swing together.

[*Hint* : Longer pendulum will go slow and shorter will go fast. After starting together they will again begin together when faster one gains over the first by complete number of oscillations. Let n be the number of oscillations the longer one makes when they again begin together.

$\therefore$ $(n+x)\ T_1=nT_2$, where x is the number of oscillations by which the first gains over the second.] (Ans. 41)

9. A simple pendulum of period 2 s is suspended from the ceiling of a car at rest. The car moves horizontally with acceleration 4·9 ms^{-2}. Find the period of oscillation of the pendulum. (Ans. 1·891 s)

[*Hint* : Bring the car to rest by applying inertia force. Consider motion of pendulum in this condition. $T=2\pi\sqrt{\dfrac{l}{\sqrt{a^2+g^2}}}$.]

10. A simple pendulum of length l is suspended from the ceiling of a moving lift. Find the time period of the pendulum when the lift is going up with acceleration f.

[*Hint* : Bring the pendulum to rest by applying inertia force (D'Alembert's Principle) and consider motion of the pendulum in this condition.]

$$\left(\text{Ans. } T=2\pi\sqrt{\frac{l}{g+f}}\right)$$

11. Show that the period of a simple pendulum is given by $T=2\pi\sqrt{\dfrac{m}{m'}\cdot\dfrac{l}{g}}$ where m is the 'inertial mass' and m' is the 'gravitational mass' of the bob.

12. Show that the time period of a simple pendulum of infinite length is given by $t=2\pi\sqrt{\dfrac{R}{g}}$ where R is the radius of the earth.

Compound Pendulum :

(A)

1. A body of mass ·42 kg oscillates about a horizontal axis distant ·25 m from its C.G. If the length of equivalent simple pendulum be ·35 m, find its moment of inertia about its axis of suspension and period of oscillation. (g=9·8 ms^{-2}) $\left(\textit{Hint}:\ L=\dfrac{k^2}{l}+l;\ I=mk^2+ml^2.\right)$

(Ans. ·03675 kgm^2; 1·1874 s)

2. A uniform square lamina of side ·3 m oscillates in a vertical plane about an axis perpendicular to the lamina. Find the minimum periodic time of oscillation. What is the locus of points about which periodic time is minimum?

[*Hint* : See Ex. 7.]

(**Ans.** ·993 s; a circle of radius $12\cdot25\times10^{-2}$ m about C.G.)

3. Calculate the time period of oscillations of a uniform rectangular plate of length 96 cm and breadth 10 cm when it is oscillating freely about one of its corners. ($g=9\cdot8\ \text{ms}^{-2}$) (**Ans.** 1·61 sec)

4. A thin uniform rod of length 120 cm is made to oscillate about an axis passing through its end. Find the period and other points about which it has the same time period. ($g=9\cdot8\ \text{ms}^{-2}$)

(**Ans.** 1·79 s, other end and 20 cm on either side of C.G.)

5. A uniform square lamina of side 10 cm is hung up by a corner and swings in its own plane. Find the length of equivalent simple pendulum.

(**Ans.** $9\cdot47\times10^{-2}$ m)

6. A disc of metal of radius '*r*' with its plane vertical can be made to swing about a horizontal axis passing through any one of a series of holes bored along a diameter. Show that the minimum period of oscillation is given by

$$t=2\pi\sqrt{\frac{1\cdot414r}{g}}.$$

7. A uniform circular disc of 25 cm radius oscillates in its own plane about point on its circumference. Calculate the time of oscillation. ($g=9\cdot8\ \text{ms}^{-2}$)

(**Ans.** 1·23 s)

(B)

8. In a Kater's pendulum the time periods about the knife edges at distances l_1 and l_2 from the centre of gravity are t and $t+\tau$ where τ is very small. Show that

$$\frac{4\pi^2(l_1+l_2)}{g}=t\left(t+\frac{2l_2\tau}{l_2-l_1}\right).$$

9. A Kater's pendulum of mass 5 kg gives an equal oscillation period of 2·004 s about knife edges at distances 10 cm and 89·4 cm on either side of the centre of gravity. Calculate the value of *g* and the moment of inertia of the pendulum about centre of gravity.

[*Hint* : Length of equivalent simple pendulum = 89·4 + 10 = 99·4 cm]

(**Ans.** $9\cdot779\ \text{ms}^{-2}$; $\cdot447\ \text{kgm}^2$)

10. A compound pendulum consists of a rigid light rod having two spherical bobs of masses m_1 and m_2 at distances l_1 and l_2 from the centre of suspension. Find the time period of oscillations of the pendulum.

$$\left(\text{Ans. } T=2\pi\sqrt{\frac{m_1l_1^2+m_2l_2^2}{m_1l_1+m_2l_2)g}}\right)$$

□□□

CHAPTER 13

ELASTICITY

1. Elasticity : When a force is applied on a fixed body, the effect of force is to change its length, volume or shape. If the force is removed the body regains its original length, volume or shape unless it is over-strained by the external force. This property of regaining original shape, length or volume after removal of straining force is called elasticity.

2. Stress and Strain : When a fixed rigid body is subjected to some external force at once a force is called into play within the body which tends to oppose the change of shape or size of the body. The internal force per unit area is called Stress. Its unit is newton per square metre (Nm^{-2}).

The ratio of change of shape or size to the original shape or size is called Strain. There are three strains :

(i) Longitudinal Strain $= \dfrac{\text{Change in length } (\triangle l)}{\text{Original length } (l)}$.

(ii) Volume Strain $= \dfrac{\text{Change in volume } (v)}{\text{Original volume } (V)}$.

(iii) Shear Strain = angle (θ) by which a line perpendicular to the fixed end is turned.

3. Hooke's law : Within elastic limit stress is proportional to strain i.e. $\dfrac{\text{Stress}}{\text{Strain}}$ = a constant called modulus of elasticity.

4. Elastic constants :

(i) Young's modulus $(Y) = \dfrac{\text{Tensile Stress}}{\text{Tensile Strain}}$.

(ii) Bulk modulus $(K) = \dfrac{\text{Volume Stress}}{\text{Volume Strain}}$.

(iii) Modulus of rigidity $(\eta) = \dfrac{\text{Shear Stress}}{\text{Shear Strain}}$.

(iv) Poisson's ratio $(\sigma) = \dfrac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$.

5. Relation of elastic constants :

(i) $Y=2\eta(1+\sigma)$, (ii) $Y=3K(1-2\sigma)$.

6. Work done in stretching a wire : Suppose a wire of length L is stretched through l by a force F. Let f be the internal force when stretching is x.

Then, $Y=\dfrac{f/A(\text{stress})}{x/L(\text{strain})}=\dfrac{fL}{Ax}$ or $f=\dfrac{YA}{L}x$ where A = area of cross-section of wire.

Elementary work done in stretching it further through $dx=fdx$

$$=\frac{YA}{L}xdx.$$

$\therefore$ Work done in stretching from $x=0$ to $x=l$ i.e. through l

$$=\int_0^l \frac{YA}{L}xdx=\frac{YA}{L}\left[\frac{x^2}{2}\right]_0^l=\frac{1}{2}\,\frac{YA}{L}l^2.$$

Now, when $x=l$, $f=F$,

$$\therefore\quad Y=\frac{FL}{Al}.$$

$\therefore$ Work done in stretching through $l=\frac{1}{2}\left(\dfrac{YAl}{L}\right)l=\frac{1}{2}Fl$

i.e. $\frac{1}{2}$ × stretching force × stretching.

Strain potential energy of a wire $=\frac{1}{2}\dfrac{YAl^2}{L}$ joule or $\frac{1}{2}Fl$ joule.

u (strain potential energy density) $=\dfrac{\frac{1}{2}Fl}{AL}=\frac{1}{2}\times\text{stress}\times\text{strain}$

$$\left(\because\ \frac{F}{A}=\text{stress and }\frac{l}{L}=\text{strain}\right).$$

7. Strain produced by thermal expansion : When a rod or wire of length l suffers change in temperature by $\triangle T$ the change in length is $l\alpha\triangle T$ where α is the coefficient of linear expansion $[\because\ l_t=l_0(1+\alpha t)]$.

$\therefore$ Strain produced by thermal expansion or contraction

$$=\alpha\triangle T.$$

EXAMPLES

Ex. 1. *A wire of length* 3 m *and diameter of* ·3 mm *is stretched by hanging a weight of* 2 kg *and the elongation produced is* 2 mm. *Calculate Young's modulus and strain potential energy of the wire.*

Sol. $$\text{Strain}=\frac{2\times10^{-3}\text{m}}{3\text{m}}=\frac{2}{3}\times10^{-3},$$

$$\text{Stress}=\frac{2\times9\cdot8}{\pi(\cdot15\times10^{-3})^2}\ \text{Nm}^{-2}$$

$$\therefore\quad Y=\frac{2\times9\cdot8}{\pi(\cdot15\times10^{-3})^2}\Big/\frac{2}{3}\times10^{-3}=\frac{2\times3\times9\cdot8}{2\pi\times\cdot15^2}\times10^9$$

$$=\frac{3\times9\cdot8}{\pi\times\cdot0225}\times10^9,\quad\text{or}\quad Y=4\cdot16\times10^{11}\ \text{Nm}^{-2}.\quad \textbf{Ans.}$$

$$\text{Strain potential energy}=\tfrac{1}{2}\times(2\times9\cdot8)\times(2\times10^{-3})$$
$$=19\cdot6\times10^{-3}\ \text{joule.}\quad \textbf{Ans.}$$

Ex. 2. *A steel wire* 2 mm *in diameter is just stretched between two fixed points at a temperature of* 20°C. *Determine its tension when its temperature falls to* 10°C. *Linear expansivity of steel* $=11\times10^{-6}/\text{K}$, *Young's modulus* $=2\times10^{11}\ \text{Nm}^{-2}$.

Sol. We have, $l_2=l_1\{1+\alpha(t_2-t_1)\}$

or $l_2-l_1=l_1\,\alpha(t_2-t_1)$, or $\dfrac{l_2-l_1}{l_1}=\alpha(t_2-t_1)$.

Longitudinal strain $=\dfrac{l_2-l_1}{l_1}=\alpha(t_2-t_1)$.

Now, Y (Young's modulus) $=\dfrac{\text{stress}}{\text{strain}}$.

$\therefore$ Stress $=Y\times$ strain $=Y\,\alpha(t_2-t_1)$.

Tension $=$ stress $\times$ area $=Y\,\alpha(t_2-t_1)\times\pi r^2$.

$$\therefore\quad \text{Tension}=2\times10^{11}\times11\times10^{-6}(20-10)\times\pi.(1\times10^{-3})^2$$
$$=22\pi=69\cdot12\ \text{newton.}\quad \textbf{Ans.}$$

Ex. 3. *Find the formula for the work done in stretching a wire and apply it to find the elastic energy stored in the wire originally* 5 *metre long and* 1 mm *in diameter which has been stretched by* ·3 mm *due to a load of* 10 kg. *Also calculate Young's modulus of the wire.* $(g=9\cdot8\ \text{ms}^{-2})$

Sol. Formula is already deduced.

Work done $= \frac{1}{2} \times$ stretching force $\times$ stretching

$$= \frac{1}{2} \times (10 \times 9{\cdot}8) \times ({\cdot}3 \times 10^{-3}) = {\cdot}0147 \text{ joule.} \quad \textbf{Ans.}$$

$$\text{Stress} = \frac{10 \times 9{\cdot}8}{\pi({\cdot}5 \times 10^{-3})^2} = \frac{9{\cdot}8 \times 10^7}{{\cdot}25\pi} \text{Nm}^{-2}.$$

$$\text{Strain} = \frac{{\cdot}3 \times 10^{-3}}{5} = \frac{3}{5} \times 10^{-4} = {\cdot}6 \times 10^{-4}.$$

$$\therefore \quad Y = \frac{9{\cdot}8 \times 10^7/{\cdot}25\pi}{{\cdot}6 \times 10^{-4}} = 2{\cdot}08 \times 10^{12} \text{ Nm}^{-2}. \quad \textbf{Ans.}$$

Ex. 4. *When a rubber cord is stretched the change in volume to change in its linear dimensions is negligible. Calculate Poisson's ratio for rubber.*

Sol. Suppose l = length of rubber cord, r = radius of cord, When stretched let l increase by $\triangle l$ and r decrease by $\triangle r$.

$$\therefore \quad \text{Longitudinal strain} = \frac{\triangle l}{l} \text{ and lateral strain} = \frac{\triangle r}{r}.$$

$$\therefore \quad \sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\triangle r/r}{\triangle l/l} = \frac{\triangle r \times l}{\triangle l \times r}.$$

Now, $\pi r^2 l = \text{volume} = \pi(r - \triangle r)^2(l + \triangle l)$

or $\pi r^2 l = \pi(r^2 - 2r\triangle r)(l + \triangle l)$, neglecting $\triangle r^2$

or $r^2 l = (r^2 l - 2r\triangle r l + r^2 \triangle l)$, neglecting $\triangle r . \triangle l$

or $2r\triangle r l = r^2 \triangle l$

or $\dfrac{\triangle r \times l}{\triangle l \times r} = \dfrac{1}{2}$; $\therefore \quad \sigma = \dfrac{1}{2} = {\cdot}5$. **Ans.**

Ex. 5. *A load of* 7·6 kg *hangs from the lower end of a steel wire which is rigidly clamped at the upper end. When the load is immersed in water, the length of wire changes by* 1 mm. *Calculate the length of the wire.* (*Young's modulus of steel* $= 2 \times 10^{11}$ Nm^{-2}; *diameter of the wire* = ·4 mm; *density of material of load* = 7600 kgm^{-3}, *and* $g = 9{\cdot}8$ ms^{-2}.)

Sol. We have, $Y = \dfrac{Mg/\pi r^2}{l/L} = \dfrac{MgL}{\pi r^2 l}$, or $M = \dfrac{Y\pi r^2}{gL} l$

$$\therefore \quad M \propto l, \quad \text{or} \quad M = kl \text{ where } k = \frac{\pi r^2 Y}{gL}.$$

Let M' = mass of load in water.

Then $$M'=kl'.$$

$$\text{Density of load}=\frac{\text{mass of load}}{\text{volume of load}}=\frac{1000M}{M-M'}$$

($\because$ density of water = 1000 kgm^{-3})

$$\therefore \quad 7600=\frac{1000\,kl}{kl-kl'}, \quad \text{or} \quad 7{\cdot}6=\frac{l}{l-l'}.$$

According to question, $l-l'=1\times10^{-3}$ m.

$$\therefore \qquad l=7{\cdot}6\times10^{-3}\text{ m.}$$

$$\therefore \qquad 7{\cdot}6=k.7{\cdot}6\times10^{-3} \text{ or } k=10^{3}$$

or $$10^{3}=\frac{\pi({\cdot}2\times10^{-3})^{2}\times2\times10^{11}}{9{\cdot}8L}$$

or $$L=\frac{\pi\times{\cdot}04\times2\times10^{2}}{9{\cdot}8}=2{\cdot}565\text{ m.}$$ **Ans.**

Ex. 6. *A steel wire of length* 1 m *and diameter* 4 mm *is stretched horizontally between two rigid supports attached to its ends. What load hung from the mid-point of the wire would be required to produce a depression of* 1 cm ? ($Y=2\times10^{11}$ Nm^{-2})

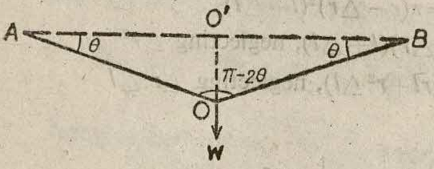


Fig. 13-1.

Sol. Suppose $2l$ = length of the wire,

x = depression produced $=OO'$,

T = tension of the wire,

W = load suspended.

$$\text{Stretched length}=2.OA=2(x^{2}+l^{2})^{\frac{1}{2}}$$

$$=2l\left(1+\frac{x^{2}}{l^{2}}\right)^{\frac{1}{2}}=2l\left(1+\tfrac{1}{2}\frac{x^{2}}{l^{2}}\right)=2l+\frac{x^{2}}{l} \qquad [\because x^{2}<<l^{2}].$$

$$\therefore \quad \text{Increase in length}=2l+\frac{x^{2}}{l}-2l=\frac{x^{2}}{l}.$$

$$\text{Strain}=\frac{x^{2}/l}{2l}=\frac{x^{2}}{2l^{2}}.$$

For equilibrium of the point O, we have $W=2T\sin\theta$

or $W = 2T\theta$ (since θ is small)

$$= 2T\frac{x}{l} \quad \text{or} \quad T = \frac{Wl}{2x}.$$

$$\therefore \quad \text{Stress} = \frac{T}{\pi r^2} = \frac{Wl}{2\pi r^2 x}.$$

$$\therefore \quad Y = \frac{\text{stress}}{\text{strain}}, \quad \therefore \quad Y = \frac{Wl/2\pi r^2 x}{x^2/2l^2} = \frac{Wl^3}{\pi r^2 x^3}.$$

Let M = mass to be suspended. Then $W = Mg$.

$$\therefore \quad 2 \times 10^{11} = \frac{M \times 9{\cdot}8 \times (\cdot 5)^3}{\pi(2 \times 10^{-3})^2(1 \times 10^{-2})^3}$$

$$\text{or} \quad M = \frac{2 \times 10^{11} \times \pi \times 4 \times 10^{-12}}{9{\cdot}8 \times \cdot 125} = \frac{8\pi \times 10^{-1}}{9{\cdot}8 \times \cdot 125} = 2{\cdot}05 \text{ kg.} \quad \textbf{Ans.}$$

Ex. 7. *A steel spiral spring has an unstretched length of* 8 cm *and when a mass is hung on it, its length becomes* 10 cm. *Calculate the periodic time of oscillation of the mass if displaced vertically.*

Sol. Let k be the 'force constant' of the spring. 'Force constant' of a spring is the force called into play per unit change of its length.

$\therefore$ Force called into play when stretched by 2 cm

$$k \times (2 \times 10^{-2}) = \cdot 02k \text{ newton.}$$

If m = mass hung on it then for equilibrium we have,

$$mg = \cdot 02k \qquad \ldots \text{(i).}$$

Let us displace the mass a little and then release it. Let x be the downward displacement at any time t. Then $\frac{d^2x}{dt^2}$ = downward acceleration of the mass at time t; additional force called into play at time $t = kx$ (upward) $= -kx$ (downward).

Force = mass × acceleration;

$$\therefore \quad -kx = m\frac{d^2x}{dt^2}, \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x.$$

$$\text{Thus} \quad \frac{d^2x}{dt^2} \propto -x.$$

Hence motion is simple harmonic and $\omega^2 = \frac{k}{m}$, or $T = 2\pi\sqrt{\frac{m}{k}}$.

$$\therefore \quad \text{Here,} \quad k = \frac{mg}{\cdot 02}.$$

$$\therefore \quad T=2\pi\sqrt{\frac{m}{\frac{mg}{\cdot 02}}}=2\pi\sqrt{\frac{\cdot 02}{9\cdot 8}}=\cdot 284 \text{ s.} \quad \textbf{Ans.}$$

Ex. 8. *Find the increment in the length of a steel wire of length 5 m and radius 6 mm under its own weight. Density of steel*$=8000$ *kgm*$^{-3}$ *and Young's modulus of the steel* 2×10^{11} *Nm*$^{-2}$.

Sol. Let l be the length of the wire and $\triangle l$ be its incresase in length. Then strain produced$=\dfrac{\triangle l}{l/2}=\dfrac{2\triangle l}{l}$, half length is taken because the weight acts at the centre of the wire.

$$\text{Stress produced}=\frac{\pi r^2 l\rho g}{\pi r^2}=l\rho g$$

$$\therefore \quad Y=\frac{\text{stress}}{\text{strain}}=\frac{l\rho g}{\frac{2\triangle l}{l}} \quad \text{or} \quad \triangle l=\frac{1}{2}\,\frac{l^2\rho g}{Y}$$

$$\therefore \quad \triangle l=\frac{5^2\times 8000\times 9\cdot 8}{2\times 2\times 10^{11}}=4\cdot 9\times 10^{-6}\text{ m.} \quad \textbf{Ans.}$$

EXERCISES

(A)

1. A copper wire 2 m long and $\cdot 5$ mm in diameter supports a mass of 10 kg. It is stretched by $2\cdot 38$ mm. Calculate Young's Modulus of the wire.

(Ans. $4\cdot 2\times 10^{11}$ Nm^{-2})

2. Calculate the strain potential energy of a horizontal rod 1 m long and $\cdot 4\times 10^{-4}$m^2 in cross-section fixed at both ends at temperature 25°C when it has been cooled to 10°C. Linear expansivity of material of rod$=11\times 10^{-6}$/K and Young's Modulus$=2\cdot 1\times 10^{11}$ Nm^{-2}.

(Ans. $\cdot 114$ joule)

3. A wire of length 3 m, diameter $\cdot 4$ mm and Young's Modulus 8×10^{10} Nm^{-2} is suspended from a point and supports a heavy cylinder of volume 10^{-3}m^3 at its lower end. Find the decrease in length when the metal cylinder is immersed in a liquid of density 800 kgm^{-3}.

(Ans. $2\cdot 339\times 10^{-3}$ m)

4. In the Young's Modulus apparatus a brick is suspended and it is found that the wire is elongated by 2 mm. When the brick is immersed in water the wire contracts by $\cdot 6$ mm. Calculate the density of the brick given that density of water is 1000 kgm^{-3}.

(Ans. 3333 kgm^{-3})

5. A bar of steel is heated to a temperature of 500°C. Its ends are then clamped at two points 1 m apart and the bar is allowed to cool. Find at what temperature it will break assuming linear expansivity of steel$=11\times 10^{-6}$/K; Young's Modulus of steel$=2\times 10^{11}$ Nm^{-2} and breaking stress for steel$=8\times 10^{8}$ Nm^{-2}.

(Ans. $134\cdot 4$ °C)

6. Find the greatest length of steel wire that can hang vertically without breaking. (Breaking stress of steel $=7{\cdot}9\times10^8$ Nm^{-2} and density of steel $=7900$ kgm^{-3}.) (Ans. $1{\cdot}02\times10^4$ m)

7. A steel wire 2 mm in diameter is just stretched between two points at a temperature of 25°C. Determine the tension when temperature falls to 15°C. (Linear expansivity of steel $=11\times10^{-6}$/K and Young's Modulus of steel $=2{\cdot}1\times10^{11}$ Nm^{-2}.) (Ans. 72·6 newton)

8. A wire of length 2 m and radius 2 mm is loaded with 5 kg when it is stretched 2·5 mm. Another load of 3 kg is suspended from below of 5 kg by a wire of the same material and length 1 m and radius 1 mm. What is the total increase in length and what are individual extensions of the wires ?

(Ans. 7 mm; 4 mm and 3 mm)

9. A wire of length 1 m and radius 1 mm is welded to another wire of length 2 m and radius 2 mm. The free end of the first is clamped and a load of 5 kg is applied at the free end of the second wire. What is the total increase of the compound wire ? (Y of both wire $=2\times10^{11}$ Nm^{-2}.)

[*Hint* : The stretching force on both is the same but not the stress.]

(Ans. $11{\cdot}7\times10^{-5}$ m)

10. A uniform pressure P is exerted on all sides of a solid cube at temperature t°C. By what amount should the temperature be raised in order to bring its volume back to the value it had before the pressure was applied. (Cubical expansivity of the material cube $=\alpha$ and the bulk modulus of elasticity is β.)

[*Hint* : $\beta=\dfrac{P}{\dfrac{\text{change in volume}}{\text{original volume}}}$; $V_t=V_0(1+\alpha t)$] $\left(\text{Ans. } \dfrac{P}{\alpha\beta}\right)$

11. Two springs have their force constants as k_1 and k_2 ($k_1>k_2$). On which spring is more work done (a) when their lengths are increased by the same amount, (b) when they are streched by the same force ? (I.I.T. 1976)

[*Hint* : W, Work done $=\frac{1}{2}kx^2$.) [Ans. (i) $w_1>w_2$, (ii) $w_1<w_2$]

(B)

12. A steel wire of length 1 m and diameter of 3 mm is stretched horizontally between supports attached at its ends. What load hung from the mid-point of the wire would be required to produce a depression of 1 cm ? ($Y=2\times10^{11}$ Nm^{-2}, $g=9{\cdot}8$ ms^{-2}) (Ans. 1·15 kg)

13. A light rod of length 2 m is suspended horizontally from the ceiling by means of two vertical wires of equal lengths tied to its ends. One of the wires is made of steel and is of cross-section ·1 sq. cm. and the other is of brass of cross-section ·2 sq. cm. Find out the position along the rod at which a weight may be hung to produce (i) equal stress in both wires, (ii) equal strain in both wires. (Y for brass $=10\times10^{10}$ Nm^{-2} and Y for steel $=20\times10^{10}$ Nm^{-2})

[Ans. (i) 1·33 m, (ii) 1 m]

14. A 4 kg block extends a spring by 16 cm from its unstretched position. The block is removed and a ·5 kg body is hung from the same spring. If the spring is then stretched and released, what is the time period of oscillation of the mass suspended ? What is the 'force constant' of the spring ?

(Ans. ·284 s; 25 kgm^{-1})

15. A uniform spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 ond l_2 where $l_1=nl_2$ and n is an integer. What are the force constants k_1 and k_2 of the two pieces in terms of n and k ?

[*Hint* : Force constant is inversely proportional to length.)

$$\left[\textbf{Ans. } \left(\frac{n+1}{n}\right)k \text{ and } (n+1)k\right]$$

16. A light elastic string is suspended vertically from a point and carries a heavy mass at its lower end, which stretches it by l. Show that the vertical oscillations of the system are simple harmonic in nature and its time period is equal to that of a simple pendulum of length l.

17. A thin uniform copper rod of length l and mass m rotates uniformly with an angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends. Determine the tension of the rod at a distance x from the rotation axis.

[*Hint* : Let T be the tension at a distance x. Consider the dynamics of circular motion of the element dx at x. Centripetal force$=-dT=\left(\frac{m}{l}\,dx\right).\,\omega^2.\,x$. Integrate and apply the condition that at $x=l$, $T=0$.) $\left[\textbf{Ans. } \frac{1}{2}m\omega^2 l\left(1-\frac{x^2}{l^2}\right)\right]$

18. A reinforced concrete column is subjected to compression by a certain load. Assuming that the modulus of elasticity of concrete is one-tenth of that of iron and area of cross-section of the iron is one-twentieth of that of concrete, find the portion of the load acting on the concrete.

[*Hint* : Here strain is the same for the iron and the concrete.]

(**Ans.** 2/3 of the load)

19. Two steel plates are soldered on the two sides of a copper plate. What tensions will arise in the plates if the temperature is increased by t°C ? All the plates have the same cross-sections. The coefficients of expansion of copper and steel are α_c and α_s and their Young's modulus are Y_c and Y_s respectively. Area of each interface$=A$.

$$\left[\textbf{Ans. } F_c=\frac{2AY_cY_s(\alpha_c-\alpha_s)t}{2Y_s+Y_s},\ F_s=\frac{F_c}{2}\right]$$

[*Hint* : The net expansion (thermal+ elastic) is the same for all the plates. The tensile force on each steel plate is half the tensile force on the copper plate.]

20. What internal pressure (in the absence of an external pressure) can be sustained by (a) a glass tube of wall thickness $\Delta r=1$ mm and radius $(r)=10$ cm, (b) a glass spherical flask of the same wall thickness and radius ? σ_b (breaking stress of glass)$=5\times10^7$ Pa.

(**Ans.** 5×10^5 Pa; 10^6 Pa)

[*Hint* : Outward force due to internal pressure is balanced by the inward elastic force.]

21. An iron ring of radius 50 cm and thickness 1 cm is heated sufficiently and then put on a wooden cart wheel of the same radius at 40°C. What is the force per unit area with which the ring grips over the wheel when cooled to 27°C ? Young's modulus of iron$=2\times10^{11}$Nm^{-2} and linear expansivity of iron $=11\times10^{-6}$K^{-1}. Neglect thermal contraction of the wooden wheel.

(**Ans.** $5{\cdot}72\times10^5$ Nm^{-2})

□□□

CHAPTER 14

SURFACE TENSION

1. Surface tension : The property of the free surface of a liquid to contract and occupy minimum possible area is called Surface Tension and is measured by the force per unit length of a line imagined to be drawn in the surface of the liquid. Unit : newton per metre (Nm^{-1}).

2. Surface energy : Due to tension the surface of a liquid possesses potential energy and this energy per unit area of surface is called Surface Energy. Unit : joule per square metre (Jm^{-2}). Surface energy is numerically equal to surface tension.

3. Excess of pressure inside a bubble or drop of a liquid : The pressure inside a bubble or drop is essentially greater than that outside of it. Due to surface tension the bubble or drop has got the tendency to contract. So unless there is greater pressure inside to balance this tendency stability of the bubble is not possible.

The excess of pressure is given by

$$P=\frac{4T}{r} \text{ in case of a bubble}$$

and $$P=\frac{2T}{r} \text{ in case of a drop.}$$

4. Angle of contact : A liquid surface is generally curved at the points where it meets a solid surface. The angle between the tangents to the solid surface and the liquid surface inside the liquid is called Angle of Contact. For pure water and clean glass angle of contact is zero.

5. Rise of a liquid in a capillary tube : Due to surface tension liquid is drawn up a capillary tube till the weight of liquid that rises in the tube balances the force due to surface tension. Let T = surface tension of liquid, r = radius of the tube, θ = angle of contact, h = height of lowest point of the meniscus, v = volume of liquid in the meniscus above the horizontal plane touching the meniscus at its lowest point.

Upward force due to surface tension $=2\pi rT\cos\theta$. Downward force due to weight of liquid in the tube $=(\pi r^2h+v)\rho g$.

For equilibrium we have

$$(\pi r^2h+v)\rho g=2\pi rT\cos\theta;$$

When permitted to neglect v as it is very small,

$$\pi r^2 h\rho g = 2\pi r T\cos\theta$$

or

$$2T\cos\theta = rh\rho g.$$

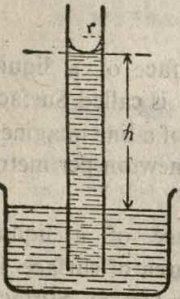


Fig. 14-1.

This is the formula that can be used when $\theta \neq 0$.

For water and glass, $\theta = 0°$ and hence meniscus is hemispherical.

Then $v = \pi r^3 - \frac{2}{3}\pi r^3 = \frac{\pi r^3}{3}$.

$$\therefore \quad \left(\pi r^2 h + \frac{\pi r^3}{3}\right)\rho g = 2\pi r\ T$$

or $T = \frac{1}{2}\rho g r\left(h + \frac{r}{3}\right)$ newton per metre (Nm^{-1}).

EXAMPLES

Ex. 1. *Calculate the amount of energy evolved when eight droplets of mercury (surface tension* ·55 Nm^{-1}) *of radius* 1 mm *each combine into one.*

Sol. Area of 8 droplets $= 8 \times 4\pi r^2 = 32\pi(1 \times 10^{-3})^2 = 32\pi \times 10^{-6}\text{m}^2$.

$\therefore$ Surface energy of 8 droplets $= 32\pi \times 10^{-6} \times \cdot 55$ joule

($\because$ Surface energy = Surface tension numerically).

Let R be the radius of the big drop formed.

Then volume of the big drop $= \frac{4\pi}{3}R^3$.

Since volume remains constant in the process of combination,

$$\therefore \quad 8\frac{4\pi}{3}(1 \times 10^{-3})^3 = \frac{4\pi}{3}R^3$$

or

$$R = 2 \times 10^{-3}\ \text{m}.$$

Surface energy of drop formed $= 4\pi(2 \times 10^{-3})^2 \times \cdot 55$

$= 16\pi \times 10^{-6} \times \cdot 55$ joule.

$\therefore$ Energy evolved $= 32\pi \times 10^{-6} \times \cdot 55 - 16\pi \times 10^{-6} \times \cdot 55$

$= 16\pi \times \cdot 55 \times 10^{-6} = 27\cdot 65 \times 10^{-6}$ joule. **Ans.**

Ex. 2. *Find the work done in blowing a soap bubble of surface tension* ·03 Nm^{-1} *from* 2 cm *diameter to* 4 cm *diameter.*

Sol. Change in surface area $=2\times 4\pi(2^2-1^2)\times 10^{-4}\ \text{m}^2$

(multiplied by 2 because a bubble has two surfaces)

$=24\pi\times 10^{-4}\ \text{m}^2$.

Hence work done $=24\pi\times 10^{-4}\times{\cdot}03$

($\because$ surface energy = surface tension numerically)

or Work done $=2{\cdot}26\times 10^{-4}$ joule. **Ans.**

Ex. 3. *A small hollow vessel which has a small hole in it is immersed in water to a depth of* 40 cm *before any water penetrates into the vessel. If the surface tension of water is* $70\times 10^{-3}\ \text{Nm}^{-1}$, *find the radius of the hole.*

Sol. Water will not enter so long excess of pressure on concave surface remains greater than hydrostatic pressure.

$$\therefore \qquad \frac{2T}{r}=P=\rho gh$$

$$\text{or} \quad r=\frac{2T}{\rho gh}=\frac{2\times 70\times 10^{-3}}{1000\times 9{\cdot}8\times 40\times 10^{-2}}=3{\cdot}72\times 10^{5}\ \text{m}. \quad \textbf{Ans.}$$

Ex. 4. *A spherical soap bubble of radius* 2 cm *becomes attached to the outside of a spherical bubble of radius* 4 cm. *Find the radius of curvature of the common surface.*

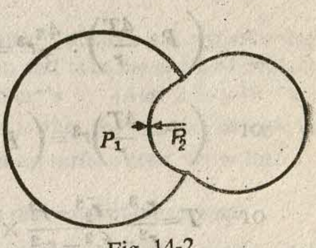


Fig. 14-2.

Sol. P_1 (excess of pressure in the first)

$$=\frac{4T}{r_1};$$

P_2 (excess of pressure in the second)

$$=\frac{4T}{r_2};$$

$$\therefore \quad P_2-P_1=4T\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\text{difference of pressure on the two sides of the common surface}$$

$$=\frac{4T}{r},$$

where r = radius of curvature of the common surface.

$$\therefore \qquad \frac{1}{r}=\frac{1}{r_2}-\frac{1}{r_1}=\frac{1}{2}-\frac{1}{4}=\frac{2-1}{4}=\frac{1}{4};$$

$$\therefore \qquad r=4\ \text{cm}=4\times 10^{-2}\ \text{m}. \quad \textbf{Ans.}$$

Ex. 5. *Two spherical bubbles of radius* 3·00 cm *and* 4·00 cm *coalesce to form another spherical bubble. Calculate the surface tension of the bubble. The radius of the bubble formed*$=$4·498 cm *and the atmospheric pressure*$=10^5$ Pa.

Sol. Let us first deduce the general formula.

Let r_1 and r_2 be the radii of bubbles and r is the radius of the bubble formed by their combination. Here in the process of combination volume does not remain constant.

Pressure of gas inside the first bubble$=P+\dfrac{4T}{r_1}$, where $P=$atmospheric pressure.

" " " " second " $=P+\dfrac{4T}{r_2}$.

Using the formula $PV=nRT$, we have

$$\left(P+\frac{4T}{r_1}\right)\frac{4\pi}{3}r_1^3=n_1RT,\ \left(P+\frac{4T}{r_2}\right)\frac{4\pi}{3}r_2^3=n_2RT$$

and for the resulting bubble $\left(P+\dfrac{4T}{r}\right)\dfrac{4\pi}{3}r^3=(n_1+n_2)RT$.

$$\therefore\ \left(P+\frac{4T}{r}\right)\cdot\frac{4\pi}{3}r^3=\left(P+\frac{4T}{r_1}\right)\cdot\frac{4\pi}{3}r_1^3+\left(P+\frac{4T}{r_2}\right)\cdot\frac{4\pi}{3}r_2^3$$

$$\text{or}\quad \left(P+\frac{4T}{r}\right)r^3=\left(P+\frac{4T}{r_1}\right)r_1^3+\left(P+\frac{4T}{r_2}\right)r_2^3$$

$$\text{or}\quad T=\frac{r_1^3+r_2^3-r^3}{r^2-r_1^2-r_2^2}\times P=\frac{3{\cdot}00^3+4{\cdot}00^3-4{\cdot}498^3}{4{\cdot}498^2-3{\cdot}00^2-4{\cdot}00^2}\times 10^5$$

$$=\cdot 189\ \text{Nm}^{-1}.\quad \textbf{Ans.}$$

EXERCISES

(A)

1. Calculate the work done in spraying a spherical drop of mercury of 1 mm radius into a million droplets of equal size; the surface tension of mercury $=550\times10^{-3}$ Nm^{-1}.

(Ans. $6{\cdot}844\times10^{-4}$ joule)

2. Two drops of water of radius 2×10^{-7} m coalesce. What rise of temperature will result ? (S.T. of water$=74\times10^{-3}$ Nm^{-1}, sp. heat capacity of water$=4200$ joule per kg per kelvin.)

(Ans. ·05°C)

3. To what height can mercury be filled in a vessel without leak occurring if there is a pin hole of diameter ·1 mm at the bottom of the vessel ? (Density of mercury$=13{\cdot}6\times10^3$ kgm^{-3}, surface tension of mercury$=550\times10^{-3}$ Nm^{-1}. Neglect angle of contact.)

[*Hint* : $T=\frac{1}{2}\rho grh$.] (**Ans.** $16{\cdot}49\times10^{-2}$ m or 16·49 cm)

4. A tube of 1 mm bore is dipped into a vessel containing a liquid of density 8×10^{-3} kgm^{-3}, surface tension 30×10^{-3} Nm^{-1} and angle of contact is zero. Calculate the length which the liquid will occupy in the tube when the tube is held (a) vertical, (b) inclined to the vertical at an angle of 30°.

[**Ans.** (a) $1{\cdot}51\times10^{-2}$ m or 1·51 cm, (b) $1{\cdot}75\times10^{-2}$ m]

5. What would be the pressure inside a spherical air bubble of ·1 mm radius situated at 1 m below the surface of water ? (Surface tension of water$=\cdot07$ Nm^{-1} and atmospheric pressure$=10^5$ Pa) [1 Pascal (Pa)$=1$ Nm^{-2}]

Density of water$=1000$ kgm^{-3}. (**Ans.** $\cdot1112\times10^5$ Pa)

6. Two soap bubbles are joined together so that they have a common surface. If their radii are 3 cm and 4 cm respectively, find the radius of the common surface. (**Ans.** 12×10^{-2} m or 12 cm)

7. By how much will the surface of a liquid be depressed in a glass tube of radius ·2 mm if the angle of contact of the liquid is 135° and the surface tension is ·547 Nm^{-1} ? Density of the liquid is $13{\cdot}5\times10^3$ kgm^{-3}.

[*Hint* : Neglecting the liquid in the meniscus we have for equilibrium

$$\left.\pi r^2h\,\rho g=2\pi rT\cos\theta,\; h=\frac{2T\cos\theta}{\rho gr}\right)$$ (**Ans.** $2{\cdot}92\times10^{-2}$ m)

8. A glass tube of radius 5 mm is dipped in water. Calculate the rise of water in the tube. (Neglect mass of water in the meniscus and take the angle of contact to be zero. Surface tension of water$=70\times10^{-3}$ Nm^{-1}) (**Ans.** $2{\cdot}86\times10^{-2}$ m)

9. Two spherical soap bubbles coalesce. If V is the consequent change in volume of the contained air and S the change in total surface area, show that

$$3PV=4ST$$

where T=surface tension of soap bubble and P is the atmospheric pressure.

10. If a number of droplets, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by $\frac{3T}{\sigma S}\left(\frac{1}{r}-\frac{1}{R}\right)$ where T=surface tension of liquid, σ=density of liquid and S=specific heat capacity of liquid in joule per kg per kelvin.

(B)

11. A glass plate of length 10 cm, breadth 4 cm and thickness ·2 cm weighs 8 gm in air. If it is held vertically with its long side horizontal and its lower end half immersed in water what will be its apparent weight ? (Surface tension $=\cdot07$ Nm^{-1} and $g=9{\cdot}8$ ms^{-2})

[*Hint* : apparent weight=weight in air−buoyancy+force due to surface tension.] (**Ans.** $5{\cdot}46\times10^{-3}$ kg or 5·46 gm)

12. Two equal spherical bubbles of radius '*a*' each coalesce to form a spherical bubble of radius '*b*'. If *P* is the atmospheric pressure, show that the surface tension of the bubble is

$$\frac{P(2a^3-b^3)}{4(b^2-2a^2)}.$$

[*Hint* : Use $P=\frac{4T}{r}$ and apply $pV=nRT$ formula.]

13. Calculate the radius of the largest drop of water that might evaporate at 0°C without heat being supplied to it. The surface energy of water at 0°C is 117×10^{-3} Jm^{-3} and the latent heat capacity of water is 606×10^3 calorie per kg at 0°C. One calorie equals 4·2 joule.

[*Hint* : Equate change in surface energy to the heat required for evaporation of mass that will disappear.] **(Ans.** $9{\cdot}2\times10^{-11}$ m)

14. A sphere of radius 6 cm weighs 2 kg in air. It is held with its lower half immersed in water. What will be its apparent weight ? (Surface tension of water $=72{\cdot}5\times10^{-3}$ Nm^{-1} and $g=9{\cdot}81$ ms^{-2}) **(Ans.** 1·5504 kg)

15. The end of a capillary tube of radius *r* is immersed into water of surface tension *T* and density ρ. What amount of heat will be evolved when water rises in the tube ? $\left(\textbf{Ans. } \frac{2\pi T^2}{\rho g}\right)$

[*Hint* : Heat evolved = work done by surface tension − gravitational energy stored in the standing column.]

16. Water rises to a height *h* in a long capillary tube. The length of the tube inside the water is *l*. The tube is taken out gently. What length of water will remain in the tube ? **(Ans.** $2h$ if $l>h$ and $l+h$ if $l<h$)

□□□

CHAPTER 15

HYDROSTATICS

1. **Pressure** : Pressure at a point is the force per unit area of surface surrounding the point.

Pressure at a point due to a column of liquid of height h is

$P = \rho gh$ pascal (Pa). (1 pascal $= 1$ Nm^{-2})

2. **Thrust on a surface** : Thrust on a surface $=$ area $\times$ pressure at the centre of the surface given by the above formula.

3. **Centre of pressure** : If the surface is vertical thrust on it acts at a point $\frac{2}{3}h$ deep where 'h' is the depth of centre of the surface from the top of liquid and if it is horizontal it acts at the centre of the surface. The point where thrust acts is called Centre of Pressure (C.P).

4. **Buoyancy** : The upward thrust of a liquid on a solid body is called Buoyancy.

Buoyancy $=$ wt. of liquid displaced $= V\rho g$ where $V =$ volume of liquid displaced, $\rho =$ density of liquid.

5. **Law of a Floating body** : A solid floats on the surface of a liquid when its weight is equal to weight of the liquid displaced by it i.e. $W = W'$ is the condition for floatation of a body.

6. **Density** : Mass per unit volume is called Density. Unit, kgm^{-3}.

$$D \text{ (density)} = \frac{M \text{ (mass)}}{V \text{ (volume)}} \text{ kgm}^{-3}.$$

7. **Relative Density** : Density of a substance relative to the density of water is called Relative Density or specific gravity. It is a pure number.

In S I, Density $=$ relative density $\times$ 1000 kgm^{-3} because density of water in S I is 1000 kgm^{-3}.

8. **Pascal's law** : When pressure is exerted at a point on a confined liquid, it is transmitted undiminished equally in all directions and acts at right angles to the wall of the container. A hydraulic press works on this principle of transmission of pressure undiminished in all directions.

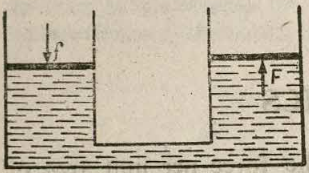


Fig. 15-1.

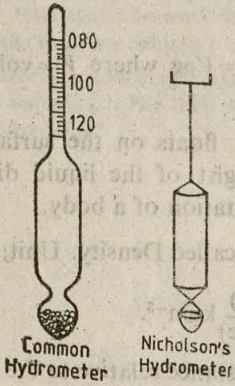
Suppose f = force applied on the shorter piston and F = force produced on the larger piston.

Then according to Pascal's law,

$\frac{f}{\alpha}=\frac{F}{\beta}$ where α and β are the area of the smaller and larger piston. According to principle of conservation of energy which is obeyed by all machines we have also $f.x=F.y$ where x = displacement (downward) of smaller piston and y = upward displacement of larger piston.

9. **Archemedes principle** : A body partly or wholly immersed in a fluid *at rest* appears to lose a part of its weight which is equal to the weight of the fluid displaced.

10. **Hydrometers** : Hydrometers are instruments to measure relative (relative to water) density of liquids. They work on the principle of a floating body, namely,



Common Hydrometer Nicholson's Hydrometer

Fig. 15-2

wt. of the hydrometer = wt. of liquid displaced,

or $W=V\rho g$ where V = volume of liquid displaced, ρ = density of the liquid.

If W remains constant as in a common hydrometer,

$V\rho$ = a constant or $\rho=\frac{k}{V}$

where k is a constant. If V is kept constant as in Nicholson hydrometer, $W\propto\rho$,

or $\rho=kW$.

11. **The Atmospheric Pressure** : The weight of a vertical column of air of unit area of cross-section extending from the surface of the earth up to where atmosphere ends is called Atmospheric Pressure. Unit, Nm^{-2} or Pa. The auxiliary unit, 'm or cm of *Hg*'.

12. **Fortin's barometer** : Fortin's barometer works on the principle that pressure at two points on the same horizontal line drawn in a liquid *at rest* is the same.

One metre long tube AB filled with pure dry mercury is inverted in a trough containing mercury. Mercury in tube stands up to certain height depending on the atmospheric pressure. Draw a horizontal line just grazing the surface of mercury in the trough and consider two points on this line E inside the tube and F outside of it according to the principle of hydrostatics stated above.

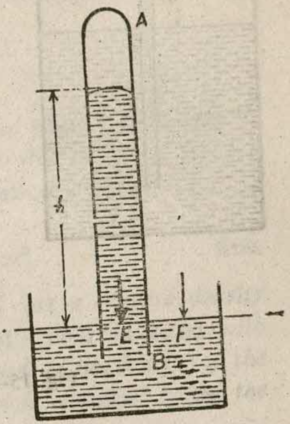


Fig. 15·3.

Pressure at E = Pressure at F.

But pressure at F = atmospheric pressure and pressure at E = pressure due to column of mercury because above mercury there is nothing except vapour of mercury which exerts very little pressure.

Atmospheric pressure $= h\rho g$ Nm^{-2} where h = height of mercury column standing in the tube, ρ = density of mercury.

13. Air Pump or Suction Pump : It is a pump by which a vessel is depleted by working the piston. The pressure inside the vessel after n strokes is $P_n = \left(\frac{V}{V+v}\right)^n \times P$ where P = atmospheric pressure, V = volume of vessel, v = volume of barrel

$$\text{or} \quad D_n = \left(\frac{V}{V+v}\right)^n \times D$$ where D = density of atmospheric air

and D_n = density of air of vessel after n strokes.

14. Compression Pump : It is a pump by which a vessel is filled with air under pressure. The preesure inside the vessel after n strokes is

$$P_n = \left(1 + n\frac{v}{V}\right) \times P \text{ and } D_n = \left(1 + n\frac{v}{V}\right) \times D.$$

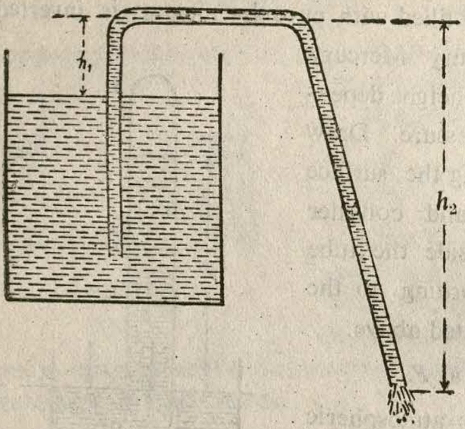


Fig. 15-4.

15. Siphon : Siphon is a device to transfer liquid from a vessel at higher level to a vessel at lower level. Conditions for working of siphon : (i) $h_2 > h_1$, (ii) h_1 be less than the height of the corresponding liquid barometer.

EXAMPLES

Ex. 1. *Find the pressure in newton per square metre* 1500 m *below the surface of the ocean. The relative density of sea water is* 1·03.

Sol. We have $P = \rho g h = (1{\cdot}03 \times 1000) \times 9{\cdot}8 \times 1500$

or $\qquad P = 1{\cdot}5 \times 10^7 \text{ Nm}^{-2}$.

This is the pressure due to water column. The atmospheric pressure is $1{\cdot}01 \times 10^5 \text{ Nm}^{-2}$ or $\cdot 01 \times 10^7 \text{ Nm}^{-2}$.

$\therefore \qquad P = 1{\cdot}51 \times 10^7 \text{ Nm}^{-2}$. **Ans.**

Ex. 2. *What is the minimum area af a block of ice* ·3 m *thick floating on water that will hold up an automobile weighing* 1134 kg ? (*Density of ice* = 910 kgm^{-3}.)

Sol. Let A be the minimum area of the ice block. Then the volume of the ice block is $A \times \cdot 3$ m^3 and its weight is $(A \times \cdot 3 \times 910)$ g.

It will displace $(\cdot 3A)$ m^3 of water whose weight is $\cdot 3A \times 100$ g. By law of floatation,

$$(\cdot 3A \times 910 \text{ g}) + 1134 \text{ g} = \cdot 3A \times 1000 \text{ g}$$

or $\qquad \cdot 3A \times 90 = 1134$

or $\qquad A = 42 \text{ m}^2$. **Ans.**

Ex. 3. *An iron casting weighs* 27 kg *in air and* 18 kg *in water. What is the volume of cavities in the casting* ? (*Density of iron* = 7800 kgm^{-3})

Sol. Let V be the outer volume of the casting and v is the volume of cavities. Then

$$27=(V-v)\,\rho_{iron}$$

and
$$18=(V-v)\rho_{iron}-V\times 1000$$
$$(\because\ \text{Density of water}=1000\ \text{kgm}^{-3})$$

$\therefore$
$$18=27-V\times 1000\quad\text{or}\quad V=\cdot 009\ \text{m}^3.$$

Substituting the value of V in the first equation we have

$$27=(\cdot 009-v)\times 7800$$

or
$$v=\cdot 009-\cdot 00346=\cdot 00554\ \text{m}^3.$$ **Ans.**

Ex. 4. *A block of wood weighs* 4 kg *and has a relative density of* ·6. *It is to be loaded with lead so that it will float in water with* ·9 *of its volume immersed. What weight of lead is needed* (a) *if the lead is on top of the wood,* (b) *if the lead is attached below the wood ?* (*density of lead* = 11,340 kgm^{-3})

Sol. (a) Let W be the required weight. By law of floatation,

$$4+W=\left(\frac{4}{\cdot 6\times 1000}\right)\cdot 9\times 1000$$

$$W=2\ \text{kg}.$$ **Ans.**

(b) By law of floatation,

$$4+W=\left(\frac{4\times\cdot 9}{\cdot 6\times 1000}+\frac{W}{11340}\right)\times 1000$$

or
$$4+W=6\cdot 000+\cdot 0882W$$

or
$$W=\frac{2\cdot 000}{\cdot 9118}=2\cdot 1935\ \text{kg}.$$ **Ans.**

Ex. 5. *After four strokes the density of air in the receiver of an air-pump is found to bear to its original density the ratio of* 256 : 625. *What is the ratio of the volume of the barrel to that of the receiver ?*

Sol. We have
$$P_n=P_0\left(\frac{V}{V+v}\right)^n$$

and
$$D_n=D_0\left(\frac{V}{V+v}\right)^n.$$

Here
$$n=4$$

and
$$\frac{D_4}{D_0}=\frac{256}{625}$$

$\therefore$
$$\frac{256}{625}=\left(\frac{V}{V+v}\right)^4\quad\text{or}\quad\left(\frac{4}{5}\right)^4=\left(\frac{V}{V+v}\right)^4$$

$$\therefore \quad \frac{4}{5}=\frac{V}{V+v} \quad \text{or} \quad \frac{v}{V}=\frac{5}{4}-1=\frac{1}{4}. \qquad \textbf{Ans.}$$

Ex. 6. *When a mercury barometer is taken to a place at a height of* 25 m *above the sea level it shows a pressure difference of* 2·5 mm *with respect to the pressure at the sea surface. If the relative density of mercury is* 13·6, *find the average density of air near the sea level.*

(Bihar 1961)

Sol. Here pressure due to 2·5 mm of mercury

=pressure due to 25 m of air

$$\therefore \quad 2{\cdot}5\times10^{-3}\times(13{\cdot}6\times1000)\,g=25\times\rho_{air}\times g$$

or $$\rho_{air}=\frac{2{\cdot}5\times13{\cdot}6}{25}=1{\cdot}36\text{ kgm}^{-3}. \qquad \textbf{Ans.}$$

Ex. 7. *A faulty barometer reads* 71·12 cm *and* 76·20 cm *when true barometer reads* 72·39 cm *and* 78·74 cm *respectively. Find the true reading when the faulty barometer stands at* 73·66 cm.

Sol. The barometer is faulty when it encloses some air in its vacuous space. Let l be the length of the tube above the fixed ivory pointer. Then volume of the enclosed air is $(l-71{\cdot}12)\times\alpha$ cm³ where α is the area of cross-section of the tube and its pressure $P=72{\cdot}39-71{\cdot}12=1{\cdot}27$ cm of mercury. When the second observation is taken the volume of air is $(l-76{\cdot}20)\alpha$ and its pressure $=78{\cdot}74-76{\cdot}20=2{\cdot}54$ cm of mercury. By Boyle's law, we have

$$(l-71{\cdot}12)\alpha\times1{\cdot}27=(l-76{\cdot}20)\alpha\times2{\cdot}54$$

or $$1{\cdot}27\,l-71{\cdot}12\times1{\cdot}27=2{\cdot}54\,l-76{\cdot}20\times2{\cdot}54$$

or $$1{\cdot}27\,l=193{\cdot}55-90{\cdot}32=103{\cdot}23$$

or $$l=81{\cdot}28\text{ cm.}$$

Let x be the true reading when the faulty one reads 73·66 cm. Then by Boyle's law again

$$(81{\cdot}28-71{\cdot}12)\times1{\cdot}27=(x-73{\cdot}66)\times(81{\cdot}28-73{\cdot}66)$$

or $$10{\cdot}16\times1{\cdot}27=(x-73{\cdot}66)\times7{\cdot}62$$

or $$x-73{\cdot}66=1{\cdot}69 \quad \text{or} \quad x=75{\cdot}35\text{ cm.} \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. A rectangular tank 2 m deep, 5 m broad and 10 m long is filled with water. Calculate the thrust on each of the sides and on the base. (Density of water=1000 kgm⁻³ and $g=9{\cdot}8$ ms⁻².)

(**Ans.** $9{\cdot}8\times10^4$ N; $19{\cdot}6\times10^4$ N and $9{\cdot}8\times10^5$ N)

2. If the atmospheric pressure is $1{\cdot}1\times10^5$ Nm^{-2} (pascal) and the diameter of Magdeburg hemispheres are 40 cm, find the force required to pull them asunder. (Ans. $13{\cdot}8\times10^3$ newton)

3. A cube floating on mercury has $\frac{1}{4}$ of its volume submerged. If enough water is added to cover the cube, what fraction of its volume will remain immersed in mercury ? (Relative density of mercury=13·6) (Ans. ·19)

4. A block of wood weighs 4 kg and has a relative density ·6. It is to be loaded with lead so that it will float completely immersed. What weight of lead is needed (a) if the lead is on top of the block, (b) if the lead is attached below the block ? Density of lead=$11{\cdot}34\times10^3$ kgm^{-3}. (Ans. 2·667 kg; 2·92 kg)

5. A metal plate measuring 2 m × 1 m is immersed in water and is held there with its shorter side parallel to the free surface at a depth 5 metre. Calculate the thrust on the plate, (i) when it is horizontal, (ii) when it is vertical, (iii) when it is at an inclination 30° with the vertical. (Ans. 98×10^3 N, $117{\cdot}6\times10^3$ N, 115×10^3 N)

6. A spherical iron shell floats almost completely submerged in water. If outer diameter is ·5 m and the relative density of iron is 8, find the inner diameter. (Ans. ·48 m)

7. A cylinder of iron floats vertically and fully immersed in a vessel containing mercury and water. Find the ratio of the length of the cylinder immersed in water to that immersed in mercury. (Relative density in mercury=13·6 and that of iron=7·78.) (Ans. ·86 : 1)

8. A stem of a common hydrometer is cylindrical and the highest graduation corresponds to relative density 1·0 and the lowest to 1·3. What relative density corresponds to a point exactly mid-way between these divisions ? (Ans. 1·13)

9. The stem of a hydrometer is divided into 100 equal parts; it reads 0 in water and 100 in a liquid of relative density ·8. Find the relative density of liquid in which it reads 50. (Ans. ·89)

[*Hint* : Consider hydrometer as a floating body.]

10. A U-tube contains water in its lower part. Now 32 cm of kerosene oil of relative density ·8 is poured in one limb and in the other limb a vegetable oil of relative density ·65. The water level on the kerosene oil side is 5 cm lower than on the other. What is the height of the column of vegetable oil ? (Ans. ·317 m)

11. Two bodies counterpoisc each other when suspended in water from the arms of a balance. The mass of one body is ·28 kg and is of relative density 5·6. If the mass of the other is ·36 kg, what is the relative density of the other ? (Ans. 2·78)

12. A narrow uniform tube of glass is sealed at one end and contains a narrow mercury pellet 5 cm long. When the tube is held vertically with the sealed end up, the length of the air column imprisoned by the mercury pellet is 25·6 cm When the tube is inverted, the air column is 22·4 cm long. What is the atmospheric pressure ? (Ans. ·75 m of mercury)

[*Hint* : Make use of Boyle's law.)

13. Calculate the depth of water where a bubble of air would just float. Density of water=1000 kgm^{-3} and density of air=1·29 kgm^{-3}. Atmospheric pressure=10^5 Pa. (Ans. 7900 m)

[*Hint* : Bubble will float when density of air is equal to density of water. Use the formula $\frac{P}{\rho}$ =a constant.]

14. A tube 1·5 m long and closed at one end is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. Find the height of mercury in the tube if the reading of the barometer is ·75 m.

(**Ans.** ·2865 m or 28·65 cm)

15. Two cylinders *A* and *B* are of the same dimensions but of different materials. *A* alone floats in a liquid with half of its length immersed. When *B* is placed over *A*, the combination just floats in the same liquid. Compare the densities of the two cylinders *A* and *B*.

(**Ans.** 1 : 3)

16. A tube 2 m long and closed at one end is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. Find the height of mercury which will stand in the tube if the reading of the barometer is ·75 m.

(**Ans.** ·31 m)

[*Hint* : $P_1 = \cdot 75$, $V_1 = 1\ \alpha\ \text{m}^3$; $P_2 = (\frac{3}{4} - x)$, $V_2 = (2 - x)\alpha\ \text{m}^3$. Use $P_1V_1 = P_2V_2$.]

17. A column of mercury 10 cm long is contained in the middle of a horizontal 1 m long tube which is closed at both ends. Both the halves contain air at a pressure ·76 m of mercury. By what distance will the column of mercury be displaced if the tube is held vertically ?

[*Hint* : $P_1 = \cdot 76$, $V_1 = (\cdot 5 - \cdot 05) = \cdot 45\alpha\ \text{m}^3$. If P_1' and P_2' are the pressures of air imprisoned in the upper and lower portion of the tube then $P_2' = P_1' + \cdot 1$ and volume (V_2) of upper portion $= (\cdot 45 + x)\ \alpha\ \text{m}^3$ and volume (V_2) of lower portion $= (\cdot 45 - x)\alpha\ \text{m}^3$. Then use $P_1V_1 = P_2V_2$.]

(**Ans.** 3 cm)

18. Find to what depth a diving bell* must be lowered into water in order that the volume of the air enclosed may be reduced by one quarter, the length of the bell being 3 m and the atmospheric pressure ·76 m of mercury and relative density of mercury = 13·6.

(**Ans.** 3·445 m)

19. A bottle whose volume is $\cdot 45 \times 10^{-3}$ m³ is sunk mouth downwards below the surface of water in a tank. How far must it be sunk so that $\cdot 75 \times 10^{-4}$ m³ of water may run up into the bottle ? The atmospheric pressure $= 10^5$ Nm^{-2} and density of water = 1000 kgm^{-3}.

(**Ans.** 2 m)

20. At what depth in a lake will a bubble of air have one-half of its volume which it will have on reaching the surface ? The height of mercury barometer at the time is ·76 m of mercury and relative density of mercury = 13·6 and density of water = 1000 kgm^{-3}.

(**Ans.** 10·34 m)

21. A faulty barometer reads 68 cm and 72 cm when true barometer reads 70 cm and 75 cm respectively. Find the true reading when the faulty barometer reads 71 cm and the length of the tube above the ivory pointer.

(**Ans.** 73·67 cm; 80 cm)

[*Hint* : A faulty barometer is that which contains a little air in the space above mercury in the tube.)

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*Treat a diving bell as a very big inverted cup pushed well below the water level.

22. The height of a mercury barometer is 75·6 cm. The barometer tube has a cross-section 1 cm^2 and the volume of the enclosed space above the mercury surface is 10 c.c. One c.c. of air at atmospheric pressure is introduced into the tube. Find the change in the reading of the barometer. (Ans. 70·6 cm)

23. The height of a mercury barometer is 76 cm at sea level and 50 cm at the top of a hill. Determine the height of the hill. (Densities of air and mercury are 1·36 kgm^{-3} and $13{\cdot}6\times10^3$ kgm^{-3})

[*Hint* : Difference of pressure $=76$ cm $-$ 50 cm $=26$ cm of mercury
$=\cdot26\times9{\cdot}8\times13{\cdot}6\times10^3$ Nm^{-2}
=pressure due to h metre of air
$=h\times9{\cdot}8\times1{\cdot}36$.] (Ans. 2600 m)

24. The height of a faulty barometer is 75 cm and the tube above mercury is 10 cm long, the correct barometer reading 76 cm. When this barometer reads 74 cm, find the true barometer reading. (Ans. 74·99 cm)

25. The neck and bottom of a bottle are 2 cm and 10 cm in diameter respectively. If, when the bottle is full of oil, the cork in the neck is pressed with a force of 1 kg, what force is exerted on the bottom of the bottle ? (Ans. 25 kg)

26. In a Bramha's press the areas of two plungers are 2 cm^2 and 60 cm^2 respectively. The pump plunger is worked by a lever whose arms are 5 cm and 75 cm. If the end of the lever is raised and lowered by 30 cm at every stroke, find the number of strokes required to raise the press plunger by 2 cm. (Ans. $n=30$ strokes)

27. A pump in the basement of a garage is used to operate a hydraulic jack on the first floor. The areas of the two pistons are 5 cm^2 and 375 cm^2 respectively and the large piston is 3 m higher than the other one. How great a load can be lifted by a force of 10 kg applied to the lower piston ? (Ans. 637·5 kg)

28. The pressure inside the receiver of an air pump is reduced to $\frac{1}{4}$ of the initial pressure after 3 strokes. What will be the pressure after 8 strokes ? (Ans. $\frac{1}{40}$ of initial pressure)

29. In an air pump, the volume of the barrel is $\frac{1}{10}$ of that of the receiver. If in the beginning the air in the receiver be atmospheric pressure, find the pressure in the receiver after 4 strokes. (Ans. ·68 atm)

30. Air is forced into a vessel by a compression pump. The barrel of the pump has $\frac{1}{10}$ of the volume of the vessel. Calculate the density of air in the vessel after 15 complete strokes. (Ans. 2·5 times initial density)

31. The cylinder of a water pump is 4 m above the surface of water in a well. If the radius of the cylinder of the pump be 5 cm, calculate the force acting on the piston while the pump is working. (Density of water $=1000$ kgm^{-3}.)

[*Hint* : Force = area $\times$ pressure due to 4 m high column of water.] (Ans. 308 newton)

32. The barrel of a common pump (pump for water) is 50 cm long and its bottom is 6 m from the surface of water. If the cross-section of the thinner pipe joining the barrel with water in well is $\frac{3}{14}$ of that of the barrel, find the height of the water in the pipe after first stroke. (Water barometer is 9 m high.)

[*Hint* : Initial volume of air imprisoned in the pipe$=\frac{8}{14}A\times6$ where A=area of cross-section of barrel, initial pressure=9 m of water.

Final volume$=A\times\frac{1}{2}+\frac{3}{14}A\times(6-x)=A\left(\frac{25}{14}-\frac{3}{14}x\right)=\frac{A}{14}(25-3x)$.

Final pressure=$(9-x)$ m of water. Use now $P_1V_1=P_2V_2$] (Ans. 1·3 m)

33. A vessel is filled with mercury which is ·9 m deep. Could the vessel be completely emptied with the aid of a siphon ? If not, to what extent can it be emptied ? (Height of mercury barometer is ·7 m)

(Ans. Because condition for working of siphon is that h_1 be less than the height of corresponding liquid barometer, hence it can be emptied only ·7 m of it.)

(B)

34. A canal lock gate is 4 m broad and depths of water of the opposite sides of the gate are 5 m and 2 m. Find the resultant thrust on the gate and the torque tending to turn the gate about its base. Density of water$=1000$ kgm^{-3} and $g=9{\cdot}8$ ms^{-2}. (Ans. 411600 newton; 764400 Nm)

35. To what height a cylinder must be filled with a liquid so that the thrust on the side may be equal to the thrust at the bottom ?

(Ans. Fill to a height equal to the radius of the cylinder)

36. Two identical cylindrical vessels with their bases at the same level contain a liquid of density ρ up to heights h_1 and h_2 respectively. Find the work done by gravity in equalising the levels when the two vessels are connected. (Area of the base of each cylinder is A.) (I.I.T. 1981)

[Ans. $\frac{1}{4}\rho A\,(h_2-h_1)^2g$]

37. An iron shell loses half its weight in water. What portion of its volume is hollow ? (Relative density of iron=7·5) (Ans. $\frac{11}{15}$)

38. A cylinder of relative density ·7 is ·5 m high and the radius of the base 1 m. It floats in water with its axis vertical. Find the length of the axis of the cylinder inside water. What force is needed to raise it by 1 cm ? Density of water=1000 kgm^{-3}. (Ans. ·35 m; 308 newton)

39. A ship with cargo sinks by x when she goes into a river from the sea. She discharges cargo and rises by y and proceeding again to sea she rises by z. If the sides of the ship be assumed to be vertical to the surface of water show that relative density of sea water is $\frac{y}{z-x+y}$, assuming relative density of river water 1.

40. A beaker containing water (weighing 150 gm) is placed on the left pan of a balance and a stone of mass 50 gm is placed on the same pan outside the beaker and the two are counterpoised by putting 200 gm on the other pan. (a) Now the stone is put in the beaker. Will they weigh same, more or less ? (b) If the stone is suspended by a thread to an external support and is allowed to sink into water without touching any part of the beaker, what weight will be required to restore balance ? (Density of stone=2000 kgm^{-3})

[Ans. (a) They weigh the same. When stone is put in beaker the level of water in beaker rises and so thrust on the bottom increases as much as there is loss in weight of stone. (b) 25 gm more will be needed to restore balance.]

41. A beaker containing water is placed on the pan of a balance which shows a reading of 200 gm. A lump of sugar 75 gm and volume 30 c.c. is now suspended inside water of beaker from an external support without touching any part of it. (a) What will be the reading just when the lump of sugar is immersed? How will the reading change as time passes on? (b) What will be the initial reading if lump is simply dropped gently into the beaker and how will the reading change?

[Ans. (a) 230 gm; reading will increase up to 275 gm. (b) 275 gm; no change in weight.]

42. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the waterlevel? (Ans. The level will fall)

[*Hint* : Assume the initial height of level H and the length of the boat immersed in water h. Apply law of floatation. Find the volume of water in the tank. Repeat the same after a stone piece is unloaded. The stone will sink to the bottom.

Assume the above quantities to be H' and h' respectively.

Make use of the fact that the volume of water remains constant.

$H-H'=\frac{V}{A}\left(\frac{\rho_s}{\rho_w}-1\right)$ where V=volume of stone, A=area of cross-section of tank, ρ_s=density of stone and ρ_w=density of water.]

43. A beaker full of water up to the brim contains a piece of ice floating in water. Will water spill as ice melts? (Ans. No)

[*Hint* : In the process total mass (mass of ice+mass of water) remains constant. Show that $H=\frac{M+m}{A\rho_w}$ where M is the mass of water, m=mass of the ice, A=area of cross-section of the beaker and ρ_w=density of water. Obviously it is a constant.]

44. A man in a boat drinks water from a lake. Will the level of water of the lake fall or rise? (Ans. No change in water level)

[*Hint* : Same as in Prob. 42.]

45. An empty boat collects a stone lying at the bottom of a lake. Will the level rise or fall? (Ans. The level will rise)

[*Hint* : Same as in Prob. 42.]

46. In problem 43 if the ice piece contains a brass piece, will water spill out? (Ans. Yes)

47. In problem 43, what difference will be observed (a) if hot water is taken, (b) if water at 4°C is taken?

[Ans. (a) no overflow, (b) there will be overflow of water]

48. In problem 43 if the ice piece contains a cork piece, will water spill out? (Ans. No)

49. An ideal gas is trapped between a mercury column and the closed lower end of a narrow vertical tube of uniform bore. The upper end of the tube is open to the atmosphere (atmospheric pressure = 76 cm of mercury). The lengths of the mercury and the trapped gas are 20 cm and 43 cm respectively. What will be the length of the gas column when the tube is tilted slowly in a vertical plane through an angle of 60° ? Assume the temperature to be constant.

(I. I. T. 1972)

(**Ans.** 48 cm)

50. A vertical cylinder of total length 100 cm is closed at the lower end and is fitted with a movable, frictionless, gas-tight disc to the other end. An ideal gas is trapped under the disc of negligible mass. Initially the height of the gas column is 90 cm when the disc is in equilibrium between the gas and the atmosphere. Mercury is slowly poured on the top of the disc and it just starts overflowing when the disc has descended through 32 cm. Find the atmospheric pressure.

(**Ans.** ·761 m of mercury)

51. Two spheres each of volume 250 cc but of densities 800 kgm^{-3} and 1200 kgm^{-3}, are connected by a string and the combination is immersed in water. What is the tension of the string ?

(**Ans.** 0·49 newton)

52. A long wooden bar with square cross-section floats on water. Show that (a) if the relative density of the wood is 0·5, the bar will float in stable equilibrium with all four faces making an angle of 45° with the water surface but (b) if the relative density of the wood is 0·2 or 0·8, the bar floats with two faces parallel to the water surface.

□□□

CHAPTER 16

HYDRODYNAMICS : BERNOULLI'S THEOREM AND VISCOSITY

1. Bernoulli's theorem : If an incompressible liquid free from viscous force flows slowly and steadily through a tube of varying cross-section and height above the ground, its total energy per unit mass remains constant.

$$\frac{1}{2}v^2+gh+\frac{p}{\rho}=\text{a constant.}$$

In horizontal flow, h remains constant.

Hence in horizontal flow, $\frac{1}{2}v^2+\frac{p}{\rho}=$ a constant.

Further since liquid is incompressible, rate of mass flow or volume flow across any section must remain constant i.e. $\alpha_1 v_1=\alpha_2 v_2$. This is known as equation of continuity.

2. Viscosity and coefficient of viscosity : The property of a liquid by virtue of which it opposes the relative motion between its different layers is known as Viscosity.

According to Newton viscous force acting tangentially on any layer is proportional to (i) A, area of the layer; (ii) v, velocity of the layer; (iii) $\frac{1}{x}$ where x is the distance of the layer from the stationary solid surface over which liquid is flowing.

or $$F \propto -\frac{Av}{x}.$$

The minus sign indicates that the direction of the force is opposite to that of motion of the liquid.

$\therefore\ F=-\eta\frac{Av}{x}$. Here η is a constant of the liquid called coefficient of viscosity. Now $\left(\frac{v}{x}\right)$ is the rate of increase of velocity with distance and hence it may be written as $\frac{dv}{dx}$.

$$\therefore \qquad F = -\eta A \frac{dv}{dx}.$$

Put $$A = 1\ \text{m}^2;\ \frac{dv}{dx} = 1\ \text{s}^{-1}.$$

Then $$F = -\eta.$$

Thus coefficient of viscosity is the viscous force per unit area of a liquid layer where velocity gradient is unity. In SI its unit is newton second per square metre (Nsm^{-2}).

3. **Velocity of flow through a capillary tube** : The rate of flow of a liquid through a tube is restricted by its viscosity. Water will flow more rapidly through a tube than glycerine. The **'rate of volume flow'** through a narrow tube is given by

$V = \frac{\pi P r^4}{8\eta l}$, where P = difference in pressure between the ends of the tube, l = length of the tube and r = radius of the tube.

4. **Critical velocity and Terminal velocity** : When a liquid flows through a tube slowly and steadily its motion is said to be streamline if its velocity at each point in the path of flow remains unchanged in magnitude and direction with time. If, however, velocity is gradually increased a stage will come when the flow will cease to be 'streamline' and become 'turbulent'. The velocity at which flow of a liquid through a tube passes on from 'streamline motion' to 'turbulent motion' is called **'critical velocity'**. It depends on density and viscosity of the liquid and radius of the tube.

$$v_c = \frac{k\eta}{\rho r},$$

where k is a constant called Reynold's number = 1000 approximately.

5. **Terminal velocity** : When a solid body moves through a viscous liquid, it experiences some viscous force. For a spherical body this is given by

$$F \quad 6\pi\eta r v$$

where η = coefficient of viscosity of liquid, r = radius of the body and v = velocity of the body. This is known as **Stokes' law of viscosity.** Due to the force forcing the body through the liquid its velocity increases and with increase of velocity viscous force also increases and soon it becomes equal to the driving force. The two forces now balance each other and the body moves with a uniform velocity already acquired by it. This velocity is called Terminal velocity.

Terminal velocity of a spherical body moving through a liquid under gravity :

Here the force driving the body downward

$$=\text{weight of the body}-\text{buoyancy}$$

$$=\frac{4\pi}{3}r^3\sigma g-\frac{4\pi}{3}r^3\rho g=\frac{4\pi}{3}r^3(\sigma-\rho)g$$

where σ = density of the body, ρ = density of the liquid, r = radius of the body.

By Stokes' law, viscous force on the body $=6\pi\eta rv$.

When terminal velocity is acquired, the driving force and the viscous force balance each other.

$$\therefore\quad 6\pi\eta rv=\frac{4\pi}{3}r^3(\sigma-\rho)g$$

or $$v=\frac{2}{9}\cdot\frac{r^2(\sigma-\rho)g}{\eta}.$$

EXAMPLES

Ex. 1. *Water is flowing through a pipe of non-uniform cross-section. The speed of flow is* ·25 ms^{-1} *at a section of radius* 3 cm. *Find the speed at the section of radius* 2 cm.

Sol. In the flow of incompressible liquid we have

$$\alpha_1 v_1=\alpha_2 v_2,\ \text{principle of continuity.}$$

$$\therefore\quad \pi(3\times10^{-2})^2\times{\cdot}25=\pi(2\times10^{-2})^2\times v_2$$

or $$v_2=56{\cdot}25\times10^{-2}\ \text{ms}^{-1}.$$ **Ans.**

Ex. 2. *A tank of water is filled up to* 1 m. *A small hole is drilled at its bottom. Calculate the velocity of efflux of water through the hole.*

Sol. Consider two tranverse sections of the liquid : (i) at the top, (ii) at the mouth of the hole.

At the top kinetic energy is zero and at the bottom potential energy is zero.

$$\therefore\ 0+gh+\frac{P}{\rho}=\frac{1}{2}v^2+0+\frac{P}{\rho}$$ where P is the atmospheric pressure

or $$v=\sqrt{2gh}.$$ This is the velocity of efflux.

$$\therefore\quad r=\sqrt{2gh}=\sqrt{2\times9{\cdot}8\times1}=\sqrt{19{\cdot}6}=4{\cdot}43\ \text{ms}^{-1}.$$ **Ans.**

Ex. 3. *A large bottle is fitted with a capillary siphon. Compare the times taken to empty the bottle when it is filled with* (i) *water,* (ii) *petroleum of relative density* ·8. *The viscocity of water and petroleum are* ·001 *and* ·002 *SI unit respectively. Density of water* $=1000$ kgm^{-3}.

Sol. We have for flow of liquid through a capillary tube

$$V \text{ (rate of volume flow)}=\frac{\pi P r^4}{8\eta l}.$$

But $P=h\rho g$ and $V=\dfrac{V_0}{t}$ where $V_0=$volume of the bottle.

$$\therefore \quad \frac{V_0}{t}=\frac{\pi h\rho g r^4}{8\eta l}.$$

Though h will change as the liquid flows out it is same for both the liquids. $\therefore \quad \dfrac{1}{t}\propto\dfrac{\rho}{\eta}$ because the rest are all same for both the liquids. $\quad\therefore \quad t\propto\dfrac{\eta}{\rho}.$

$$\therefore \quad \frac{t_1}{t_2}=\frac{\eta_1}{\eta_2}.\frac{\rho_2}{\rho_1}=\frac{\cdot 001}{\cdot 002}\times\frac{\cdot 8\times 1000}{1000}=\frac{2}{5}. \textbf{ Ans.}$$

Ex. 4. *Two equal drops of water are falling through air with a steady velocity of* 10 cm *per second. If drops coalesce, what will be the new velocity* ?

Sol. The terminal velocity is given by

$$v=\frac{2}{9}.\frac{r^2(\sigma-\rho)g}{\eta}; \quad \therefore \quad v\propto r^2, \text{ or } v=kr^2$$

$$\therefore \quad 10\times 10^{-2}=kr^2 \qquad \ldots (1).$$

When the two drops coalesce mass remains constant.

$$\therefore \quad 2\frac{4\pi}{3}r^3\sigma=\frac{4\pi}{3}R^3\sigma, \text{ or } R=2^{\frac{1}{3}}r.$$

If v be the velocity of drop formed then

$$v=k.R^2=k2^{\frac{2}{3}}r^2$$

$$\therefore \quad \frac{v}{10\times 10^{-2}}=\frac{kr^2 2^{\frac{2}{3}}}{kr^2}=2^{\frac{2}{3}}$$

$$\therefore \quad v=2^{\frac{2}{3}}\times 10\times 10^{-2}=15{\cdot}88\times 10^{-2} \text{ ms}^{-1}. \quad \textbf{Ans.}$$

Ex. 5. *A cylinder of radius* 10 cm *is filled with a liquid up to a height of* 20 cm. *Calculate the time in which the cylinder will be emptied half by a capillary tube of length* 10 cm *and radius* 1 mm. *Viscosity of liquid* = ·5 *SI units. Density of liquid* = 700 kgm^{-3}.

Sol. Let A = area of vessel; h = height up to which the vessel is filled with liquid initially.

Let the level go down by x in time t.

Then at time t height of liquid $= h - x$.

$\therefore$ Pressure difference between the ends of capillary tube

$$= (h-x)\rho g.$$

$\therefore$ Rate of volume flow at time $t = \dfrac{\pi(h-x)\rho g r^4}{8\eta l}$.

Let the level further go down by dx in time dt, then rate of volume flow $= \dfrac{dV}{dt} = \pi R^2 \dfrac{dx}{dt}$, where R = radius of the cylindrical vessel.

$$\therefore \quad \pi R^2 \frac{dx}{dt} = \frac{\pi(h-x)\rho g r^4}{8\eta l}$$

or
$$R^2 \frac{dx}{dt} = \frac{(h-x)\rho g r^4}{8\eta l}$$

or
$$dt = \frac{8\eta R^2 l}{\rho g r^4} \cdot \frac{dx}{h-x}.$$

$$\therefore \quad \int_0^T dt = \frac{8\eta R^2 l}{\rho g r^4} \int_0^{h/2} \frac{dx}{h-x}$$

or $$T = \frac{8\eta R^2 l}{\rho g r^4} \left[-\log(h-x)\right]_0^{h/2} = \frac{8\eta R^2 l}{\rho g r^4} . \log_e 2.$$

$$\therefore \quad t = \frac{8 \times \cdot 5 \times (10 \times 10^{-2})^2 . (10 \times 10^{-2})}{700 \times 9{\cdot}8 \times (1 \times 10^{-3})^4} \times 2{\cdot}3 \times \log_{10} 2$$

$$= \frac{8 \times \cdot 5 \times 2{\cdot}3 \times \cdot 3010}{7 \times 9{\cdot}8} \times 10^7 = 4 \times 10^5 \text{ s.} \quad \textbf{Ans.}$$

EXERCISES

(A)

1. The velocity of flow of water in the main pipe of water supply is 1·0 ms^{-1}. What is the velocity in the delivery pipe if area of cross-section of supply pipe is sixteen times that of delivery pipe ? **(Ans. 16 ms^{-1})**

2. Calculate the velocity of efflux of kerosene oil from a tank in which pressure is 4×10^5 Nm^{-2} above the atmospheric pressure. The density of kerosene oil is 800 kgm^{-3}. (**Ans.** 31·62 ms^{-1})

[*Hint* : $v=\sqrt{2gh}$.]

3. The reading of a pressure meter attached with a closed water pipe is $3{\cdot}5\times10^5$ Nm^{-2}. On opening the valve of the pipe, the reading of pressure meter reduces to 3×10^5 N^{-2}. Calculate the speed of water flowing in the pipe.

[*Hint* : Use Bernoulli's theorem.] (**Ans.** 10 ms^{-1})

4. A metal plate 100 sq. cm in area rests on a layer of castor oil 2 mm thick whose coefficient of viscosity is 1·55 SI units. Calculate the horizontal force required to move the plate with a speed of 3 cm per second. (**Ans.** ·2325 N)

$$\left[\textit{Hint}: F=-\eta A\frac{dv}{dx}. \text{ Here } \frac{dv}{dx}=\frac{3\times10^{-2}\ \text{ms}^{-1}}{2\times10^{-3}}=15\ \text{s}^{-1}.\right]$$

5. Calculate the critical velocity of water through a narrow tube of radius ·3 mm. Coefficient of viscosity of water is ·0011 SI units and density of water 1000 kgm^{-3}. Take Reynold's number 1000. (**Ans.** 3·67 ms^{-1})

6. Water is conveyed through a tube 8 cm in diameter and 4 kilometre in length at the rate of 120 litres per minute. Calculate the pressure difference required to maintain the flow. Coefficient of viscosity of water = ·001 SI units.

$$\left[\textit{Hint}: V=\frac{\pi P r^4}{8\eta l}.\right]$$

(**Ans.** $7{\cdot}96\times10^3$ Pa)

7. Find the terminal velocity acquired by a steel ball of diameter 3·2 mm falling under gravity through an oil of density 920 kgm^{-3} and viscosity 1·64 Nsm^{-2}. The density of steel may be taken as 7820 kgm^{-1}.

(**Ans.** $2{\cdot}35\times10^{-2}$ ms^{-1})

8. A steel ball-bearing, diameter 3 mm falls through glycerine and covers a distance of 25 cm in 10 sec. The sp. gr. of steel and glycerine are 7·8 and 1·26 respectively. Obtain value for the viscosity of glycerine. Density of water = 1000 kgm^{-3}. (**Ans.** 1·282 SI units)

9. An air bubble of radius 1 cm is found to rise in a cylindrical vessel of large radius at a steady rate of ·2 cm per second. If the density of liquid is 1470 kgm^{-3}, find its viscosity. $g=9{\cdot}8$ ms^{-2}.

[*Hint* : $\eta=\frac{2}{9}\cdot\frac{r^2(\sigma-\rho)g}{v}$. Here $\sigma=0$ as density of air is negligible in comparison to that of the liquid. Then $\eta=-\frac{2}{9}\cdot\frac{r^2\rho g}{v}$. The negative sign shows that bubble is going up.] (**Ans.** $1{\cdot}6\times10^2$ SI units)

10. Calculate the velocity with which rain drops reach us if diameter of rain drops is 1 mm and viscosity of air 18×10^{-6} SI unit. (**Ans.** 30·2 ms^{-1})

11. A bubble of air of 2 mm diameter rises in a liquid of viscosity ·075 SI unit and density 1350 kg per cubic metre. Find the terminal velocity of the bubble.

(**Ans.** $3{\cdot}96\times10^{-2}$ ms^{-1})

12. A vessel fitted with a horizontal capillary tube near its bottom is

alternately filled with two liquids of sp. gr ·8 and ·9. It empties itself in 40 and 30 minutes respectively. Calculate the ratio of their coefficient of viscosities. (Ans. 1·185 : 1)

(B)

13. A vessel of cross-section 20×10^{-4} square metre has at the bottom a capillary tube of length 10 cm and internal radius ·5 mm. It is initially filled with water to a height of 20 cm above the capillary tube. Find the time taken by the vessel to empty one-half its contents, given that the viscosity of water is ·001 SI units. Density of water = 1000 kg per cubic metre.

[*Hint* : See Ex. 5.] (Ans. 9 minutes 36 sec)

14. A tank is filled with water to a height *H*. A hole is punched in the wall at a depth *h* below the water surface. Find the distance *x* from the foot of the wall at which the stream strikes the floor. Could a hole be punched at another depth so that this second stream would have the same range ? If so at what depth ? [Ans. $2\sqrt{h(H-h)}$; Yes, a distance *h* above the bottom]

51. In a horizontal oil pipe line of constant cross-sectional area the pressure decrease between two points 300 km apart is 5000 pascal (1 pascal = 1 newton per square metre). What is the loss of energy per unit volume per unit distance ?

[*Hint* : Loss in energy $= \frac{P_1}{\rho} - \frac{P_2}{\rho}$ because $\frac{P}{\rho}$ is pressure energy per unit mass and loss in energy per unit volume $= P_1 - P_2$.] (Ans. ·01667 joule)

16. Show that a vessel filled with a liquid of density ρ up to a height *h* is emptied by a capillary tube exponentially with time according to the equation

$$x = h(1 - e^{-kt}) \text{ where } k \text{ is a constant}$$

depending on the radius of the vessel and the tube, coefficient of viscosity and density of the liquid and the length of the tube. Assume the tube to be connected at the bottom of the vessel.

□□□

CHAPTER 17

ERRORS AND THEIR MEASUREMENTS

1. Kinds of error : All experimental measurements involve certain uncertainty or error. We cannot make our measurements with all certainty and claim that measurements are hundred per cent accurate because neither instruments may be supposed to be 'perfect' nor any experimenter may be 'perfect'. Hence errors in measurements are to be taken as natural as anything else.

There are two types of errors—(i) systematic and (ii) non-systematic or random.

(i) Systematic error : Systematic errors are those which creep in measurements in a regular way due to imperfection of instruments, improper adjustment, neglect of some factor etc. We can always find the cause of such errors and due attention may be paid to them and minimised.

(ii) Random error : These are errors which creep in due to unkonwn causes beyond the control of the experimenter. These errors are inherently random in nature and hence are to be treated by statistical methods.

2. Propagation of errors : Sometimes we measure certain quantities and make calculation of something with the help of measured quantities. For example, in a simple pendulum we measure length and time of the pendulum and hence make calculation of g.

Errors in the measurement of observed quantities lead to error in the calculated results. This is called propagation of error.

(i) If $C = A \pm B$, $\triangle C = \triangle A + \triangle B$.

That is, the absolute error (not the percentage error) in a sum or difference of two quantities is the sum of the individual absolute errors of the quantities.

(ii) If $C = A \times B$ or $\frac{A}{B}$, $\frac{\triangle C}{C} = \frac{\triangle A}{A} + \frac{\triangle B}{B}$

or $\frac{\triangle C}{C} \times 100 = \frac{\triangle A}{A} \times 100 + \frac{\triangle B}{B} \times 100.$

That is, the percentage error (not the absolute error) in a product or quotient of two quantities is equal to the sum of the individual percentage errors of the observed quantities.

For proof of the above relations, see author's Introductory Physics Part I, chapter 17.

3. Computation of 'error' in a physical quantity :

Three errors are defined : (i) average error, (ii) standard or root mean square error, (iii) probable error.

Average error : Let $x_1, x_2, x_3 \ldots x_n$ be the result of n observations, n being very large and $\overline{x}$ be their arithmetic mean. Then $\delta_1 = x_1 - \overline{x}$, $\delta_2 = x_2 - \overline{x}$, $\delta_3 = x_3 - \overline{x}$, ..., $\delta_n = x_n - \overline{x}$ are called deviations or residuals of x. The deviations or residuals may be positive or negative.

The average of mod of deviations is defined as the average deviation or error.

$$\therefore \quad \overline{\delta} \text{ (average deviation)} = \frac{|\delta_1| + |\delta_2| + |\delta_3| + \ldots + |\delta_n|}{n}.$$

Standard deviation (S. D.) or r.m.s. deviation is defined as

$$\sigma = \sqrt{\frac{\delta_1^2 + \delta_2^2 + \delta_3^2 + \ldots + \delta_n^2}{n}} = \sqrt{\frac{\Sigma \delta^2}{n}}.$$

The Standard deviation of a physical quantity from a 'sample' of observations, say 10 observations only is given by

$$\sigma = \sqrt{\frac{\Sigma \,|\delta|^2}{(n-1)}}.$$

Probable error is defined as $\sigma_p = \dfrac{\cdot 8\ \overline{\delta}}{\sqrt{n-1}}$.

Average deviation of the mean $= \dfrac{\overline{\delta}}{\sqrt{n-1}}$.

4. Propagation of standard deviations :

(i) If $A = B + C$,

$$\sigma_A^2 = \sigma_B^2 + \sigma_C^2.$$

(ii) If $A = B \times C$,

$$\left(\frac{\sigma_A}{A}\right)^2 = \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_C}{C}\right)^2.$$

(iii) If $A = B/C$,

$$\left(\frac{\sigma_A}{A}\right)^2 = \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_C}{C}\right)^2.$$

(iv) If $A = B^m C^n$ then $\left(\frac{\sigma_A}{A}\right)^2 = m^2\left(\frac{\sigma_B}{B}\right)^2 + n^2\left(\frac{\sigma_C}{C}\right)^2$.

For proof of the above relations, see author's Introductory Physics Part I, chapter 17.

5. Significant Figures : The number of significant figures in the measurement of a physical quantity is an indication of its relative accuracy. 26·0 cm implies 3 significant figures and 26 cm implies 2 significant figures.

6. Addition (subtraction) and multiplication (or division) of physical quantities of different significant figures : Addition or subtraction of physical quantities of different significant figures should be carried only up to minimum places of decimals. To add 21·8315 to 0·13, proceed only up to second place of decimal. It is 21·86 after the usual procedure of rounding off.

In a multiplication or a division, the number of significant figures in the result should be equal to that combined in the factor which has the least number of significant figures. The multiplication 11·232 by 10·2 should be of 3 significant figures after the usual procedure of rounding off. It is 115.

The number of significant figures in the errors should be the least number of significant figures of the components.

EXAMPLES

Ex. 1. *A student takes ten readings for the focal length of a convex lens and he finds* 16·20, 15·90, 15·98, 16·01, 16·03, 15·90, 15·93, 16·30, 16·25 *and* 16·00. *Calculate the following :*

(*i*) *Average error,* (*ii*) *Standard deviation.*

Sol.

f	$\overline{f}=\frac{(\Sigma f)}{n}$	$\Delta f=f-\overline{f}$	$\lvert\Delta f\rvert$	$(\Delta f)^2$
16·20	16·05	+·15	·15	225×10^{-4}
15·90		−·15	·15	225×10^{-4}
15·98		−·07	·07	049×10^{-4}
16·01		−·04	·04	016×10^{-4}
16·03		−·02	·02	004×10^{-4}
15·90		−·15	·15	225×10^{-4}
15·93		−·12	·12	144×10^{-4}
16·30		+·25	·25	625×10^{-4}
16·25		+·20	·20	400×10^{-4}
16·00		−·05	·05	025×10^{-4}

$\Sigma\mid\Delta f\mid=1{\cdot}2 \quad \Sigma(\Delta f)^2=1938\times10^{-4}$

$$\therefore \quad \delta=\frac{\Sigma\mid\Delta f\mid}{n}=\frac{1{\cdot}2}{10}={\cdot}12. \qquad \textbf{Ans.}$$

$$\sigma=\sqrt{\frac{\Sigma(\Delta f)^2}{n-1}}=\sqrt{\frac{1938\times10^{-4}}{9}}=14{\cdot}67\times10^{-2}={\cdot}15. \qquad \textbf{Ans.}$$

Ex. 2. *The measures of length and breadth of a rectangle are : $l=(30{\cdot}0\pm{\cdot}2)$ cm and $b=(10{\cdot}0\pm{\cdot}1)$ cm. What is the percentage error and absolute error in area ?*

Sol. We have, length $=30{\cdot}0\pm{\cdot}2=30{\cdot}0\pm{\cdot}67\%$

breadth $=10{\cdot}0\pm{\cdot}1=10{\cdot}0\pm1\%$

Area = length × breadth.

Since in a product % errors get added,

$\therefore$ Area $=300\pm1{\cdot}57=300\pm5.$

(i) $\pm1{\cdot}67$, (ii) ±5 cm^2. **Ans.**

Ex. 3. *The measure of the radius of a sphere is $(3{\cdot}00\pm2\%)$ cm. Calculate the percentage error in volume and area. Also calculate absolute error in volume and surface area.*

Sol. We have, $V=\frac{4\pi}{3}r^3, \quad \therefore \quad \partial V=\frac{4\pi}{3}3r^2\partial r.$

$$\therefore \quad \frac{\partial V}{V}=\frac{4\pi r^2\partial r}{V}=\frac{4\pi r^2\partial r}{4\pi/3.r^3}=3\frac{\partial r}{r}.$$

$\therefore$ % error of volume $=3\times$ % error of radius $=3\times{\cdot}2={\cdot}6\%.$

$\therefore$ Volume of sphere $=(113\pm{\cdot}6\%)$ cm$^3=(113\pm{\cdot}7)$ cm^3.

We have, $S=4\pi r^2$; $\therefore\ \partial S=4\pi 2r\partial r$

or
$$\frac{\partial S}{S}=\frac{2\pi 2r\partial r}{4\pi r^2}=\frac{2\partial r}{r}$$

$\therefore$ % error of area $=2\times$ % error of radius $=2\times{\cdot}2={\cdot}4$.

$\therefore$ $S=(113\pm{\cdot}4\%)\ \text{cm}^2=(113\pm{\cdot}5)\ \text{cm}^2$.

$\therefore$ ·6%, ·7 cm^3 and ·4% and ·5 cm^2. **Ans.**

Ex. 4. *Show that the percentage error in the measurement of resistance by a metre bridge is minimum when the null point is near about the centre of the wire.*

Sol. The working formula for the calculation of unknown resistance in a metre bridge is

$$\frac{X}{R}=\frac{l}{100-l}\quad\text{or}\quad X=\frac{l}{100-l}\times R.$$

$$\partial X=R\partial[l(100-l)^{-1}]=R\partial l(100-l)^{-1}+Rl\partial(100-l)^{-1}$$

$$=\frac{R\partial l}{(100-l)}+\frac{Rl\partial l}{(100-l)^2},\ \text{(ignoring -- sign)}$$

$\therefore$
$$\frac{\partial X}{X}=\frac{R\partial l}{(100-l)X}+\frac{Rl\partial l}{(100-l)^2X}$$

$$=\frac{\partial l}{l}+\frac{\partial l}{(100-l)}=\partial l\left(\frac{1}{l}+\frac{1}{100-l}\right)$$

or
$$\frac{\partial X}{X}=\partial l.\frac{100}{l(100-l)}\quad\text{or}\quad\frac{\partial X}{X}\times 100=\frac{\partial l\times 100^2}{l(100-l)}.$$

Obviously % error is minimum when $l(100-l)$ is maximum.

Now, $l(100-l)=100l-l^2=50^2-50^2+100l-l^2$

or
$$l(100-l)=50^2-(50-l)^2$$

$\therefore$ $l(100-l)$ is maximum when $(50-l)^2$ is minimum. This is a positive quantity, its minimum value is zero.

$\therefore$ % error is minimum when $(50-l)^2=0$ or $l=50$. **Proved.**

EXERCISES

(A)

1. A student makes measurement of the diameter of a wire by a screw gauge and he finds the following readings : ·38, ·40, ·39, ·37, ·41. ·40, ·38, ·39, ·40 and ·41 mm. Calculate average error and standard deviation of his measurement.

[**Ans.** (i) ·011, (ii) ·013]

2. Two students A and B make measurement of the focal length of the same lens :

Readings of A : 16·35, 16·30, 16·27, 16·35, 16·50, 16·25, 16·30, 16·50, 16·20 and 16·40 cm.

Readings of B : 15·83, 16·38, 15·88, 16·22, 16·30, 16·43, 16·00, 15·80, 16·40 and 16·30.

Whose measurement is more reliable and why ?

(Ans. S. D. of $A = \cdot 100$ and S. D. of $B = \cdot 250$. Hence readings of A are more reliable than that of B.)

3. The ammeter and voltmeter connected in series and parallel with a resistor read as follows :

Voltmeter	2·50	2·00	1·50	1·00	0·52
Ammeter	0·52	0·42	0·29	0·21	0·10

Write a report about the resistance of the resistor in terms of S. D.

[*Hint* : Calculate R using Ohm's law formula, $R=\frac{V}{C}$ and then proceed as in Ex. 1.] [Ans. Resistance of resistor is : (4·94 ± ·22) ohm]

4. Time of fall of a body $= (10\cdot 0 \pm \cdot 5)$ sec.

Acceleration due to gravity $= (9\cdot 80 \pm \cdot 02)\ \text{ms}^{-2}$.

What are (i) percentage error in the distance described,

(ii) absolute error in the distance described ?

(Ans. % error $= \pm 10\cdot 2$; Absolute error $= \pm 50$ cm)

5. Mass of a body $= (100 \pm 2\%)$ kg,

Volume of the body $= (10 \pm 3\%)\ \text{m}^3$.

What are (i) the percentage error in density,

(ii) absolute error in density ?

[Ans. (i) $\pm 5\%$, (ii) $\pm \cdot 5\ \text{kg m}^{-3}$)

6. The measures of the length and breadth of a rectangle are $(30 \pm 3\%)$ m and $(10\cdot 0 \pm \cdot 5\%)$ m. Calculate percentage error in the perimeter of the rectangle. (Ans. 2·4%)

[*Hint* : Perimeter = 2 (length + breadth).]

7. In a simple pendulum $l = (100\cdot 0 \pm \cdot 5)$ cm and $t = (2\cdot 00 \pm \cdot 01)$ sec. What is the percentage error in g ? (Ans. $\pm 1\cdot 5\%$)

8. The modulus of rigidity of a wire is given by the formula

$$Wg = \frac{n\pi r^4 \theta}{2l}.$$

What of these quantities should be measured very accurately and why ?

(Ans. Radius 'r' because the greater the power of a quantity the greater is the propagation of % error in the estimated quantity.)

9. Show that the percentage error in the measurement of m/B by deflection magnetometer is minimum when deflection is near about 45°.

(B)

10. In a prism $\mu = \dfrac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$. Find the formula giving propagation of

error in μ due to error in *A* and *D*.

$$\left[\text{Ans. } \partial\mu = \frac{\sin\left(\frac{D}{2}+A\right)}{1-\cos A}\partial A + \frac{\cos\frac{+}{2}}{2\sin\frac{A}{2}}\cdot\partial D\right]$$

11. The voltage across a resistor $8\cdot0 \pm 0\cdot2$ volt and the current through it is $(3\cdot8 \pm 0\cdot1)$ ampere. What is the power consumed ? The errors quoted are the standard deviations in the measurements.
(**Ans.** $30\cdot4 \pm 1\cdot1$ watt)

$$\left[\textit{Hint}: \text{Use } W = V.I. \text{ and } \left(\frac{\sigma_W}{W}\right)^2 = \left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{\sigma_I}{I}\right)^2\right]$$

12. Calculate the volume of a sphere of radius $(3\cdot0 \pm 0\cdot1)$ cm, if the error quoted is the standard deviation in the measurement.
[**Ans.** (113 ± 11) cm^3]

$$\left[\textit{Hint}: V = \frac{4\pi}{3}r^3;\ \left(\frac{\partial V}{V}\right)^2 = 3^2.\left(\frac{\partial r}{r}\right)^2.\right]$$

□□□

CHAPTER 1

THERMOMETRY

1. **Perfect gas scale** : A scale of temperature is defined by some property of matter. The scale of temperature defined by 'expansion property of a perfect gas' on heating is called perfect gas scale.

For a perfect gas we have $V_t = V_0(1+\alpha t)$ where α is the coefficient of expansion of a perfect gas at constant pressure and t is the temperature on centigrade scale. At $t = -\frac{1}{\alpha}$°C, $V_t = 0$. This temperature at which volume of any gas will become zero is called absolute zero on this scale and the temperature of a body on this scale is called its absolute temperature. The temperature on absolute scale is written by $T°$.

$\therefore$ T (temperature on perfect gas scale) $= t + \frac{1}{\alpha}$,

where t is the corresponding temperature on centigrade.

For a perfect gas $\alpha = \frac{1}{273{\cdot}15}$.

$$\therefore \quad T = t + 273{\cdot}15.$$

2. **Thermodynamic scale or Kelvin scale** : Kelvin defined a scale of temperature making use of the fact that efficiency of an ideal Carnot engine depends only on the temperatures between which it works and does not depend on the working substance of the engine. This scale is called kelvin scale or thermodynamic scale of temperature. It is this scale which is adopted in SI system. The temperature of a body on this scale coincides with the temperature of the same body on perfect gas scale. Hence, whatever is the temperature of a body on perfect gas scale that is also temperature of the body on the thermodynamic scale.

3. **The Degree Celsius (°C)** : The old centigrade scale of temperature used the ice point at 0°C and the boiling point of water at one atmospheric pressure as the upper fixed point as 100°C. The Celsius scale temperature is the same scale of temperature with the zero shifted to the ice point on thermodynamic scale which is

273·15°K. Therefore if T°K be the corresponding temp. of θ°C on the thermodynamic scale then $\theta = T - 273{\cdot}15$.

Henceforth by '°C' we will mean celsius scale and read it as *celsius* and 'K' as *kelvin*.

4. Constant volume thermometer : In a constant volume thermometer, volume is kept constant and pressure is measured at the ice point (0°C) and at the boiling point of water (100°C) at normal pressure. The unknown temperature on celsius scale is given by

$$t = \frac{P_t - P_0}{P_{100} - P_0} \times 100.$$

5. Constant pressure thermometer : In this thermometer pressure is kept constant and volume is measured at the ice point and at the boiling point of water at normal pressure. The unknown temperature on celsius scale

$$t = \frac{V_t - V_0}{V_{100} - V_0} \times 100.$$

6. Platinum resistance thermometer : The resistance of a platinum wire at any temperature t°C is given by

$R_t = R_0\,(1 + \alpha t + \beta t^2)$ where R_0 and R_t are resistances at 0°C and t°C and α, β are two constants called first and second temperature coefficient of resistance. The resistance of the wire is measured at the ice point (0°C), boiling point of water (100°C) at normal pressure and boiling point of sulphur (444·4°C) at normal pressure to calculate the unknowns α and β.

To simplify the process of calculation a new scale of temperature is defined on the simplified formula $R_t = R_0\,(1 + \alpha t)$ and it is called the platinum scale. The unknown temperature on this scale is given by

$$t_p = \frac{R_t - R_0}{R_{100} - R_0} \times 100.$$

The correction required is given by

$$\text{correction} = t - t_p = \delta\left(\frac{t}{100}\right)\left(\frac{t}{100} - 1\right),$$

where $\delta = -\dfrac{100^2\beta}{\alpha + 100\beta}$. The value of δ is 1·5 for pure platinum.

In general if X is any property and X_{tr} be its value at the triple point, then enforcing a linear relation of it with abolute temperature T, we have on the thermodynamic scale

$$T=\frac{X}{X_{tr}}\times 273{\cdot}16°\text{K}$$

and on the celsius scale

$$t=\frac{X-X_0}{X_{100}-X_0}\times 100°\text{C}.$$

This temperature is not the correct value of the unknown temperature. Hence the scale is calibrated against the 'universal perfect gas scale' to find the correct temperature. This is exactly what is done in platinum resistance thermometer.

EXAMPLES

Ex. 1. *The pressure of air in a constant volume gas thermometer is* ·8 m *and* 1·093 m *at* 0°C *and* 100°C *respectively. When the bulb is placed in some hot bath, the pressure is* 1 m. *Calculate the temperature of the bath.*

Sol. We have, $t \; \frac{P_t-P_0}{P_{100}-P_0}\times 100.$

$$\therefore\quad t=\frac{1-{\cdot}8}{1{\cdot}093-{\cdot}8}\times 100=68{\cdot}3°\text{C}.\ \textbf{Ans.}$$

Ex. 2. *The resistance of a certain platinum resistance thermometer is found to be* 2·56 *ohm at* 0°C, 3·56 *ohm at* 100°C *and* 6·78 *ohm at* 444·6°C, *the boiling point of sulphur on gas scale of temperature. Calculate the temperature of the bath in which the platinum resistance is* 5·06 *ohm.*

Sol. We have, $t_p=\frac{R_t-R_0}{R_{100}-R_0}\times 100.$

Temperature of sulphur point on platinum scale is

$$t_p=\frac{6{\cdot}78-2{\cdot}56}{3{\cdot}56-2{\cdot}56}\times 100=422.$$

But actual temperature corresponding to boiling point of sulphur is 444·6.

Hence from $t-t_p=\delta\frac{t}{100}\left(\frac{t}{100}-1\right)$

we have, $$444{\cdot}6-422=\delta.\,\frac{444{\cdot}6}{100}\left(\frac{444{\cdot}6}{100}-1\right)$$

or $$\delta=\frac{22{\cdot}6}{4{\cdot}446\times3{\cdot}446}=1{\cdot}48.$$

Temperature on platinum scale is

$$t_p=\frac{5{\cdot}06-2{\cdot}56}{3{\cdot}56-2{\cdot}56}\times100=250.$$

Calculation of correction by the method of successive approximation : As first approximation, let us put $t=t_p=250$.

Then correction $=1{\cdot}48\times\frac{250}{100}\left(\frac{250}{100}-1\right)=1{\cdot}48\times2{\cdot}5\times1{\cdot}5=5{\cdot}55.$

$\therefore$ Correct temperature after 1st approximation $=250+5{\cdot}55$
$=255{\cdot}55.$

For second approximation we put $t=255{\cdot}55$.

$$\therefore\quad \text{Correction}=1{\cdot}48\times\frac{255{\cdot}55}{100}\left(\frac{255{\cdot}55}{100}-1\right)$$
$$=1{\cdot}48\times2{\cdot}556\times1{\cdot}556=5{\cdot}87.$$

$\therefore$ Correct temperature after 2nd approximation $=255{\cdot}87$.

For third approximation we put $t=255{\cdot}87$.

$\therefore$ Correction $=1{\cdot}48\times2{\cdot}559\times1{\cdot}559=5{\cdot}89.$

$\therefore$ Correct temperature after third approximation $=255{\cdot}89$.

Thus after third approximation we get sufficiently steady value. Hence correct temperature is 255·89°C. **Ans.**

Alternative method : Assume any two temperatures near t_p and calculate t_p corresponding to them using correction formula. We illustrate the whol process :

Let us assume $t=250$.

Then, $$250-t_p=1{\cdot}48\times\frac{250}{100}\left(\frac{250}{100}-1\right)$$
$$=1{\cdot}48\times2{\cdot}5\times1{\cdot}5=5{\cdot}55;$$
$$\therefore\quad t_p=244{\cdot}45.$$

Next assume $t=260$.

Then, $260-t_p=1{\cdot}48\times2{\cdot}6\times1{\cdot}6=6{\cdot}16;$

$\therefore$ $t_p=253{\cdot}84.$

Thus we have, when $t_p=253{\cdot}84,\ t=260$
$t_p=244{\cdot}45,\ t=250.$

With this at our hand we have to answer$\longrightarrow$when $t_p=250,\ t=?$

Difference of t_p by 9·39 ≡ Difference of t by 10;

$$\therefore\ \text{,,}\quad \text{,,}\quad \text{,,}\quad 1 \equiv\ \text{,,}\quad \text{,,}\quad \text{,,}\quad \frac{10}{9{\cdot}39}$$

$$\therefore\ \text{,,}\quad \text{,,}\quad \text{,,}\quad 3{\cdot}84 \equiv\ \text{,,}\quad \text{,,}\quad \text{,,}\quad \frac{10\times 3{\cdot}84}{9{\cdot}39}=4{\cdot}09.$$

$\therefore$ Correct temperature $=260-4{\cdot}09=255{\cdot}91°\text{C}.$ **Ans.**

Ex. 3. *If the ideal gas temperature at the steam point is 373·15°K, what is the limiting value of the ratio of the pressures of a gas at the steam point and at the triple point of water when the gas is kept at constant volume ?*

Sol. On the 'pressure property' of an ideal gas

$$T=\frac{p}{p_{tr}}\times 273{\cdot}16°\text{K}.$$

But gases in real existence are not perfect. They become perfect as they become thinner and thinner that is, their volume tends to infinity and pressure tends to zero. Hence the above result correctly reads

$$T=273{\cdot}16 \lim_{p_{tr}\to 0} \frac{p}{p_{tr}}$$

$$\therefore \qquad 373{\cdot}15=273{\cdot}16 \lim_{p_{tr}\to 0} \frac{p_{steam}}{p_{tr}}$$

$$\text{or} \qquad \lim_{p_{tr}\to 0} \frac{p_{steam}}{p_{tr}}=\frac{373{\cdot}15}{273{\cdot}16}. \qquad \textbf{Ans.}$$

Ex. 4. *The susceptibility of a paramagnetic gas varies as* $\chi=\frac{c}{T}$ *where T is the absolute temperature of the substance. If the value of its susceptibility at the triple point is* 140×10^{-8} *and at unknown temperature* 428×10^{-8}*, what is the unknown temperature defined on this property (call it 'magnetic scale') ? What is the correct temperature on the absolute scale and celsius scale ?*

Sol. On the susceptibility property

$$T=\frac{\chi}{\chi_{tr}}\times 273{\cdot}16=\frac{428\times 10^{-8}}{140\times 10^{-8}}\times 273{\cdot}16=835. \qquad \textbf{Ans.}$$

$$\because \quad \chi=\frac{c}{T},\ T=\frac{\chi_{tr}}{\chi}\times 273{\cdot}16=\frac{140\times 10^{-8}}{428\times 10^{-8}}\times 273{\cdot}16=89{\cdot}35°\text{K}.$$

$$\therefore \quad t=T-273{\cdot}15=89{\cdot}35-273{\cdot}15=-183{\cdot}8°\text{C}.$$

Ex. 5. *A certain platinum resistance thermometer has a resistance of* 90·35 *ohms when its bulb is placed in a triple-point cell. What is the temperature on the platinum scale when its resistance is* 96·28 *ohms? What is the correct temperature on the celsius and thermodynamic scale if the resistance varies as* $R=R_0(1+{\cdot}004\,t)$?

Sol. $T=\dfrac{R}{R_{tr}}\times 273{\cdot}16=\dfrac{96{\cdot}28}{90{\cdot}35}\times 273{\cdot}16=291{\cdot}09.$ **Ans.**

$\because\quad R=R_0\,(1+{\cdot}004\,t),\ 96{\cdot}28=90{\cdot}35\,(1+{\cdot}004\,t)$ or $t=16{\cdot}41°\text{C}.$ **Ans.**

$\therefore\quad T=t+273{\cdot}15=16{\cdot}41+273{\cdot}15=289{\cdot}56°\text{K}.$ **Ans.**

EXERCISES

(A)

1. The pressures indicated by a constant volume hydrogen thermometer are 2·235 m, ·75 m and 1·024 m of mercury when the bulb is immersed in a hot bath, ice and steam respectively. What is the temperature of hot bath ? (Ans. 524°C)

2. The volumes of a perfect gas enclosed in the bulb of a constant pressure thermometer are 1 m^3 and 2 m^3 at the ice point and boiling point of water at normal pressure. What is the temperature of the bath in which its volume is ·75 m^3 ?
(Ans. −25°C)

3. The pressures indicated by a constant volume hydrogen thermometer are 23·5 cm, 75 cm and 102·4 cm in liquid air, ice and steam respectively. What is the temperature of liquid air ?
(Ans. −187·95°C)

4. A constant volume air thermometer is used to determine the temperature of a furnace and the excess pressure in the bulb is found to be equal to 152×10^{-2} m of mercury. At 0°C the pressure in the bulb is equal to that of the atmosphere. If the barometric height throughout the experiment is 76×10^{-2} m of mercury, calculate the temperature of the furnace.

[*Hint* : Use formula $P_t=P_0\left(1+\dfrac{t}{273{\cdot}15}\right)$.] (Ans. 546°C)

5. The resistances of a platinum thermometer at 0°C, 100°C and 203°C are found to be 3·5, 5·2 and 6·9 ohm respectively. Find the temperature in which the resistance of thermometer is 9·4 ohms.
(Ans. Platinum scale temperature=347·1 and correct temperature=360·2°C)

6. The platinum wire has a resistance of 2 ohm at the temperature of melting ice, 2·778 ohm at temperature of boiling water under standard pressure and 2·54 ohm at the boiling point of a liquid. Assume $\delta=1{\cdot}5$ for pure platinum.
(Ans. $t_p=69{\cdot}41°$C and $t=69{\cdot}09°$C)

7. Calculate the temperature at which platinum scale temperature does not require any correction.
(Ans. 0°C and 100°C)

(B)

8. A certain mass of gas has a volume of 3×10^{-4} cubic metre when its pressure is 1 m of mercury and its temperature is 0°C. When heated to 100°C, the

volume of gas becomes $3{\cdot}2\times10^{-4}$ m^3 and the pressure 1·29 m. What is the temperature when the volume is $3{\cdot}3\times10^{-4}$ m^3 and pressure 1·4 m of mercury ?

[*Hint*: Use formula $PV=RT$ where T=absolute temperature$=t+\frac{1}{\alpha}$ where α=coefficient of cubical expansion of gas.] (Ans. 143·6°C)

9. A platinum resistance thermometer has 11 ohm resistance at the ice point, 15·247 ohm at the steam point and 28·887 ohm at the sulphur point (444·6°C). Find the first and second temperature coefficient of platinum.

(Ans. $\alpha=3{\cdot}92\times10^{-3}\text{K}^{-1}$; $\beta=-5{\cdot}89\times10^{-7}\text{K}^{-2}$)

10. If the resistance of a platinum thermometer at 0°C, 100°C and at the boiling point of sulphur (444·6°C) be 3·6, 4·6 and 7·82 ohm respectively, calculate the true temperature at which the resistance of the thermometer is 6·6 ohm.

(Ans. 309·6°C)

11. The magnetic susceptibility of a paramagnetic substance changes with absolute temperature as $\chi=\frac{c}{T-223^\circ}$ when $T>223$ and c is a constant. Derive an expression for celsius temperature t based on this property and establish the relation between t and T. What is the value of t corresponding to $T=423^\circ$K ? Take ice point$=273^\circ$K.

[*Hint* : Use $t=\frac{\chi-\chi_0}{\chi_{100}-\chi_0}\times100$] (Ans. $t=\frac{T-273}{T-223}\times150$; 112·5°C)

12. The thermo e m fs of a thermocouple at the triple point and at steam point are 4 mV and 5·4 mV respectively. Calculate the temperature on the thermoelectric scale. What are the corresponding temperatures on thermodynamic and celsius scale if the variation of thermo e m fs is governed by $e=\cdot014t+4$ when the e m fs are in millivolts ? (Ans. 368·77; 100°C, 373·15°K)

* * *

CHAPTER 2

EXPANSION OF SOLIDS, LIQUIDS AND GASES

1. Expansion of Solids : (i) The linear expansion of a solid is given by $l_t = l_0(1+\alpha t)$, where α is a constant called linear expansivity of the solid.

Unit, K^{-1} or $°C^{-1}$.

(ii) The superficial expansion of a solid is $S_t = S_0(1+\beta t)$, where β is another constant called superficial expansivity.

Unit, K^{-1} or $°C^{-1}$.

(iii) The cubical expansion (volume expansion) is given by

$$V_t = V_0(1+\gamma t)$$

where γ is a third constant of the solid called cubical expansivity.

Unit, K^{-1} or $°C^{-1}$.

Relation of α, β and γ : $\beta = 2\alpha$ and $\gamma = 3\alpha$.

2. Expansion of Liquids : In case of liquids there is no question of linear and superficial expansion; liquids have only cubical expansion. There are two cubical expansivity of liquids : (i) real expansion given by $V_t = V_0(1+\gamma_r t)$ and (ii) apparent expansion given by $V_t = V_0(1+\gamma_a t)$.

γ_r and γ_a are related by the relation

$$\gamma_r = \gamma_a + \gamma_g,$$

where γ_g = cubical expansivity of vessel.

Density variation with temperature :

We have, $V_t = V_0(1+\gamma t)$,

$$M \text{ (mass)} = V_t D_t = V_0 D_0,$$

$$\therefore \quad D_0 = D_t(1+\gamma t).$$

Weight thermometer : It is a device of determining γ_a and also γ_r provided γ_g is known or given.

$$\gamma_a = \frac{\text{mass expelled}}{\text{mass remaining} \times \text{rise in temperature}}.$$

Dulong and Petit's method : This is a method of determining γ_r by balancing a column of liquid at temperature t°C against another column of the same liquid at 0°C by taking it in a U-tube.

$$\gamma_r = \frac{h_t - h_0}{h_0 \times t}.$$

3. Expansion of Gases : The expansion of gases is governed by external condition under which gas is heated.

At constant pressure, $V_t = V_0(1+\gamma_p t)$ where γ_p is a constant of gas called cubical expansivity at constant pressure.

Unit→K^{-1} or $°C^{-1}$.

At constant volume, $P_t = P_0(1+\gamma_v t)$, where γ_v is a constant called pressure expansivity of gas.

The peculiarity with expansion of gases is that cubical expansivity of all ideal gases is the same and it is $\frac{1}{273{\cdot}15}$ whereas in case of liquids and solids it varies from substance to substance.

At constant temperature, $PV =$ a constant. This is known as 'Boyle's law' and the above two are known as 'Charles' law' and 'Pressure law' or Gay-Lussac's law.

4. Perfect Gas equation : Any gas obeying Charles' law and Boyle's law is termed as perfect gas or ideal gas. If p is the pressure, V is the molar volume and T is the temperature on absolute scale, then we have

$$pV = RT,$$

where R is a universal constant called gas constant. If the amount of gas is not one mole but say n mole then

$$pV = nRT.$$

It is to be noted here that V in this formula refers to the volume of n mole of a gas and R is the universal gas constant for a mole. Standard value of $R = 8{\cdot}3$ joule $mol^{-1}\ K^{-1}$. For any mass of a gas

$$pV = mrT$$

where r is the gas constant per unit mass. It is not a universal constant.

EXAMPLES

Ex. 1. *A circular hole in an aluminium plate is* 2·500 *cm at* 0°*C. What is its diameter when its temperature is raised to* 100°*C ?*

(*The linear expansivity of aluminium* $= 23 \times 10^{-6}\ K^{-1}$)

Sol. We have $l_t = l_0(1+\alpha t)$

∴ $l_{100} = 2{\cdot}50(1+23\times10^{-6}\times100)$

or $l_{100} = 2{\cdot}506$ cm. **Ans.**

Ex. 2. *A steel metre scale is to be ruled so that the millimetre intervals are accurate within about* 5×10^{-5} *mm at a certain temperature. What is maximum temperature variation allowable during the ruling ?* (α *for steel* $=11\times10^{-6}\ {}^\circ C^{-1}$)

Sol. We have, $\triangle l$ (change in length) $=\alpha l \triangle T$

∴ $5\times10^{-5}\text{ mm} = 11\times10^{-6}\times1\text{ mm}\times\triangle T$

or $\triangle T = 5^\circ$C. **Ans.**

Ex. 3. *A weight thermometer contains* 500 *gm of mercury at* $0^\circ C$. *If the temperature is raised to* $100^\circ C$, *how much mercury will be expelled ?* (*Cubical expansivity of mercury* $=182\times10^{-6}\ {}^\circ C^{-1}$ *and that of glass* $=27\times10^{-6}\ {}^\circ C^{-1}$.)

Sol. We have, $\gamma_a = \dfrac{\text{mass expelled}}{\text{mass remaining}\times\text{rise in temperature}}$.

Here $\gamma_a = 182\times10^{-6} - 27\times10^{-6} = 155\times10^{-6}\ {}^\circ\text{C}^{-1}$

$$\therefore\quad 155\times10^{-6} = \frac{x}{(500-x)\times100}$$

or $x = 155\times10^{-4}(500-x)$

or $x = 7{\cdot}63$ gm. **Ans.**

Ex. 4. *The density of air at N.T.P. is* $1{\cdot}293\ kgm^{-2}$. *Calculate its density when its temperature is* $16{\cdot}6^\circ C$ *and pressure is* 75 *cm of mercury.*

Sol. We have $PV = RT$ for a perfect gas

or $\dfrac{PV}{T} = \text{a constant.}$

∵ $D = \dfrac{M}{V}$,

∴ $\dfrac{P}{DT} = \text{a constant}$

$$\therefore\quad \frac{76\text{ cm of mercury}}{1{\cdot}293\times273} = \frac{75\text{ cm of mercury}}{D\times(273+16{\cdot}6)}$$

or $D = \dfrac{273}{289{\cdot}6}\times\dfrac{75}{76}\times1{\cdot}293 = 1{\cdot}203\text{ kgm}^{-3}$. **Ans.**

Ex. 5. *A piece of metal weighs 46 gm in air; when it is immersed in a liquid of relative density 1·24 at 27°C, it weighs 30 gm. When the temperature of the liquid is raised to 42°C, the metal piece weighs 30·5 gm. Specific gravity of the liquid at 42°C is 1·20. Calculate the linear expansivity of the metal.* (I. I. T. 1974)

Sol. A solid loses a part of its weight which is equal to the weight of the liquid displaced.

$$\therefore \quad 30 = 46 - \frac{46}{D_{27}} \times 1{\cdot}24 \text{ or } D_{27} = 3{\cdot}565$$

and

$$30{\cdot}5 = 46 - \frac{46}{D_{42}} \times 1{\cdot}20 \text{ or } D_{42} = 3{\cdot}561.$$

We have from the relation

$$D_1 = D_2\{1 + \gamma(t_2 - t_1)\},$$

$$3{\cdot}565 = 3{\cdot}561\{1 + \gamma(42 - 27)\}$$

or $$\gamma = {\cdot}00007.$$

$$\because \quad \gamma = 3\alpha,$$

$$\therefore \quad \alpha = {\cdot}000023 = 23 \times 10^{-6}\ {}^\circ\mathrm{C}^{-1}.$$ **Ans.**

Ex. 6. *A vessel of volume $V = {\cdot}03\ m^3$ contains an ideal gas at 0°C. After a portion has been let out, the pressure in the vessel decreased by $\Delta p = 0{\cdot}78$ atm (the temperature remaining constant). Find the mass of the released gas. The density under normal conditions*

$$\rho_0 = 1{\cdot}3\ kg\ m^{-3}.$$

Sol. We have for a perfect gas, $pV = RT$ for a mole or $pV = mrT$ for any mass of gas.

In terms of density, the equivalent form is $p = \rho rT$.

$$\therefore \quad p_0 = \rho_0 rT \text{ and } p = \rho rT$$

$$\therefore \quad \Delta p = p_0 - p = (\rho_0 - \rho) rT.$$

$$\text{Mass let off} = V\rho_0 - V\rho = V.\frac{\Delta p}{rT} = \frac{V\rho_0 \Delta p}{p_0} \quad \left(\because \quad rT = \frac{p_0}{\rho_0}\right)$$

$$= \frac{{\cdot}03 \times 1{\cdot}3 \times {\cdot}78 \text{ atm}}{1 \text{ atm}}$$

$$= {\cdot}03 \text{ kg.}$$ **Ans.**

Ex. 7. *A vessel of volume V contains n_1 mole of oxygen and n_2 mole of carbon dioxide at temperature T. Assuming the gases to be ideal, find :*

(a) *the pressure of the mixture;*

(b) *the mean molar mass of the mixture.*

Sol. (a) We have $pV=nRT$

$$\therefore \quad p_1V=n_1RT \text{ and } p_2V=n_2RT$$

and for the mixture $pV=nRT$.

$$\because \quad n=n_1+n_2, \quad \therefore \quad p=\frac{(n_1+n_2)RT}{V}. \qquad \textbf{Ans.}$$

(b) Molar mass of the mixture $=\dfrac{\text{mass of the mixture}}{\text{number of moles}}$.

$$\therefore \quad M=\frac{m_1+m_2}{n_1+n_2}=\frac{n_1M_1+n_2M_2}{n_1+n_2}$$

$$\left(\because \quad n_1=\frac{m_1}{M_1} \text{ and } n_2=\frac{m_2}{M_2}\right)$$

where M_1 and M_2 are molar masses of oxygen and carbon dioxide.

EXERCISES

(A)

1. A circular hole in an aluminium plate is 4 cm at 0°C. What is its diameter when the temperature of the plate is raised to 100°C ? (Linear expansivity of aluminium $=23\times10^{-6}$ K^{-1}) (Ans. 4·0092 cm)

2. An iron scale is correct at 0°C and at this temperature zinc rod measures 1 m on this scale. What will be the length of the zinc rod as measured by iron scale when both are at 100°C ? (Liner expansivity of zinc $=26\times10^{-6}$ K^{-1} and linear expansivity of iron $=12\times10^{-6}$ K^{-1}) (Ans. 1·0014 m)

3. If a clock with an uncompensated pendulum of steel keeps correct time at 15°C, how many seconds would it lose in 24 hours at a temperature of 25°C ? (Linear expansivity of steel $=12\times10^{-6}$ K^{-1}) (Ans. 5·2 s)

4. The linear expansivity of glass is 8×10^{-6}/K and cubical expansivity of mercury is 18×10^{-5}/K. What volume of mercury must be placed inside a flask of 300 c.c. so that the volume of the flask not occupied by mercury may remain the same at all temperatures ?

[*Hint* : $V_g\gamma_g t=V_m\gamma_m t$, or $V_g\gamma_g=V_m\gamma_m$] (Ans. 40 c.c.)

5. A porcelain crucible is filled with 1 kg of mercury at room temperature (20°C). The crucible is then placed inside a furnace, kept for sufficient time there and finally removed. It is found that it contains only ·8 kg of mercury. Calculate the temperature of the furnace. (Average cubical expansivity of mercury $=18{\cdot}2\times10^{-5}$ K^{-1} and that of porcelain $=2\times10^{-5}$ K^{-1}) (Ans. 1523°C)

6. The scale of a barometer is made of brass and is correct at 15°C. The reading of the barometer is ·754 m at 5°C. Correct this reading to 0°C. (Linear expansivity of brass $=18\times10^{-6}$ K^{-1} and cubical expansivity of mercury $=18\times10^{-5}$ K^{-1})

(*Hint* : 1 m at 15°C=1 m; $\therefore$ 1 m at 5°C$=(1-18\times10^{-6}\times10)$ m; $\therefore$ ·754 m at 5°C=·754 $(1-18\times10^{-5})$ m. This is the correct height at 5°C. Then use the formula, $P=h_0\rho_0 g=h_8\rho_t g$ where h_0=height of zero degree cold mercury barometer and h_t=height of t-degree hot mercury barometer. Standard pressure is the height of zero-degree cold mercury.] **(Ans. ·7532 m)**

7. A sinker of weight w_0 has an apparent weight w_1 when weighed in a liquid at temperature t_1 and w_2 when weighed in the same liquid at temperature t_2. The coefficient of cubical expansion of material of the sinker is β. What is the coefficient of volume expansion of the liquid ?

[*Hint* : Loss in wt.$=w_0-w_1$=wt. of liquid displaced$=V_1\rho_1$.]

$$\left[\textbf{Ans. } \gamma=\frac{w_2-w_1}{(w_0-w_2)(t_2-t_1)}+\left(\frac{w\ {}_0w_1}{w_0-w_2}\right)\beta\right]$$

8. A vessel of volume ·02 m³ contains a mixture of hydrogen and helium, at a temperature 20°C and 2 atm of pressure. The mass of the mixture is equal to 5 gm. Find the ratio of the mass of hydrogen to that of helium in the given mixture. **(Ans. ·5)**

9. A vessel contains 7 gm of nitrogen and 11 gm of carbon dioxide at a temperature T=290°K and pressure 1 atm. Find the density of this mixture, assuming the gases to be ideal. **(Ans. 1·5 kgm^{-3})**

(B)

10. A glass tube is attached to the bottom of a thin iron rod of length 1 m. How high must mercury be poured into the glass tube so that the centre of mass of the pendulum formed will not rise or fall with changes of temperature ?

Linear expansivity of glass$=9\times10^{-6}$ K^{-1}; that of iron$=12\times10^{-6}$ K^{-1} and cubical expansivity of mercury$=18{\cdot}2\times10^{-5}$ K^{-1}). **(Ans. ·146 m)**

11. A mercury hermometer reads 99·25°C when immersed in a steam bath in such a way that the whole of its stem above 0° graduation is outside the bath and the temperature of the room is 20°C. When the whole of its stem is pushed inside the bath, the thermometer reads 100·5. Calculate coefficient of cubical expansivity of mercury. (Given that linear expansivity of glass$=8\times10^{-6}$ K^{-1})

(Ans. 18×10^{-5} K^{-1})

12. A glass hydrometer reads sp. gr. 9200 in a liquid at 45°C. What would be the reading at 15°C ? (Cubical expansivity of liquid$=52{\cdot}5\times10^{-5}$/K and that of glass$=2{\cdot}4\times10^{-5}$/K) **(Ans. ·9338)**

13. A vessel of glass of constant volume contains 340 gm of mercury at 0°C. On putting few spherical iron balls 85 gm of mercury over-flow at 0°C. When the system is heated to 100°C, 4·8 gm of mercury flow out further. Calculate the coefficient of linear expansion of iron.

($\gamma_{mercury}=180\times10^{-6}$ K^{-1}) **(Ans. $11{\cdot}8\times10^{-6}$ K^{-1})**

[*Hint* : V_0 (Volume of iron balls)$=\dfrac{\cdot085}{\rho_0}$, volume of vessel$=\dfrac{\cdot34}{\rho_0}$ where ρ_0 is the density of mercury at 0°C.

At 100°C volume of the vessel=volume of mercury remaining+volume of iron balls$=\dfrac{\cdot 2502}{\rho_{100}}+V_{100}$, where ρ_{100}=density of mercury at 100°C and V_{100} is the volume of iron balls at 100°C. Since the vessel is of constant volume,

$$\therefore \quad \frac{\cdot 340}{\rho_0}=\frac{\cdot 2502}{\rho_{100}}+V_{100}.\Bigg]$$

14. A vertical cylinder closed at both ends is divided into two parts by a frictionless piston, each part containing one mole of air. At temperature 300°K the volume of the upper part is 4 times than that of the lower part. At what temperature will the volume of upper part be three times than that of the lower part ? (Ans. 422°K)

15. Two identical vessels are connected by a tube with a valve letting the gas pass from one vessel into the other if the pressure difference, $\Delta p \geqslant 1\cdot 1$ atm. Initially there is a vacuum in one vessel while the other contained an ideal gas at a temperature 27°C and 1 atm pressure. Then both vessels are heated to 107°C. Up to what value of pressure in the first vessel (which had vacuum initially) increase ? (Ans. ·083 atm)

□□□

CHAPTER 3

CALORIMETRY

1. Quantity of Heat : Energy in transit due to difference in temperature is called Heat. Heat is measured in joule (J) in SI. But other units are also in use, notably the calorie. The calorie is the amount of heat required to raise the temperature of 10^{-3} kg, that is, 1 gm of water by 1°C. Where quantities of heat are expressed in calories, it is recommended that the conversion factor to convert to joule be stated. (1 calorie = 4·2 joule)

2. Specific Heat Capacity : The amount of heat required to raise the temperature of 1 kg of a substance by 1 K is called Specific Heat Capacity. Unit, joule per kilogramme per kelvin ($Jkg^{-1}K^{-1}$).

3. Molar Heat Capacity : This is the amount of heat required to raise the temperature of 1 mole of a substance through 1 K. Unit, joule per mole per kelvin ($J\ mol^{-1}\ K^{-1}$).

4. Heat Capacity : The amount of heat required to raise the temperature of a body through 1 K. Unit, joule per kelvin (JK^{-1}).

Water equivalent : The mass of water having the same total heat capacity as the given body. Unit, kg.

5. Specific Latent Heat of Fusion : The specific latent heat of fusion of a substance is the amount of heat required to convert 1 kg of it from the solid state to the liquid state at its melting point without change of temperature. Unit, joule per kilogramme (Jkg^{-1}).

Specific latent heat of ice is $336 \times 10^3\ Jkg^{-1}$ or 80×10^3 calorie kg^{-1}.

6. Specific Latent Heat of Vaporisation : The quantity of heat required to convert 1 kg of a substance from liquid to vapour at a constant temperature (called boiling point) is called specific latent heat of vaporisation of that substance. Unit, joule per kilogramme (Jkg^{-1}).

Specific latent heat of vaporisation of water is $2250 \times 10^3\ Jkg^{-1}$ at 100°C.

7. Principle of Calorimetry : 'Heat lost = Heat gained' is the principle of Calorimetry.

Heat lost or gained by a body $= mst$ where $m =$ mass of body, $s =$ sp. heat capacity and $t =$ fall or rise of temperature.

Methods : (*i*) *Electrical method* (*Callendar and Barnes continuous flow method*) : In this method energy in the electrical form is converted into heat energy in a resistance wire and that heat is absorbed by the body surrounding the wire. Specific heat capacity is calculated by equating heat absorbed to the energy supplied.

Heat supplied $= VIt$ joules or I^2Rt joules

where $V =$ P. D. in volt, I is current in ampere and $t =$ time in second.

Heat absorbed $= ms\theta$ joule.

$$VIt = ms\theta.$$

Hence s is calculated.

(*ii*) *Method of Cooling* : This method is based on Newton s law of cooling which states that the rate of cooling of a hot body in a surrounding of constant temperature is proportional to the excess of its temperature above that of the surroundings.

If $m =$ mass of the body, $\theta =$ its temperature at time t, $s =$ specific heat capacity and $\theta_0 =$ temperature of the surrounding, then rate of cooling of the body $= ms\left(-\frac{d\theta}{dt}\right)$.

According to Newton's law of cooling,

$$-ms\frac{d\theta}{dt} \propto (\theta - \theta_0), \quad \text{or} \quad -ms\frac{d\theta}{dt} = k(\theta - \theta_0)$$

where k is a constant depending on (i) area of the body and (ii) nature of the surface.

$$\text{or} \qquad -\frac{d\theta}{dt} = c(\theta - \theta_0) \text{ where } c = \frac{k}{ms}.$$

Thus c depends on area, nature of surface and also on material of the body.

EXAMPLES

Ex. 1. *A platinum ball of mass* 100 gm *is removed from a furnace and immersed in a copper vessel of mass* 100 gm *containing water of mass* 390 gm *at* 30°C. *The temperature of water rises to* 40°C. *Calculate the temperature of the furance. (Given that the specific* *heat capacity of platinum* $=168$ J kg K^{-1}, *specific heat capacity of copper* $=420$ J kg^{-1} K^{-1} *and specific heat capacity of water* $=4200$ J kg^{-1} K^{-1}.

Sol. Heat lost by platinum $=100\times10^{-3}\times168(\theta-40)$ joule.

Heat gained by water and copper vessel

$$=100\times10^{-3}\times420(40-30)+390\times10^{-3}\times4200(40-30)$$

Heat lost $=$ Heat gained.

$$\therefore\quad 100\times10^{-3}\times168(\theta-40)$$
$$=100\times10^{-3}\times420(40-30)+390\times10^{-3}\times4200(40-30)$$

or $\theta=1040°\text{C}$. **Ans.**

Ex. 2. *Three liquids of equal masses are thoroughly mixed. The specific heat capacities of liquids are* s_1, s_3 *and* s_3 *and their temperatures* t_1, t_2 *and* t_3 *respectively. Find the temperature of the mixture.*

Sol. Let t be the temperature of the mixture.

Heat lost by 1st liquid $=ms_1(t_1-t)$

„ „ „ 2nd „ $=ms_2(t_2-t)$

„ „ „ 3rd „ $=ms_3(t_3-t)$

Net loss by all the liquids $=ms_1(t_1-t)+ms_2(t_2-t)+ms_3(t_3-t)$.

If we consider 'loss' as negative gain then principle of calorimetry 'Heat lost $=$ Heat gained' also means net gain of heat or net loss of heat $=0$. This follows from principle of conservation of energy.

$$\therefore\quad ms_1(t_1-t)+ms_2(t_2-t)+ms_3(t_3-t)=0$$

or
$$t=\left(\frac{s_1t_1+s_2t_2+s_3t_3}{s_1+s_2+s_3}\right).$$
Ans.

Ex. 3. *How* 1 kg *of water at* 50°C *should be divided so that one part of it when turned into ice at* 0°C *would by this change of state give out a quantity of heat that will be sufficient to vaporise the other part?* (*Specific latent heat of ice* $=336\times10^3$ Jkg^{-1}; *specific latent heat of steam* $=2250\times10^3$ Jkg^{-1} *and specific heat capacity of water* $=4200$ Jkg^{-1})

Sol. Let x kg of ice when turned into ice give out sufficient heat to vaporise remaining $(1-x)$ kg of water.

Heat lost by x kg of water $=x\times336\times10^3+x\times4200\times50$

$$=336x\times10^3+210x\times10^3$$
$$=546x\times10^3 \text{ joules.}$$

Heat gained by $(1-x)$ kg water at 50°C

$$=(1-x)\times2250\times10^3 \text{ joules.}$$

$$\therefore \quad (1-x)\times 2250\times 10^3 = 546x\times 10^3$$

$$\therefore \quad x = \cdot 812 \text{ kg.}$$

·812 kg for ice and ·188 kg for steam. **Ans.**

Ex. 4. *A spherical iron ball is placed on large block of dry ice at* 0°C *into which it sinks until half submerged. What was the temperature of the iron ?*

(*Density of iron* $= 7{\cdot}7\times 10^3$ kg m^{-3}; *density of ice* $= 920$ kg m^{-3}; *specific heat capacity of iron* $= 504$ Jkg^{-1} K^{-1}, *specific latent heat of fusion of ice* $= 336\times 10^3$ *joule* kg^{-1})

Sol. Let r be the radius of the spherical ball. Since the block is large, final temperature is 0°C. If ρ is the density of the ball and s is its specific heat capacity, then

$$\text{heat lost by iron} = \frac{4\pi}{3}r^3\rho\times s\times\theta$$

and heat absorbed by ice $= \dfrac{2\pi}{3}r^3\rho'\times L$ where $\rho' =$ density of ice

$$\therefore \quad \frac{2\pi}{3}r^3\rho' L = \frac{4\pi}{3}r^3\rho s\theta$$

$$\therefore \quad \theta = \frac{\rho' L}{2\rho s} = \frac{920\times 336\times 10^3}{2\times 7700\times 504}$$

($\because$ density of water in SI $= 1000$ kg m^{-3})

$$= 39{\cdot}82°\text{C}.$$ **Ans.**

Ex. 5. *A body cools from* 50°C *to* 40°C *in* 5 *minutes in surroundings maintained at* 20°C. *What will be its temperature after further* 5 *minutes ?*

Sol. From Newton's law of cooling we have, $-\dfrac{d\theta}{dt} \propto (\theta-\theta_0)$

$$\therefore \quad \frac{d\theta}{dt} = -c(\theta-\theta_0) \text{ where } c \text{ is a constant}$$

$$\text{or} \quad \frac{d\theta}{\theta-\theta_0} = -c\,dt, \quad \text{or} \quad \int\frac{d\theta}{\theta-\theta_0} = -c\int dt + A \text{ (a constant)}$$

$$\text{or} \quad \log(\theta-\theta_0) = -ct + A.$$

Here at
$$\left.\begin{array}{ll} t=0, & \theta=50 \\ t=5, & \theta=40 \\ t=10, & \theta=? \\ \multicolumn{2}{l}{\theta_0=20^\circ\text{C given.}} \end{array}\right\}$$

$\therefore$ We have

$\log(50-20)=A$ or $A=\log 30$

$\therefore$ $\log(\theta-\theta_0)=-ct+\log 30.$

Again putting second set of values

$\log(40-20)=-c.\,5+\log 30$

or $\log 20=-5c+\log 30$

or $5c=\log 30-\log 20$

or $c=\frac{1}{5}\log\frac{3}{2}$

$\therefore$ $\log(\theta-\theta_0)=-(\frac{1}{5}\log\frac{3}{2})t+\log 30$

$\therefore$ $\log(\theta-20)=-(\frac{1}{5}\log\frac{3}{2})\times 10+\log 30$

or $\log(\theta-20)=-2\cdot\log\frac{3}{2}+\log 30$

or $\log(\theta-20)=\log 30-\log(\frac{3}{2})^2$

or $\log(\theta-20)=\log 30-\log\dfrac{9}{4}=\log\dfrac{30\times 4}{9}$

$\therefore$ $\theta-20=\frac{120}{9}$, or $9\theta=300$ or $\theta=33{\cdot}33^\circ\text{C}.$ **Ans.**

Alternative Method :

Rate of fall of temperature $=\dfrac{50-40}{5}=2^\circ\text{C}$ per minute.

Average excess of temperature $=\dfrac{40+50}{2}-20=45-20=25.$

According to Law of cooling,

Rate $\propto$ excess of temperature

$\therefore$ $2=k.25,$ or $k=2/25.$

Let θ = temperature after another 5 minutes.

Then rate of fall of temperature $=\dfrac{40-\theta}{5}$

and average excess of temperature $=\dfrac{\theta+40}{2}-20=\dfrac{\theta}{2}$

$\therefore$ $\dfrac{40-\theta}{5}=k.\dfrac{\theta}{2}$

or $\dfrac{40-\theta}{5}=\dfrac{2}{25}.\dfrac{\theta}{2}$ (putting the value of k obtained above)

or $40-\theta=\theta/5$ or $200-5\theta=\theta$ or $6\theta=200$ **Ans.**

$\therefore$ $\theta=\frac{100}{3}=33{\cdot}33^\circ\text{C}.$

Ex. 6. 200 gm *of water and an equal volume of another liquid of mass* 250 gm *are placed in turn in the same calorimeter of mass* 100 gm *and specific heat capacity* 420 J kg^{-1} K^{-1}. *The liquids which are constantly stirred are found to cool from* 60°C *to* 40°C *in* 180 s *and* 140 s *respectively. Find the specific heat capacity of the liquid. The temperature of the surroundings* = 20°C.

Sol. Rate of cooling by water and calorimeter

$$=\frac{200\times10^{-3}\times4200(60-20)+100\times10^{-3}\times420\times(60-20)}{180}$$

$$=\frac{35280}{180}=\frac{3528}{18}\text{ Js}^{-1}.$$

Rate of cooling by liquid and calorimeter

$$=\frac{250\times10^{-3}\times s(60-20)+100\times10^{-3}\times420(60-20)}{140}$$

$$=\frac{10s+1680}{140}=\frac{s+168}{14}\text{ Js}^{-1}.$$

According to Newton's law of cooling the two rates are equal.

$$\therefore\quad \frac{s+168}{14}=\frac{3528}{18}\quad\text{or}\quad s=2576\text{ Jkg}^{-1}.\qquad\textbf{Ans.}$$

Ex. 7. *In the constant flow method of Callendar and Barnes it was found that when rate of flow of water was* 11 gm *per minute the heating current was* 2 *ampere and the difference in potential between the ends of the heating wire was* 1 *volt, the rise in temperature was* 2·5°C. *On increasing the rate of flow to* 25·4 gm *per minute, the heating current to* 3 *ampere and the p.d. to* 1·51 *volt, the rise of temperature of water was still* 2·5°C. *Calculate the value of J.*

Sol. Rate of supply of energy $=VC$ Js^{-1} $=VC$ watt.

Rate of absorption of heat by flowing water $=ms(\theta_2-\theta_1)$ calories per second where s = specific heat capacity of water in calories per kg per kelvin.

$\therefore\ VC=Jms(\theta_2-\theta_1)+h$ where h = rate of loss of heat by radiation.

Here two sets of readings are taken to eliminate h :

$$V_1C_1=m_1sJ(\theta_2-\theta_1)+h$$

and $$V_2C_2=m_2sJ(\theta_2-\theta_1)+h.$$

$$\therefore\quad V_2C_2-V_1C_1=(m_2-m_1)sJ(\theta_2-\theta_1),\quad\text{or}\quad J=\frac{V_2C_2-V_1C_1}{(m_2-m_1)s(\theta_2-\theta_1)}$$

$$\therefore\quad J=\frac{1{\cdot}51\times3-1\times2}{\dfrac{(25{\cdot}4-11)\times10^{-3}}{60}\times10^3\times2{\cdot}5}$$

($\because$ sp. heat capacity of water $=10^3$ caloric per kg per kelvin)

or $$J=\frac{2{\cdot}53\times60}{14{\cdot}4\times2{\cdot}5}=4{\cdot}216.$$ **Ans.**

Ex. 8. *Find the result of mixing* ½ kg *of ice at* 0°C *with* 2 kg *of water at* 30°C. ($L=336\times10^3$ Jkg^{-1} *and specific heat capacity of water* $=4200$ Jkg^{-1} K^{-1})

Sol. Let t°C be the final temperature.

Heat lost by water $=2\times4200(30-t)$;

Heat gained by ice $=\frac{1}{2}\times4200t+\frac{1}{2}\times336\times10^3$.

$\therefore\quad 2\times4200(30-t)=\frac{1}{2}\times4200t+\frac{1}{2}\times336\times10^3$.

Hence $t=8$°C.

(**Ans.** The result is 2·5 kg of water at 8°C)

Ex. 9. *What will be the result of mixing* 2 kg *of copper at* 100°C *with* ·75 kg *of ice at* 0°C ? (*Specific heat capacity of copper* $=378$ Jkg^{-1}K^{-1}; *specific heat capacity of water* $=4200$ Jkg^{-1}K^{-1} *and specific latent heat of fusion of ice* $=336\times10^3$ Jkg^{-1}.)

Sol. Let t be the final temperature.

$$2\times378\times(100-t)={\cdot}75\times336\times10^3+{\cdot}75\times4200\times t.$$

Hence $t=-45$°C which is absurd because final temperature cannot be less than 0°C and greater than 100°C. Hence we conclude that whole of ice will not melt and final temperature will be 0°C. Let x be the mass of ice that will melt.

$$\therefore\qquad x\times336\times10^3=2\times378\times100$$

Hence $$x={\cdot}225\text{ kg}.$$

(**Ans.** The result is ·225 kg of water at 0°C and ·425 kg of ice.)

Ex. 10. *When* 45 gm *of a metal heated to* 100°C *is dropped into an ice calorimeter, the contraction in volume of ice is observed to be* ·4596 $\times10^{-6}$ m^3. *What is the specific heat capacity of the metal ?* ($L=336\times10^3$Jkg^{-1} *and relative density of ice* $=$ ·917)

Sol. Density of water $=1000$ kgm^{-3}

and density of ice $={\cdot}917\times1000$ kgm^{-3}.

$\therefore$ Volume of 1 kg of water $= \frac{1}{1000}$ m^3

and volume of 1 kg of ice $= \dfrac{1}{\cdot 917 \times 1000}m^3 = \dfrac{1}{917}$m^3

$\therefore$ Contraction in volume when 1 kg of ice melts

$$= \left(\frac{1}{917} - \frac{1}{1000} \right)$$

$$= \tfrac{83}{917000} \text{ m}^3$$

$\therefore$ 1 m^3 contraction in volume $\equiv$ melting of $\dfrac{917000}{83}$ kg.

$\therefore$ $\cdot 4596 \times 10^{-6}$ m^3 contraction in volume

$$\equiv \text{melting of } \frac{917000}{83} \times \cdot 4596 \times 10^{-6} \text{ kg.}$$

Heat lost by solid $= 45 \times 10^{-3} s \times 100$ joule

Heat gained by ice $= \dfrac{917000}{83} \times \cdot 4596 \times 10^{-6} \times 336 \times 10^3$ joule

$$\therefore \qquad 45 \times 10^{-3} \times s \times 100 = \frac{917 \times 4596 \times 336 \times 10^{-4}}{83}$$

or $$s = \frac{917 \times 4596 \times 336}{83 \times 45} \times 10^{-3}$$

or $$s = 379 \cdot 1 \text{ Jkg}^{-1}\text{K}^{-1}.$$ **Ans.**

Ex. 11. *An alloy consists of n metals of masses* m_1, m_2, $m_3 \cdots \cdot$ m_n *and specific heat capacities* s_1, s_2, $s_3, \ldots, s_n$. *What is the specific heat capacity of alloy* ?

Sol. Suppose that m_1, m_2, $m_3, \ldots, m_n$ masses of metals are heated through t°C by Q_1, Q_2, $Q_3, \ldots, Q_n$ amounts of heat respectively. Then $Q_1 = m_1 s_1 t$; $Q_2 = m_2 s_2 t$; $Q_3 = m_3 s_3 t, \ldots, Q_n = m_n s_n t$. If now we give $(Q_1 + Q_2 + Q_3 + \ldots + Q_n)$ amount of heat to the alloy it will be heated through same temperature.

$$\therefore \qquad (Q_1 + Q_2 + Q_3 + \ldots . + O_n) = (m_1 + m_2 + m_3 + \ldots . + m_n) \quad s.t$$

where s is specific heat capacity of alloy.

or $$m_1 s_1 t + m_2 s_2 t + m_3 s_3 t + \ldots . + m_n s_n t$$
$$= (m_1 + m_2 + m_3 + \ldots . + m_n) st$$

$$\therefore \qquad s = \left(\frac{m_1 s_1 + m_2 s_2 + m_3 s_3 + \ldots . + m_n s_n}{m_1 + m_2 + m_3 + \ldots . + m_n} \right).$$ **Ans.**

EXERCISES

(A)

1. A copper vessel weighing 190 gm contains 300 gm of water at 0°C and 50 gm of ice at 0°C. Find the quantity of steam at 100°C, that must be passed into the vessel to raise its temperature and that of its contents by 10°C. [Specific heat capacity of copper $=420\ \text{Jkg}^{-1}\ \text{K}^{-1}$; L (of steam) $=2250\times10^{3}\ \text{Jkg}^{-1}$ and L (of ice) $=336\times10^{3}\ \text{Jkg}^{-1}$]
(**Ans.** $12{\cdot}27\times10^{-3}$ kg)

2. Find the result of mixing 1 kg of ice at 0°C with 1·5 kg of water at 45°C. (Specific latent heat of fusion of ice $=336\times10^{3}\ \text{Jkg}^{-1}$)
(**Ans.** ·16 kg of ice and 2·34 kg of water at 0°C)

3. What would be the final temperature of the mixture when 1 kg of ice at −10°C is mixed with 4·4 kg of water at 30°C ? (Specific heat capacity of ice $=2100\ \text{Jkg}^{-1}\ \text{K}^{-1}$.)
(**Ans.** 8·7°C)

4. An alloy consists of 40% copper and 60% nickel. A piece of alloy of mass ·1 kg is placed in a calorimeter whose water equivalent is 10×10^{-3} kg and which contains 90×10^{-3} kg of water at 10°C. If the final temperature is 20°C, calculate the original temperature of the alloy. (Specific heat capacity of copper $=399\ \text{Jkg}^{-1}\ \text{K}^{-1}$ and that of nickel $=462\ \text{Jkg}^{-1}\ \text{K}^{-1}$.)
(**Ans.** 116·2°C)

5. 4 gm of steam are added to a mixture of 35 gm of ice and 35 gm of water in a copper calorimeter weighing 50 gm. What is the final temperature ? Specific heat capacity of copper $=420\ \text{Jkg}^{-1}\ \text{K}^{-1}$, L (of ice) $=336\times10^{3}\ \text{Jkg}^{-1}$ and L (of steam) $=2268\times10^{3}\ \text{Jkg}^{1}$.)
(**Ans.** 3×10^{-3} kg of ice and 67×10^{-3} kg of water at 0°C)

6. How should 2 kg of water at 60°C be divided so that one part of it when turned into ice at 0°C would by this change give out sufficient heat to vaporise the other part ? (Specific latent heat of fusion of ice $=336\times10^{3}\ \text{Jkg}^{-1}$ and specific latent heat of vaporisation of steam $=2250\times10^{3}\ \text{Jkg}^{-1}$)
(**Ans.** 1·609 kg and ·391 kg)

7. The specific heat capacities of n liquids are $s_1, s_2, s_3, \ldots\ldots, s_n$ respectively and their respective temperatures $t_1, t_2, t_3, \ldots\ldots t_n$. Equal mass of each liquid is taken and mixed. Find the temperature of the mixture.

$$\left(\textbf{Ans. } \frac{s_1t_1+s_2t_2+s_3t_3+\ldots\ldots+s_nt_n}{s_1+s_2+s_3+\ldots\ldots+s_n}\right)$$

8. If a kg of ice at 0°C contracts by $91\times10^{-6}\ \text{m}^3$ on melting, calculate the specific heat capacity of a solid when 40 gm of it at 60°C dropped into an ice calorimeter cause a change in volume by $\cdot273\times10^{-6}\ \text{m}^3$. (Specific latent heat of fusion of ice $=336\times10^{3}\ \text{Jkg}^{-1}$)
(**Ans.** $420\ \text{Jkg}^{-1}\ \text{K}^{-1}$)

9. The relative density of ice is ·917; 10×10^{-3} kg of a metal at 100°C are immersed in a mixture of ice and water, and the volume of the mixture is found to be reduced by $\cdot125\times10^{-6}\ \text{m}^3$ without change in temperature. Find the specific heat capacity of the metal.
(**Ans.** $464\ \text{Jkg}^{-1}\ \text{K}^{-1}$)

10. When 20×10^{-3} kg of water at 15°C is placed in the tube of an ice calorimeter, it is found that the mercury thread moves through 29 cm. If a metal of mass 12 gm at 100°C is placed in the tube the mercury thread contracts by 12 cm. Find the specific heat capacity of metal. (Specific heat capacity of water $=4200\ \text{Jkg}^{-1}\ \text{K}^{-1}$)
(**Ans.** $434{\cdot}5\ \text{Jkg}^{-1}\ \text{K}^{-1}$)

11. A calorimeter whose water equivalent is 5×10^{-3} kg is filled with 50×10^{-3} kg of water at 80°C. The temperature falls to 75°C in 4 minutes. When filled with another liquid, the weight being 40×10^{-3} kg the time to fall through same range of temperature (from 80°C to 75°C) is 130 second. Find the specific heat capacity of liquid. (**Ans.** 2603 $\text{Jkg}^{-1}\,\text{K}^{-1}$)

12. A body cools from 50°C to 40°C in a surrounding of constant temperature 20°C in 5 minutes. What is the time in which it will cool by another 10°C ? (**Ans.** in next 8·33 minutes)

13. A copper calorimeter blackened outside is filled with some hot liquid and placed on a table. It is found to cool from 60°C to 50°C in 4 minutes and 40°C and 30°C in 8 minutes. What is the temperature of the surroundings ? Why is it blackened ? (**Ans.** 15°C; to promote radiation)

[*Hint* : From Newton's law of cooling we have

$$\frac{ms(\theta_1-\theta_2)}{t}=k\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$$
]

14. The temperatue of a body falls from 30°C to 20°C in 5 minutes. The air temperature is 13°C. Find the temperature of the body after further 5 minutes. (**Ans.** 15·88°C)

15. A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. What will be its temperature in 15 minutes and what is the temperature of its surroundings ? (**Ans.** 43°C and 16°C)

16. A calorimeter of water equivalent 10×10^{-3} kg contains 70×10^{-3} kg of a substance at its melting point and the substance is found to solidify completely in 21 minutes. A similar calorimeter containing 80×10^{-2} kg of water cools at the rate of 1·5°C per minute at the same temperature, the room temperature being same in both cases. What is the latent heat of fusion of the substance ? (Specific heat capacity of water = 4200 $\text{Jkg}^{-1}\,\text{K}^{-1}$) (**Ans.** 170×10^3 Jkg^{-1})

17. In continuous flow method of Callendar and Barnes the potential difference across the wire was 2 volt and the current 2 ampere. The temperature of in-going water was 20°C and that of outgoing water 22·7°C and 300 gm of water were collected in 10 minutes. When the p.d. was increased to 3·75 volt and the current to 2·5 ampere, the flow was adjusted to maintain the same temperature difference and 240 gm of water were collected in 5 minutes. Calculate the mean specific heat capacity of water at the mean temperature 21·35°C. (**Ans.** 4167 $\text{Jkg}^{-1}\,\text{K}^{-1}$)

18. The resistance of the wire of Callendar and Barnes apparatus is 10 ohm at 20°C. A cell of steady voltage 2·2 volts and 1 ohm internal resistance is connected to it. A liquid is slowly and steadily forced through it and the temperatures of the incoming and outgoing liquid were found to be 18°C and 22°C respectively in the steady state and 120 gm of the liquid were collected in 40 minutes. Find the specific heat capacity of the liquid. Neglect loss of heat by radiation. (**Ans.** 2000 $\text{Jkg}^{-1}\,\text{K}^{-1}$)

19. In the first experiment of Callendar and Barnes method the voltage across the heating wire was 2·01 volt and the current 7·81 ampere 501·2 gm of water passed through the tube of the apparatus in 22 minutes and the rise was 9·37°C. In the second experiment (to eliminate radiation effect) the voltage was 2·21 and the current 8·52 ampere. The flow of water was adjusted to give the same temperature difference and 607 gm passed through in the same time as before. Calculate specific heat capacity of water. **(Ans.** 4166 Jkg^{-1} K^{-1})

20. A liquid of density 8·50 kgm^{-3} flows through a calorimeter, at the rate of 8×10^{-6} cubic metre per second. Heat is added by means of a 250 watt electric heating coil and a temperature difference of 15°C is established in the steady conditions between the inflow and outflow points. Find the specific heat capacity of the liquid. **(Ans.** 2451 Jkg^{-1} K^{-1})

(B)

21. The temperatures of three different liquids *A*, *B* and *C* are 14°C, 24°C and 34°C respectively. On mixing equal masses of *A* and *B*, the temperature of the mixture is 20°C. On mixing equal masses of *B* and *C*, the temperature of the mixture is 31°C. Supposing equal masses of *A* and *C* were mixed, what would be the temperature of the mixture ? (I. I. T. 1976) **(Ans.** 29·6°C)

22. A copper calorimeter of negligible thermal capacity is filled with a liquid. The mass of the liquid equals 250 gm. A heating element of negligible thermal capacity is immersed in the liquid. It is found that the temperature of the calorimeter and its contents rises from 25°C to 30°C in 5 minutes when a current of 2·05 ampere is passed through it at potential difference of 5 volts. The liquid is thrown off and the heater is again switched on. It is now found that the temperature of the calorimeter alone is constantly maintained at 32°C when the current through the heater is ·7 A at the potential difference 6 volts. Calculate the specific heat capacity of the liquid. The temperature of the surroundings is 25°C. **(Ans.** 2100 Jkg^{-1} K^{-1})

23. A calorimeter contains ice. Determine the heat capacity of the calorimeter if 2·1 kJ of heat is required to heat it together with its content from 270 to 272°K, and 69·72 kJ of heat is required from 272 to 274°K.

L of ice $=336\times10^3$ Jkg^{-1}; specific heat capacity of ice $=2100$ Jkg^{-1} K^{-1}.

(Ans. 630 JK^{-1})

24. A heat-proof envelope contains 100 gm of ice at 1°C. It is compressed to 1200 atm. Find the mass of the melted ice if the melting point of ice is lowered by 1°C for increase of pressure by 138 atm.

L of ice $=336\times10^3$ Jkg^{-1} and specific heat capacity of ice $=2100$ Jkg^{-1} K^{-1}.

(Ans. 5·4 gm)

25. Assume that a planet radiates heat at the rate proportional to the fourth power of its surface temperature *T* and that the planet assumes such a steady temperature that this loss of heat is exactly compensated by the heat gained from the sun. Show that other things remaining the same, a planet's surface temperature varies inversely as the square root of its distance from the sun.

(JEE, WB 1982)

□□□

CHAPTER 4

SATURATED AND UNSATURATED VAPOUR : DALTON'S LAW OF PARTIAL PRESSURE AND RELATIVE HUMIDITY AND DEW POINT

1. Saturated and Unsaturated Vapour Pressure Law : The saturated vapour pressure (S. V. P.) of a liquid at a given temperature is solely dependent on **temperature** and independent of mass and volume of vapour. This is why if the volume of saturated vapour is increased in presence of the liquid more liquid will evaporate till saturation is attained and if the volume is decreased vapour will condense leaving that much vapour which is sufficient to saturate that space. Every time pressure will be maintained at a particular value characteristics of the temperature alone. Unsaturated vapour behaves to some extent like a perfect gas. Hence its pressure and volume will change in accordance with perfect gas law i.e. $pV=nRT$ where $n=$mass of vapour in mole.

2. Dalton's Law of Partial Pressure : It states that the pressure of a mixture (mixture of gases or gases and vapours or vapours having no chemical action) is equal to the sum of the pressures called their partial pressures which they would separately exert if they were allowed to occupy the whole space at the same temperature.

$$p=p_1+p_2+p_3+\cdots\cdots$$

3. Relative Humidity and Dew Point :

Dew point : The temperature at which atmosphere will become saturated with water vapour already present in it is called **Dew Point.**

Relative humidity : The degree of wetness of atmosphere is expressed by the term relative humidity. It is defined as the ratio of mass of water vapour present in any volume at present to the mass of water vapour that would saturate the same volume at the present temperature.

For a gas or vapour, Pressure $\propto$ Mass when temperature and volume remain constant.

$$\therefore \text{ Relative humidity}=\frac{\text{Vapour pressure at } t°\text{C}}{\text{Saturated vapour pressure at } t°\text{C}}.$$

Vapour pressure at t°C = S.V.P. at dew point.

$$\therefore \text{ Relative humidity} = \frac{\text{S. V. P. at dew point}}{\text{S. V. P. at present temperature}}.$$

Absolute humidity : The amount of water vapour present in one cubic metre of air is called Absolute humidity. Unit, kgm^{-3}.

EXAMPLES

Ex. 1. *A jar contains a gas and a few drops of water at T°K. The pressure in the jar is* 830 mm *of mercury. The temperature of the jar is reduced by* 1%. *The staturated vapour pressure of the two temperatures are* 30 mm *and* 25 mm *of mercury. Calculate the new pressure in the jar.*

(*I.I.T. 1980*)

Sol. By Dalton's law of partial pressure, the pressure of the gas at T°K is (830 − 30) mm of mercury and the pressure of the same gas at $\left(T - \frac{T}{100}\right)$°K is $(P-25)$ mm of mercury where P is the new pressure in the jar. Applying the ideal gas equation

$$PV = nRT \text{ to the gas}$$

$$\text{or} \quad 800 \times V = nRT$$

$$\text{and } (P-25) \times V = nR\left(T - \frac{T}{100}\right) = nRT.\frac{99}{100}.$$

Dividing

$$\frac{P-25}{800} = \frac{99}{100} \text{ or } P = 817 \text{ mm of mercury.}$$ **Ans.**

Ex. 2. *On a particular day when the atmospheric temperature is* 18·5°C *the dew point is found to be* 8·6°C. *Determine the relative humidity of air if the maximum vapour pressure of water be* 8·04, 8·61, 15·46 *and* 16·46 mm *at* 8°, 9°, 18° *and* 19° *respectively.*

Sol. Here Difference by 1°C ≡ (8·61 − 8·04) mm of mercury

∴ „ by ·6°C ≡ ·57 × ·6 = ·342 mm of mercury

∴ Saturated vapour pressure at 8·6°C = 8·04 + ·342

= 8·38

Similarly, saturated vapour pressure at 18·5°C = 15·46 + ·5

= 15·96

$$\therefore \text{ Relative humidity} = \frac{8\cdot38}{15\cdot96} \times 100 = 52\cdot5\%.$$ **Ans.**

Ex. 3. *If the temperature of air whose relative humidity is* 60% *falls from* 20° *to* 5°C, *calculate the fraction of the mass of water* *vapour contained in air which will condense into drops. (Saturated vapour pressure of water at* 20°C = 17·5 mm *and at* 5°C = 6·5 mm)

Sol. Since relative humidity

$$= \frac{\text{mass of water vapour present in a certain volume}}{\text{mass of saturated water vapour in the same volume}},$$

$\therefore \quad \frac{60}{100} = \frac{m'}{m}$ or $m' = \cdot 6m$ where m is the mass of saturated vapour present in a certain volume V at pressure 17·5 mm of mercury and temperature 20°C. From the perfect gas equation

$$PV = \frac{m}{M} RT \qquad \text{we have}$$

$$m = \frac{17 \cdot 5\, k \times VM}{R\,(273+20)} = \frac{17 \cdot 5\, VkM}{293\, R}$$

where 1 mm of $Hg = k$ Nm^{-2}.

$$\therefore \quad m' = \frac{6 \times 17 \cdot 5\, VkM}{293\, R} = \cdot 03584\, \frac{VkM}{R}.$$

The mass of saturated vapour present in the same volume at pressure 6·5 mm and temperature 5°C is given by

$$m'' = \frac{6 \cdot 5\, VkM}{R\,(273+5)} = \frac{6 \cdot 5\, VkM}{278\, R} = \cdot 02338\, \frac{VkM}{R}$$

$$\therefore \quad \text{mass of water vapour condensed} = (\cdot 03584 - \cdot 02338) \frac{VkM}{R}$$

$$= \cdot 01246 \frac{VkM}{R}$$

$$\therefore \quad \text{The required fraction} = \frac{\cdot 01246 \frac{VkM}{R}}{\cdot 03584 \frac{VkM}{R}} = \cdot 35. \qquad \textbf{Ans.}$$

Ex. 4. *The temperature of atmosphere is observed to be* 27°C *and the dew point* 18°C. *If the temperature falls to* 22°C, *what will be the new dew point ? The saturated vapour pressure at* 18°C = 15·46 mm *of mercury, at* 22°C = 20·88 mm *and at* 17°C = 14·78 mm *of mercury.*

Sol. We have unsaturated vapour pressure at 27°C
= saturated vapour pressure at 18°C = 15·46 mm of mercury

$$\frac{\text{vapour pressure at } 22°\text{C}}{\text{vapour pressure at } 27°\text{C}} = \frac{273+22}{273+27}$$

[∵ pressure ∝ temperature (absolute) at constant volume]

$$\therefore \text{ Vapour pressure at } 22°\text{C} = \frac{295}{300} \times 15{\cdot}46 = 15{\cdot}20.$$

This is much below the saturated vapour pressure at 22°C. Hence when the temperature falls to 22°C, the atmosphere is still unsaturated. The corresponding dew point is the temperature at which saturated vapour pressure is 15·20 mm of mercury.

Here difference at pressure by ·68 ≡ difference in temperature by 1°C

∴ ,, ,, ,, ,, ·42 ≡ ,, ,, $\frac{1}{{\cdot}62} \times 4{\cdot}2 = {\cdot}68$

∴ The new dew point = 17 + ·68 = 17·68°C. **Ans.**

EXERCISES

(A)

1. A mass of air is saturated with water vapour at 100°C. On raising the temperature to 200°C without change of volume, the mixture exerts a pressure of 2×10^5 pascal (Pa). What was the pressure of air alone in the initial condition ? (Saturation vapour pressure of water at 100°C = 10^5 Pa) (**Ans.** $1{\cdot}54 \times 10^5$ Pa)

2. The temperature of air in a closed space is observed to be 15°C and the dew point is 8°C. If the temperature falls to 10°C, how will the dew point be effected ? (Saturation vapour pressure at 7°C = 7·49 mm of mercury and that at 8°C = 8·02)

(*Hint* : Vapour pressure at 15°C = saturated vapour pressure at 8°C. Since volume remains constant, $\frac{\text{vapour pressure at } 15°\text{C}}{\text{vapour pressure at } 10°\text{C}} = \frac{288}{283}$; or, vapour pressure at 10°C = 7·88 mm of *Hg*. Find the temperature where 7·88 is the saturated vapour pressure.) (**Ans.** 7·74°C)

3. On a certain day the temperature of air is 30°C and relative humidity 50%. What fraction of the mass of water vapour would condense if the temperature falls to 10°C ?

(Saturated vapour pressure at 30°C = 31·7 mm and at 10°C = 9·2 mm)

[*Hint* : Relative humidity = $\frac{\text{mass of vapour present at } 30°\text{C}}{\text{mass of vapour when saturated at } 30°\text{C}}$

$$50 = \frac{m'}{m} \times 100, \text{ or } m' = \frac{1}{2} m.$$

Again, $\frac{\text{mass of saturated vapour at } 10°\text{C}}{\text{mass of saturated vapour at } 30°\text{C}} = \frac{\text{S. V. P. at } 10°\text{C}}{\text{S. V. P. at } 30°\text{C}} = \frac{9{\cdot}2}{31{\cdot}7}$

∴ Mass of saturated vapour at 10°C = $\frac{9{\cdot}2}{31{\cdot}7} \times m = m''$ (say).

This mass will remain and $(m' - m'')$ will condense. Hence calculate the required fraction which is $\frac{m' - m''}{m}$.] (**Ans.** ·42)

4. Find the mass of 1 cubic metre of moist air at 27°C and 759·2 mm of pressure, the dew point being 10°C. (Saturated vapour pressure at 10°C $=9{\cdot}2$ mm. Density of dry air at S.T.P. $=1{\cdot}293$ kgm^{-3}. Density of moisture at any temperature and pressure $=\frac{5}{8}$ of density of dry air at the same temperature and pressure)

(Ans. 1·1699 kg)

5. Water is introduced into the vacuum space of Fortin's barometer at 20°C. What mass of vapour will collect in the space ? (Volume of the space $=50\times10^{-6}$ m^3; Saturated vapour pressure at 20°C $=2{\cdot}2\times10^3$ Nm^{-2}, $R=8{\cdot}3$ J mol^{-1} and weight of vapour $=18$.)

(*Hint* : $PV=nRT$)

(Ans. $\cdot81\times10^{-3}$ gm)

6. A certain quantity of vapour of a liquid mixed up with air is contained in a vessel of constant volume. The pressure shown at 20°C is 80 cm of mercury and at 40°C it is 100 cm. The vapour pressure of the liquid at 20°C is 15 cm of mercury. Calculate the same at 40°C.

(Ans. 30·6 cm of mercury)

7. Calculate the mass of 1 litre of moist air at 27°C when the barometer reads 753·6 mm of mercury and the dew point is 16·1°C. (Saturation vapour pressure of water at 16·1°C $=13{\cdot}6$ mm of mercury; density of air at N.T.P. $=1{\cdot}293$ kgm^{-3} and density of saturated water vapour at N.T.P. $=\cdot808$ kgm^{-3}) (I.I.T. 1977)

(Ans. $1{\cdot}1589\times10^{-3}$ kg)

(B)

8. Two chambers containing m_1 and m_2 kg of a gas at pressures p_1 and p_2 are put into communication. What will be the pressure of the mixture ?

(*Hint* : Use $PV=nRT$)

$$\left[\text{Ans. } \frac{(m_1+m_2)p_1p_2}{m_1p_2+m_2p_1}\right]$$

9. Two gases which are at pressure p_1, volume V_1 and temperature T_1 and p_2, V_2 and T_2 respectively are forced into a vessel of volume V at temperature T. Calculate pressure of the mixture.

(*Hint* : Use Dalton's law of partial pressure.)

$$\left[\text{Ans. } \frac{T}{V}\left(\frac{p_1V_1}{T_1}+\frac{p_2V_2}{T_2}\right)\right]$$

10. Two vessels of equal capacity are connected together by a narrow tube of negligible volume. They contain a gas at atmospheric pressure at 27°C. One is cooled to 0°C and the other is heated to 100°C. Calculate the resulting pressure assuming that the volume of the vessels remain unchanged.

(Ans. 79·9 cm of mercury)

[*Hint* : This is a problem on equilibrium of two gases due to diffusion i.e. mass distribution. Let each vessel contain 'm' mole of the gas at 27°C then from $pV=mRT$, $m=\dfrac{76\ V}{300\ R}$. When the two parts are maintained at different temperatures, mass distribution takes place and an equilibrium is reached. Let p be the resulting pressure. Then from $pV=mRT$, m_1' (mass in mole in the vessel at 0°C) $=\dfrac{PV}{273\ R}$ and $m'_2=\dfrac{PV}{373\ R}$. The total mass remains same, $\therefore\ 2m=m_1'+m_2'$.]

* * *

CHAPTER 5

FIRST LAW OF THERMODYNAMICS AND MECHANICAL EQUIVALENT OF HEAT

1. Heat and Work : The energy in transit due to temperature difference is called 'heat' and the energy in transit due to any other cause except temperature difference is called 'work'. The mechanical work done by a system is given by

$$W=\int p dV = \text{area between } p\text{-}V \text{ curve and } V\text{-axis.}$$

2. The First Law of Thermodynamics : The principle of conservation of energy applied to thermodynamical processes is specially called the first law of thermodynamics.

If dQ is the 'inflow' of heat to a system, dW is the work done by the system, that is, the 'outflow' of work from the system and dU is the change in internal energy of the system, then

$$dQ=dU+dW. \qquad .. \quad \text{(i)}$$

This is the first law of thermodynamics. In this equation all the quantities are in the same energy units, namely, joules. Here dQ is positive if heat is gained by the system, dW is positive if work is done by the system on the environment.

If, however, the inflow of heat is dQ and dW is the work outflow from the system and no energy is retained by the system, then

$$dQ=dW.$$

But when dQ is **not** in joules but, say; in calories, then

$$dQ=J\,dW \qquad .. \quad \text{(ii)}$$

where J is a universal constant called Joule's mechanical equivalent of heat. Its value is 4·186 joules per calorie. In SI Joule's mechanical equivalent of heat does not exist.

3. Internal energy of an ideal gas : At constant volume

$$dW=0 \text{ and } dQ=C_v dT.$$

$\therefore\ C_v\ dT=dU$. Integrating, $C_vT=U+c$ where c is a constant. At $T=0$, $U=0$. Therefore $c=0$.

Thus $$U=C_vT \text{ for mole of a gas.} \qquad .. \quad \text{(i)}$$

If M is the molar mass of a gas, then for any mass m of a gas

$$U=\frac{m}{M}C_vT \qquad .. \quad \text{(ii).}$$

Now $$C_p - C_v = R \quad \text{or} \quad \frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

$$\text{or} \quad C_v = \frac{R}{\gamma - 1}.$$

Thus for an ideal gas $C_v = \dfrac{R}{\gamma-1}$ and $C_p = \dfrac{\gamma R}{\gamma - 1}$

$$\therefore \quad U = \frac{m}{M} \cdot \frac{RT}{\gamma - 1} \quad \left(\because \; C_v = \frac{R}{\gamma - 1} \right) \qquad \text{.. (iii)}$$

$$\text{or} \quad U = \frac{pV}{\gamma - 1} \qquad \left(\because \; pV = \frac{m}{M} RT \right) \qquad \text{.. (iv).}$$

The above various expressions give the internal energy of an ideal gas in terms of temperature, pressure and volume of the gas.

EXAMPLES

Ex. 1. *A waterfall whose vertical height is* 100 *m discharges water into a pool below the fall. Calculate the rise in temperature of water assuming that all the heat remains in the water.* (*Specific heat capacity of water* = 4200 *joule per kg per kelvin.*)

Sol. Here W (mechanical work done) = Loss in potential energy $= mgh = m \times 9{\cdot}8 \times 100$ J.

Heat produced $= m \times 4200 \times \theta$ joule.

$$\therefore \quad m \times 9{\cdot}8 \times 100 = m \times 4200 \times \theta$$

or $$\theta = \frac{9{\cdot}8 \times 100}{4200} = {\cdot}234°\text{C}.$$ **Ans.**

Ex. 2. *A bullet of mass* 20×10^{-3} *kg enters into a fixed block of wood with a velocity of* 100 ms^{-1} *and is brought to rest in the wood. Calculate the rise in temperature of the bullet if two-third of the heat produced is absorbed by the bullet.* (*Specific heat capacity of lead*= $= 32$ *cal* kg^{-1} K^{-1} *and* $J = 4{\cdot}2$ *J/calorie.*)

Sol. W (work done) $= \dfrac{1}{2} mv^2$ joule $= \dfrac{mv^2}{2J}$ calorie.

But all the heat is not absorbed by the bullet, only $\frac{2}{3}$ of heat is absorbed by the bullet.

$$\therefore \quad \text{Heat absorbed by the bullet} = ms\theta \text{ calorie} = \frac{2}{3} \cdot \frac{mv^2}{2J}$$

or $$\theta = \frac{v^2}{3Js} = \frac{100^2}{3 \times 4{\cdot}2 \times 32} = 25°\text{C}.$$ **Ans.**

Ex. 3. *How much work is done in supplying heat necessary to convert* 10 *gm of ice at* $-5°C$ *into steam at* $100°C$? (*Specific heat capacity of ice* $= 2100\ Jkg^{-1}\ K^{-1}$, *sp. latent heat of fusion of ice* $= 336 \times 10^3\ Jkg^{-1}$ *and that of steam* $= 2250 \times 10^3\ Jkg^{-1}$.)

Sol. Heat required to raise temperature of 10 gm of ice from $-5°C$ to $0°C = 10 \times 10^{-3} \times 2100 \times 5 = 105$ J.

Heat required to convert 10 gm of ice at 0°C into water at 0°C

$$= 10 \times 10^{-3} \times 336 \times 10^3 = 3360 \text{ J.}$$

Heat required to raise temperature from 0°C to 100°C

$$= 10 \times 10^{-3} \times 4200 \times 100$$

$(\because$ Sp. heat capacity of water is $4200\ \text{Jkg}^{-1}\ \text{K}^{-1})$

$$= 4200 \text{ J.}$$

Heat required to convert 10 gm of water at 100°C into steam at $100°C = 10 \times 10^{-3} \times 2250 \times 10^3 = 22500$ J

$\therefore$ Total heat required $= 105 + 3360 + 4200 + 22500$

$= 30165$ joule. **Ans.**

Ex. 4. *A meteorite of* 10^4 *kg enters into atmosphere with a velocity of* $10^3\ ms^{-1}$. *How many calories of heat is produced when it is stopped by friction with atomospheric air* ? ($J = 4{\cdot}2$ *joule per calorie*.)

Sol. Kinetic energy $= \frac{1}{2}\ mv^2$ joule

$$= \frac{1}{2}\ \frac{mv^2}{J} \text{ calorie} = \frac{1}{2} \times \frac{10^4 \times (10^3)^2}{4{\cdot}2}$$

$= 1{\cdot}19 \times 10^9$ calorie. **Ans.**

Ex. 5. *In the determination of J by Joule's experiment the falling weights each of mass* 1 *kg were allowed to fall through* 1 *m. When they fell* 80 *times the temperature of water in the calorimeter increased by* 3°*C. Calculate J if the time of each fall of weights is* 1 *second and water equivalent of calorimeter and its contents is* 100×10^{-3} *kg*; *Sp. heat capacity of water* $= 1000\ cal\ kg^{-1}\ K^{-1}$.

Sol. Loss in potential energy of weights per fall $= 2\ Mgh$.

Gain in kinetic energy of weights per fall $= 2.\frac{1}{2}\ Mv^2 = Mv^2$.

$\therefore$ Work done by weights per fall $= (2Mgh - Mv^2)$.

$\therefore$ Work done by weights in n falls $= n\ (2Mgh - Mv^2)$ joule.

Heat produced $= ms\theta$ cal.

$$\therefore \quad J=\frac{n(2Mgh-Mv^2)}{ms\theta} \text{ joule per calorie (J cal}^{-1}).$$

Now, if t = time of fall then $v=ft$ and $h=\frac{1}{2}ft^2$.

$$\therefore \quad v=\frac{2h}{t}.$$

$$\therefore \quad J=\frac{n\left(2Mgh-\dfrac{M4h^2}{t^2}\right)}{ms^2}$$

$$\therefore \quad J=\frac{80\left(2.1.9{\cdot}8.1-\dfrac{1.4.2^2}{1^2}\right)}{100\times10^{-3}\times1000\times3}=4{\cdot}16.$$

Ans. 4·16 J cal^{-1}.

Ex. 6. *In the Searle's friction cone experiment the hanging mass which was 250×10^{-3} kg remained stationary when the wheel was rotated at the rate of ½ revolution per second. It was found that the temperature of water in the cones increased by 3°C in 27 minutes. The total water eqvivalent of cones and water taken was 150 gm. Diameter of wooden disc = 30 cm; Sp. heat capacity of water = 1000 cal kg^{-1} K^{-1}. Calculate J.*

Sol. Couple balancing moment of frictional force

$$=Mg\times\frac{D}{2}=\frac{1}{2}MgD,$$

where D = diameter of wooden disc, M = hanging mass.

Work done in rotational motion = couple × angle of rotation

$$=\tfrac{1}{2}MgD\times2\pi n=Mg\pi nD,$$

where n = number of rotations.

Heat produced = $ms\theta$.

$$\therefore \quad J=\frac{Mg\pi nD}{ms\theta}$$

$$=\frac{250\times10^{-3}\times3{\cdot}142\times(\frac{1}{2}\times27\times60)\times30\times10^{-2}}{150\times10^{-3}\times1000\times3}$$

$=4{\cdot}24$ J cal^{-1}. **Ans.**

Ex. 7. *An immersion type electric heater of 250 watt is immersed in water of 5 kg contained in a bucket. What is the time in which temperature of water will rise by 10°C ? Sp. heat capacity of water = 1000 cal kg^{-1} K^{-1} and $J=4{\cdot}1$ J cal^{-1}. Neglect heat capacity of bucket.*

Sol. In electricity, Power $= VC$ watt $=$ Rate of supply of energy.

$$1 \text{ watt} = 1 \text{ Js}^{-1}; \quad \therefore \quad 250 \text{ watt} = 250 \text{ Js}^{-1}.$$

Let t be the time in which temperature will rise by t°C.

$$\text{Energy supplied} = 250t \text{ joule}$$

$$\text{Energy absorbed} = 5 \times 4100 \times 10 \text{ joule}$$

$$250t = 5 \times 4100 \times 10, \quad t = 820 \text{ sec} = 13 \text{ min } 40 \text{ sec.}$$ **Ans.**

Ex. 8. *A cyclic process for an ideal monatomic gas* ($C_V = 12{\cdot}5$ $Jmol^{-1}$ K^{-1}) *is represented in the Fig.* 5.1. *The temperatures at* 1, 2 *and* 3 *are* 300°*K*, 600°*K and* 455°*K respectively. Compute the values of* $\triangle Q$, $\triangle U$ *and* $\triangle W$ *for each of the processes. The process from* 2 *to* 3 *is adiabatic.*

Sol. In the process from 1 to 2,

$\triangle W = \int p\,dV = 0$ (volume remains constant)

$$\triangle Q = \int_{T_1}^{T_2} C_V dT = C_V(T_2 - T_1)$$
$$= 12{\cdot}5(600-300)$$
$$= 3750 \text{ joules.}$$ **Ans.**

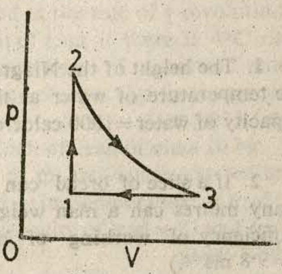

Fig. 5.1

By the first law of thermodynamics,

$$\triangle Q = \triangle U + \triangle W \text{ or } \triangle U = \triangle Q - \triangle W$$
$$= 3750 - 0 = 3750 \text{ joules.}$$

In the process 2 to 3, $\triangle Q = 0$ (since the process is adiabatic)

$$\triangle W = \frac{R(T_2 - T_3)}{\gamma - 1} \text{ (see calculation of this in Chapter 7)}$$

$$= C_V(T_2 - T_3) \qquad \left(\because \; C_V = \frac{R}{\gamma - 1}\right)$$

$$= 12{\cdot}5(600-455) = 12{\cdot}5 \times 145 = 1812{\cdot}5 \text{ joules.}$$

$$\therefore \quad \triangle U = \triangle Q - \triangle W = 0 - 1812{\cdot}5 = -1812{\cdot}5 \text{ joules.}$$

In the process from 3 to 1, $\triangle W = \int_V^{V_1} p dV = p(V_1 - V_3) = pV_1 - pV_2$

or $$\triangle W = R(T_1 - T_3) \qquad (\because \; pV = RT)$$
$$= 8{\cdot}31(300-455) = -1288 \text{ joules.}$$

$$\triangle Q = \int_3^{T_1} C_p \, dT = C_p(T_1 - T_3) = 1{\cdot}67 \times 12{\cdot}5 \times (300-455)$$

$$= -3235{\cdot}6 \text{ joules.} \qquad \left(\because \; \gamma = \frac{C_p}{C_v}\right)$$

By the first law of thermodynamics, $\triangle Q = \triangle U + \triangle W$

$$\therefore \quad \triangle U = \triangle Q - \triangle W = (-3235{\cdot}6) - (-1288) = -1989{\cdot}1 \text{ joules.}$$

Ex. 9. *Find the internal energy of air in a room of volume 40 m^3 at one standard atmospheric pressure.*

Sol. We have $U = \dfrac{pV}{\gamma - 1}$ for a perfect gas.

Air is diatomic and therefore its γ is 1·4.

$$\therefore \quad U = \frac{10^5 \times 40}{1{\cdot}4 - 1} \ (p = 1 \text{ atm} = 10^5 \text{ Nm}^{-2}) = 10^7 \text{ joules.}$$

Ans.

EXERCISES

(A)

1. The height of the Niagra falls is 50 m. Calculate the difference between the temperature of water at the top and the bottom of the fall. Specific heat capacity of water = 1000 calorie per kg and $J = 4{\cdot}2$ joule per calorie)
(Ans. ·116°C)

2. If a slice of bread can supply 100 kilocalorie of heat to a man, how many metres can a man weighing 60 kg can climb up by using this energy ? (Efficiency of working of human body = 30%; $J = 4{\cdot}2$ joule per calorie and $g = 9{\cdot}8$ ms^{-2}.)
(Ans. 214·3 m)

3. How much work is needed to convert 5 gm of ice at −3°C to steam at 100°C ? (Sp. heat capacity of ice = 500 cal kg^{-1} K^{-1}; Sp. latent heat of fusion of ice = 80×10^3 cal kg^{-1}; Sp. latent heat of vaporisation of water = 536×10^3 cal kg^{-1} and $J = 4{\cdot}2$ joule per calorie.)
(Ans. 15067·5 joule)

4. A tube 2 m long containing a little mercury and closed at both ends is rapidly inverted 50 times. What is the maximum rise in temperature expected ? (Specific heat capacity of mercury = 30 cal kg^{-1} K^{-1} and $J = 4{\cdot}2$.) (Ans. 7·78°C)

5. How much work is done against uniform pressure when 1 gm of water at 100°C is converted into steam ? Express your result in calories. ($J = 4{\cdot}2$ joule per calorie; volume of 1 kg of steam at 100°C = 1650×10^{-3} m^3 and 1 atmospheric pressure = 10^5 Nm^{-2}.)

[*Hint* : Work done = pressure × change in volume.] (Ans. 39·26 J)

6. A meteorite weighing 2000 kg enters into earth's atmosphere with a velocity of 1000 km per second. How many calories of heat will be produced ? ($J = 4{\cdot}19$ joule per calorie.)
(Ans $2{\cdot}38 \times 10^{14}$ calorie)

7. If a lead bullet be suddenly stopped and all its energy employed to heat it, with what velocity must the bullet be fired to raise the temperature through 100°C ? (the specific heat capacity of lead being 31·4 calorie per kg per kelvin and $J = 4{\cdot}2$ joule per calorie.)
(Ans. 162·4 ms^{-1})

8. From what height must a block of ice fall to be just melted by the impact assuming that half of the heat generated is absorbed by ice ? ($J = 4{\cdot}2$ joule per calorie and $L = 80$ kilocalorie per kg.)
(Ans. $6{\cdot}857 \times 10^4$ m)

9. With what velocity must a lead bullet at 50°C strike against an obstacle in order that the heat produced by the arrest of the motion, if all heat produced remain within the bullet, must be sufficient to melt it ?

(Specific heat capacity of lead=31 cal. $kg.^{-1}$ K^{-1}; melting point of lead=335°C; specific latent heat capacity of lead=5370 cal kg^{-1} and $J=4{\cdot}2$ joule cal^{-1}.) (Ans. 345 ms^{-1})

10. A block of ice dropped into a well of water, both ice and water being at 0°C. From what height must ice fall in order that 1/100 of it may melt? (L of ice$=80\times10^3$ cal kg^{-1}, $J=4{\cdot}2$ joule cal^{-1} and $g=9{\cdot}8$ ms^{-2}.) (Ans. 343 m)

11. In Joule's experiment the falling weights which were $1\frac{1}{2}$ kg each fell on average through 1 m. When they fell 84 times, the temperature of water in the calorimeter rose through 4°C. Calculate Joule's mechanical equivalent of heat if 150 gm is the water equivalent of calorimeter and its contents. The time of fall of the weights were recorded and found to be 2 seconds. Sp. heat capacity of water is 1000 cal kg^{-1} K^{-1}. (Ans. 3·91 joule cal^{-1})

12. In the friction cone experiment the hanging mass which was 250 gm remained stationary when the wheel was rotated at the rate of $\frac{1}{2}$ revolution per second. Calculate Joule's mechanical equivalent of heat if there is 4°C rise in temperature in 15 minutes. The total water equivalent of cones and water in them is 100 gm. Diameter of the wooden disc=50 cm, Sp. heat capacity of water=1000 cal per kg per kelvin. (Ans. 4·33 J cal^{-1})

13. A hole s drilled by a driller in a block of lead of mass 10 kg. The driller is driven by an electric motor of 30 r. p. m. and couple exerted by the motor on the driller is 10 Nm. Calculate the rise in temperature of lead in 10 minutes. $J=4{\cdot}2$ J cal^{-1}. Relative specific heat capacity of lead=·03 and sp. heat capacity of water 1000 cal kg^{-1} K^{-1}. (Ans. 14·96°C)

14. A bullet of lead melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C.

(Melting point of lead=327°C, specific heat capacity of lead =30 cal kg^{-1} K^{-1} and specific latent heat of fusion of lead=6000 cal kg^{-1}, $J=4{\cdot}2$ joules per calorie.) (Ans. 410 ms^{-1}) (**I. I. T. 1981**)

15. A thermally insulated vessel containing a gas whose molar mass is equal to M and the specific heat capacity ratio γ moves with a velocity V. Find the temperature rise resulting from the sudden stoppage of the vessel.

[*Hint* : $\triangle U=C_V\ \triangle T$.] [Ans. $\frac{1}{2}Mv^2(\gamma-1)/R$]

16. Gaseous hydrogen contained initially under standard conditions in a sealed vessel of volume 5×10^{-3} m^3 was cooled by 55°K. Find how much the internal energy will change and what amount of heat will be lost by the gas. (Ans. 252 joules; 252 joules)

(B)

17. Two lead spheres of masses 10 kg and 30 kg approach each other with speed 10 ms^{-1} and 20 ms^{-1} and collide completely inelastically. What is the heat produced by the collision? What is the rise in temperature if all the heat produced is retained by the spheres? (Specific heat capacity of lead=31 cal kg^{-1} K^{-1} and $J=4{\cdot}2$ joule cal^{-1})

[*Hint* : Consider conservation of momentum only. In complete inelastic collision the two bodies stick together after collision. Kinetic energy is not conserved in inelastic collision.] (Ans. 3375 joule; ·65°C)

18. A copper calorimeter of mass ·1 kg is filled with 52 gm of kerosene oil. A resistance wire of resistance 2 ohm is immersed in it and it is observed that there is 5°C rise in temperature in 6 minutes when 1 ampere of current is passed through it. Calculate the value of J. Sp. heat capacity of copper $=100$ cal kg^{-1} K^{-1} and Sp. heat capacity of kerosene oil$=500$ cal kg^{-1} K^{-1}.

[*Hint* : Power$=VC=C^2R$. Also see Ex. 7.] **(Ans.** 4·11 J cal^{-1})

19. The anode of a diode valve is bombarded by a stream of electrons each of mass 9×10^{-31} kg moving with velocity 10^7 ms^{-1}. If the mass of anode is ·5 gm and its specific heat capacity 100 cal. per kg ($J=4{\cdot}2$ J cal^{-1}) and 3×10^{17} electrons hit it per second, calculaie the rate at which its temperature rises. **(Ans.** 64·3°C per sec)

20. Water oozes out from a porous pot, the pressure inside being 20 atmosphere more than outside. If the temperature of water inside be 10°C, what would it be on coming out? ($J=4{\cdot}2$ joule cal^{-1}; atmospheric pressure $=1{\cdot}05\times10^5$ N m^{-2})

$$\left[\textit{Hint}: \text{Pressure energy per unit mass}=\frac{P}{\rho}.\right]$$

(Ans. 10·5°C)

21. As a result of heating a mole of an ideal gas at constant pressure by 72°C, a heat flow by an amount 1600 joules takes place. Find the work performed by the gas, the increment of its internal energy, and the value of γ.

[*Hint* : $\Delta Q=\Delta U+\Delta W$ or $C_p\,\Delta T=C_V\Delta T+\Delta W$
or $\Delta W=(C_p-C_V)\,\Delta T=R\Delta T$.]

(Ans. 597·6 J; 1002·4 J; 1·6)

22. What amount of heat is to be transferred to nitrogen in a heating process at constant pressure so that the gas may perform 2 joule of work? **(Ans.** 7 joules)

23. Two moles of a certain ideal gas at a temperature 300 K were cooled at a constant volume so that the pressure is reduced to half of the initial value. Then, as a result of heating at a constant pressure, the gas expanded till its temperature got back to the initial value. Find the total amount of heat absorbed by the gas in the process. **(Ans.** $2{\cdot}5\times10^3$ J)

24. Calculate the value of γ for a gaseous mixture consisting of n_1 moles of oxygen and n_2 moles of carbon dioxide. The values of γ for oxygen and carbon dioxide are γ_1 and γ_2 respectively. Assume the gases to be ideal.

$$\left[\textit{Hint}: U_m=U_1+U_2 \text{ and } U=\frac{nRT}{\gamma-1}.\right]$$

$$\left[\textbf{Ans. } \gamma=\frac{\gamma_1\gamma_2(n_1+n_2)-(n_1\gamma_1+n_2\gamma_2)}{n_1(\gamma_2-1)+n_2(\gamma_1-1)}\right]$$

25. Find how much heat is necessary to do internal work in converting 1 gm of water at the normal boiling point into steam at the same temperature. Latent heat capacity of steam$=540\times10^3$ cal kg^{-1}; volume of 1 kg of steam at 100°C$=1{\cdot}65$ m^3. **(Ans.** 500 calorie)

[*Hint* : $\Delta Q=\Delta U+\Delta W$; $\Delta W=p\ (V_2-V_1)$. Here $\Delta Q=L$, specific latent heat capacity and ΔU=change in internal energy=internal work done.]

26. Three moles of an ideal gas being initially at a temperature $T_0=273$ K were isothermally expanded $n=5$ times its initial volume and then isochorically heated so that pressure in the final state became equal to that in the initial state. The total quantity of heat transferred to the gas during the process equals $Q=80$ k J. Represent the whole process on p-V diagram. Find the adiabatic exponent of the gas. **(Ans.** 1·4)

□□□

CHAPTER 6

SPECIFIC HEAT CAPACITY OF GASES

1. There are two specific heat capacities of gases :

(i) Specific heat capacity at constant volume (c_v) : The amount of heat required to raise the temperature of 1 kg of a gas at constant volume through 1°C is called specific heat capacity of gas at constant volume.

(ii) Specific heat capacity at constant pressure (c_p) : The amount of heat required to raise the temperature of 1 kg of a gas through 1°C at constant pressure is called specific heat capacity of gas at constant pressure.

2. Relation of c_p and c_v : The specific heat capacity at constant pressure is greater than specific heat capacity at constant volume.

$c_p - c_v = r/J$ where r is the gas constant for 1 kg of a gas and c_p and c_v are in calorie. If however c_p and c_v are in joule then

$$\mathbf{c_p - c_v = r}.$$

If we consider 1 mole of an ideal gas then

$C_p - C_v = \frac{R}{J}$ where C_p and C_v are molar specific heat capacities at constant pressure and volume. Here C_p and C_v are in calories. If, however, C_p and C_v are in joule then $\mathbf{C_p - C_v = R}$.

3. Ratio of two specific heat capacities : The ratio of the two heat capacities of a gas is a constant of that gas and it depends on atomicity of the molecules of the gas. This ratio is denoted by γ.

Thus $$\gamma = \frac{\text{Specific heat capacity at constant pressure } (C_p)}{\text{Specific heat capacity at constant volume } (C_v)}.$$

For monatomic gas $\gamma = 1{\cdot}67$; for diatomic $\gamma = 1{\cdot}41$;
for triatomic $\gamma = 1{\cdot}33$.

4. Internal Energy of a perfect gas :

$$U = \frac{m}{M} C_v T = \frac{m\,RT}{M(\gamma - 1)} = \frac{pV}{\gamma - 1}$$

$$C_v = \frac{R}{\gamma - 1};\ C_p = \frac{\gamma R}{\gamma - 1}.$$

EXAMPLES

Ex. 1. *If the density of air at N.T.P. is* 1·293 kgm^{-3} *and its sp. heat capacity at constant volume is* 169 *cal.* kg^{-1} K^{-1}, *calculate the specific heat capacity at constant pressure.* (*Density of mercury* = 13·6 × 10^3 kgm^{-3} *at* 0°C *and* $J = 4{\cdot}2$ *joule cal*$^{-1}$.)

Sol. Volume of 1 kg of air $= \dfrac{1}{1{\cdot}293}$ m^3. $\left(\because \quad V = \dfrac{M}{D}\right)$

Now we have, $pV = rT$ for a gas of mass 1 kg.

$$r = \frac{pV}{T} = \frac{(\cdot 76 \times 13{\cdot}6 \times 10^3 \times 9{\cdot}8)\left(\dfrac{1}{1{\cdot}293}\right)}{273}$$

or $$r = \frac{\cdot 76 \times 9{\cdot}8 \times 13{\cdot}6 \times 10^3}{273 \times 1{\cdot}293}$$

$$= 287{\cdot}2 \text{ Jk}^{-1}$$

Now $$c_p - c_v = \frac{r}{J}, \text{ or } c_p = c_v + \frac{r}{J} = 169 + \frac{287{\cdot}2}{4{\cdot}2}$$

or $$c_p = 169 + 68{\cdot}4 = 237{\cdot}4 \text{ cal kg}^{-1}\text{ K}^{-1}.$$ **Ans.**

Ex. 2. *In Regnault's method of determining specific heat capacity of gas at constant pressure gas was stored in a reservoir* 50 *litre capacity at* 10°C *and* 6 *atmospheric pressure. The gas was heated to* 100°C *and passed through a calorimeter at* 10°C. *When the temperature rose to* 30°C *the calorimeter was detached from the heating bath. The pressure of the reservoir was found to fall to* 4 *atmospheric pressure in the mean time. Calculate the specific heat capacity of gas at constant pressure if water equivalent of calorimeter and its contents was* 50 gm. *Mass of* 1 *litre of gas at N.T.P.* = 1·25 × 10^{-3} kg.

Sol. Suppose m = mass of gas that escaped through calorimeter.

Then heat lost by gas $= mc_p\left(\theta - \dfrac{\theta_1 + \theta_2}{2}\right)$

$$= mc_p\left(100 - \frac{30 + 10}{2}\right) = mc_p \, 80 \text{ joule}.$$

Heat gained by calorimeter and its contents

$= 50 \times 10^{-3} \times 4200\,(30 - 10)$ ($\because$ Sp. heat capacity of water $= 4200$ Jkg^{-1} K^{-1})

$= 4200$ joule.

To calculate mass 'm' we note that for a perfect gas we have

$$\frac{P}{\rho T}=\text{a constant.}$$

$\therefore \quad \frac{P_1}{\rho_1 T}=\frac{P_2}{\rho_2 T}=\frac{P_0}{\rho_0 T}$ where P_1 = initial pressure, P_2 = final pressure, P_0 = normal pressure (1 atmosphere). T_0 = normal temperature (273°K) and T = temperature of reservoir.

$$\therefore \quad \rho_1=\frac{P_1\rho_0 T_0}{P_0 T} \text{ and } \rho_2=\frac{P_2\rho_0 T_0}{P_0 T}.$$

$\therefore \quad m$ = initial mass − mass remaining

$= V\rho_1 - V\rho_2 = V(\rho_1-\rho_2)$ where V = volume of reservoir

$$\text{or} \quad m=V\rho_0\frac{P_1-P_2}{P_0}\cdot\frac{T_0}{T}.$$

Here, $V=50$ litre $=50\times10^{-3}$ m^3; $\rho=1{\cdot}25$ kgm^{-3}

$$\therefore \quad m=50\times10^{-3}\times1{\cdot}25\left(\frac{6\text{ atmos.}-4\text{ atmos.}}{1\text{ atmos.}}\right)\cdot\frac{273}{273+10}$$

$$=50\times10^{-3}\times1{\cdot}25\times2\times\frac{273}{283}\text{ kg.}$$

$$\therefore \quad c_p=\frac{4200\times283}{80\times50\times10^{-3}\times1{\cdot}25\times273}=903\text{ Jkg}^{-1}\text{ K}^{-1}.$$ **Ans.**

Ex. 3. *In Jolly's differential calorimeter one of the spheres of volume* 1 *litre was filled with experimental gas at* 10 *atmospheric pressure. When steady state was attained the excess of steam that condensed on this sphere was* ·378 gm. *Calculate the specific heat capacity of gas at constant volume.* (*initial temperature of gas* = 15°C *and L of steam at* 100°C = 540×10^3 *cal* kg^{-1}, $J=4{\cdot}2$ J *cal*$^{-1}$ *and density of gas at* 1 *atmospheric pressure* = ·8 kgm^{-3}.)

Sol. Density of gas at 10 atmospheric pressure $=10\times{\cdot}8=8$ kgm^{-3} because pressure is proportional to density.

$\therefore$ mass of gas $=8\times1000\times10^{-6}=8\times10^{-3}$ kg ($\because \; M=V\times D$)

Heat gained by gas $=8\times10^{-3}\times c_v\,(100-15)=8\times85\times10^{-3}\,c_v$ J.

Heat lost by steam $={\cdot}378\times10^{-3}\times4{\cdot}2\times540\times10^3$ joule

$={\cdot}378\times4{\cdot}2\times540$ joule.

$$\therefore \quad 8\times85\times10^{-3}\,c_v={\cdot}378\times4{\cdot}2\times540$$

$$\text{or} \quad c_v=\frac{{\cdot}378\times4{\cdot}2\times540}{8\times85\times10^{-3}}$$

$$\text{or} \quad c_v=1260\text{ Jkg}^{-1}\text{ K}^{-1}.$$ **Ans.**

EXERCISES

(A)

1. The volume of each sphere of a Jolly differential steam calorimeter is 500 c.c. and the excess of steam condensed is ·1 gm. Find the specific heat capacity of the gas at constant volume. The initial temperature of gas is 15°C and density of gas 6 kgm^{-3}. The latent heat capacity of steam $=540\times10^3$ cal kg^{-1}. (Ans. 211·8 cal kg^{-1} K^{-1})

2. Find the value of Joule's mechanical equivalent of heat from the following data : density of hydrogen at N. T. P. is ·09 kg m^{-3}; its specific heat capacity at constant pressure=3400 cal kg^{-1} K^{-1} and ratio of specific heat capacities $(\gamma)=1\cdot4$.

$\left[\textit{Hint} : \text{Use } \gamma=\frac{c_p}{c_v} \text{ and } c_p-c_v=\frac{r}{J}.\right]$ (Ans. $J=4\cdot244$ J cal^{-1})

3. The specific heat capacity of helium at constant pressure is 1250 cal kg^{-1}k^{-1}. Assuming that the gas is monatomic calculate the mechanical equivalent of heat. Density of gas at N. T. P.=·1785 kg m^{-3}.

$\left[\textit{Hint} : \gamma \text{ for monatomic gas}=5/3. \text{ Use formula } c_p-c_v=\frac{r}{J}.\right]$

(Ans. 4·16 J cal^{-1})

4. The difference in the two molar specific heat capacities of gas is 2 calorie. Calculate the mechanical equivalent of heat assuming that molar volume of a gas at N. T. P. is ·0224 m^3. ($g=9\cdot81$ ms^{-2} and density of mercury$=13\cdot6\times10^3$ kg m^{-3}.)

[*Hint* : $C_p-C_v=\frac{R}{J}$ and use $PV=RT$.] (Ans. 4·16 J cal^{-1})

5. The velocity of sound through CO_2 at 0°C is 259 ms^{-1} and its specific heat capacity at constant pressure is 220 cal kg^{-1} K^{-1} and $\gamma=1\cdot31$. Calculate the value of J.

$\left[\textit{Hint} : C \text{ (velocity of sound)}=\sqrt{\frac{\gamma P}{\rho}};\ \frac{P}{\rho T}=r.\right]$

(Ans. 3·6 joule cal^{-1})

6. One cubic metre of air at 27°C and 10^5 Nm^{-2} pressure weighs 1·18 kg. Calculate the value of gas constant for 1 kg of gas and calculate c_p of air if c_v of it is 168 cal kg^{-1} K^{-1} and $J=4\cdot2$ J cal^{-1}.

(Ans. 282·5 J kg^{-1} K^{-1}; 235·3 cal. kg^{-1} K^{-1})

7. The density of a gas at N. T. P.=2·468 kgm^{-3} and $c_p=156$ cal kg^{-1} K^{-1}. Find the ratio of specific heat capacities of the gas. Is it diatomic or triatomic ? (Ans. 1·3; triatomic)

8. In the determination of sp. heat capacity at constant pressure by Regnault's method the gas was supplied from a reservoir of volume ·03 m^3 at 10°C and 6 atmospheric pressure. The pressure of the gas was reduced to 2 atmosphere at the end of the experiment, the temperature remaining constant at 10°C. The temperature of oil bath of the apparatus was maintained at 150°C.

The hot gas was led into a calorimeter at 10°C. The final temperature of the calorimeter and contents was 31·5°C and its water equivalent 210 gm. If the density of the gas is ·089 kg per cubic metre at N. T. P., calculate the specific heat capacity of the gas at constant pressure.

(**Ans.** 3391 calories per kg per kelvin)

9. One cubic metre of hydrogen at 0°C and 76 cm of *Hg* weighs ·0896 kg. The specific heat capacities of hydrogen at constant pressure and volume are 3409 and 2411 cal per kg per kelvin respectively, Calculate the value of *J*. ($g=9{\cdot}81$ ms^{-2}; density of mercury$=13{\cdot}6\times10^3$ kg per cubic metre.) (**Ans.** 4·15)

(B)

10. The gram-molecular sp. heat capacity of hydrogen at constant pressure $=6{\cdot}865$ cal and volume$=0{\cdot}0224$ cubic metre and coefficient of expansion of hydrogen at constant pressure $(\alpha)=\frac{1}{273{\cdot}3}$ per kelvin. Calculate the value of *J*. (1 atmospheric pressure$=1{\cdot}013\times10^5$ newton per square metre.)

[*Hint* : Use $\gamma=\frac{C_p}{C}$. Hydrogen being diatomic $\gamma=1{\cdot}4$; $PV=RT$; *use* $T=t+\frac{1}{\alpha}$ to calculate the value of temperature on perfect gas scale;

$$C_p-C_v=\frac{R}{J}.\Big]$$

(**Ans.** 4·232 J cal^{-1})

11. Two thermally insulated vessels 1 and 2 are filled with air and connected by a short tube equipped with a valve. The volumes of the vessels, the pressures and temperatures of air in them are V_1, p_1, T_1 and V_2, p_2, T_2 respectively. Find the air temperature after the opening of the valve.

$$\left(\textbf{Ans. } T=\frac{T_1T_2(p_1V_1+p_2V_2)}{(p_1V_1T_2+p_2V_2T_1)};\ p=\frac{p_1V_1+p_2V_2}{V_1+V_2}\right)$$

[*Hint* : Use the facts that $U=U_1+U_2$ and $m=m_1+m_2$.)

12. If the kinetic energy of molecules in 5 litres of helium at 2 atm is *E*, what is the kinetic energy of molecules of 15 litres of oxygen at 3 atm in terms of *E* ? $\left[\textit{Hint} : \text{Use } U=\frac{RT}{\gamma-1}\right]$ (**Ans.** 7·5 *E*)

13. One mole of oxygen is mixed with one mole of helium. What is γ of the mixture ? (**Ans.** 5/4)

□□□

CHAPTER 7

ISOTHERMAL AND ADIABATIC PROCESSES

1. Isothermal Process : When a gas expands or shrinks with heat entering or leaving it but its temperature remaining constant, it is said to undergo an isothermal process. For a perfect gas

$$\mathbf{pV = a\ constant} \qquad \ldots \quad (1)$$

in an isothermal process. The constant is RT where R is the gas constant and T is the absolute temperature.

$$\textbf{Work done in isothermal change} = \int_{V_1}^{V_2} pdV = \int \frac{RT}{V}\, dV$$

$$= RT \log_e \frac{V_2}{V_1}. \qquad \ldots \quad (2)$$

When a gas is expanded or compressed **slowly**, the process is isothermal.

2. Adiabatic Process : When a gas expands or shrinks so that heat can neither enter nor leave it, the process is said to be adiabatic.

For adiabatic change we have, $\mathbf{pV^{\gamma} = a\ constant\ (k)}$ $\qquad \ldots \quad (3)$
where γ is the ratio for the specific heat capacity of gas at constant pressure to specific heat capacity at constant volume. We have $pV = RT$ always.

Eliminating p, we have $\quad \mathbf{TV^{\gamma-1} = a\ constant} \qquad \ldots \quad (3a)$

Eliminating V, we have $\quad \mathbf{T^{\gamma}p^{1-\gamma} = a\ constant} \qquad \ldots \quad (3b)$

$$\text{Work done in adiabatic process} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} kv^{-\gamma}dV$$

$$= k\,\frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{1-\gamma}$$

$$= \frac{p_2V_2^{\gamma}.V_2^{1-\gamma} - p_1V_1^{\gamma}.V_1^{1-\gamma}}{1-\gamma} \quad (\because\ p_1V_1^{\gamma} = p_2V_2^{\gamma} = k)$$

$$\text{Work done in adiabatic process} = \frac{R(T_1 - T_2)}{\gamma - 1} \qquad \ldots \quad (4)$$

When a gas is compressed or expanded **suddenly**, the change is adiabatic.

EXAMPLES

Ex. 1. *A motor car tyre is pumped up to a pressure of two atmospheres at* 15°C *when it suddenly bursts. Calculate the resulting drop in temperature of the escaping air* ($\gamma = 1{\cdot}4$).

Sol. As it bursts suddenly, the change is adiabatic.

We have, $T^{\gamma}.p^{1-\gamma} = \text{a constant.}$

$\therefore$ $T_1{}^{\gamma}p_1{}^{1-\gamma} = T_2{}^{\gamma}p_2{}^{1-\gamma}$

or $(273+15)^{\gamma}(2p_0)^{1-\gamma} = T_2{}^{\gamma}\ (p_0)^{1-\gamma}$ where $p_0 = 1$ atmosphere

or $288^{1{\cdot}4}.2^{1-1{\cdot}4}\ p_0{}^{1-\gamma} = T_2{}^{1{\cdot}4}\ p_0{}^{1-\gamma}$

or $288^{1{\cdot}4}2^{-{\cdot}4} = T_2{}^{1{\cdot}4}$

or $$T_2{}^{1{\cdot}4} = \frac{288^{1{\cdot}4}}{2^{{\cdot}4}}$$

or $1{\cdot}4 \log T_2 = 1{\cdot}4 \log 288 - {\cdot}4 \log 2$

or $$\log T_2 = \log 288 - \frac{{\cdot}4 \log 2}{1{\cdot}4}$$

$$= 2{\cdot}4594 - \frac{{\cdot}4 \times {\cdot}3010}{1{\cdot}4}$$

$$= 2{\cdot}4594 - {\cdot}0860 = 2{\cdot}3734$$

$\therefore$ $T_2 = 206{\cdot}3$ K

$\therefore$ Drop in temperatue $= 288 - 236{\cdot}3 = 51{\cdot}7$ K. **Ans.**

Ex. 2. *A certain volume of dry air at* 20°C *is expanded to three times of its volume* (i) *slowly,* (ii) *suddenly. Calculate in each case the final pressure and temperature. Atmospheric pressure* $= 10^5 \text{Nm}^{-2}$, γ *of air* $= 1{\cdot}4$.

Sol. When process is slow, temperature remains constant and we may take it as an isothermal process for which we have

$$pV = \text{a constant.}$$

$\therefore$ $10^5.V = p_2.3V$ or $p_2 = \dfrac{10^5}{3} = 3{\cdot}3 \times 10^4 \text{ Nm}^{-2}$. **Ans.**

When process is sudden, it is adiabatic.

In adiabatic change temperature-volume relation is

$$TV^{\gamma-1} = \text{a constant}$$

$$(273+20)\;V^{1\cdot4-1}=(273+t)(3V)^{1\cdot4-1}$$

or $\quad 293V^{\cdot4}=(273+t)3^{\cdot4}V^{\cdot4}$, or $(273+t)=\dfrac{293}{3^{\cdot4}}$

$\therefore \quad \log(273+t)=\log293-\log3^{\cdot4}=\log293-{\cdot}4\log3$

or $\quad \log(273+t)\;\; 2\cdot4669-{\cdot}4\times{\cdot}4771=2\cdot2761$

$\therefore \quad 273+t=\text{anti}\log2\cdot2761=188\cdot8$

$\therefore \quad t=-84\cdot2°\text{C}.$

To find pressure we have to use 'pressure-volume' relation in adiabatic change which is $pV^{\gamma}=\text{constant}$.

$\therefore \quad 10^5\,V^{1\cdot4}=p_2(3V)^{1\cdot4}$, or $10^5\,V^{1\cdot4}=p_2 3^{1\cdot4}.V^{1\cdot4}$

or $\quad p_2=\dfrac{10^5}{3^{1\cdot4}}$

$\therefore \quad \log p_2=\log10^5-1\cdot4\log3$

$\qquad =5-1\cdot4\times{\cdot}4771=5-{\cdot}6479=4\cdot3521$

$\therefore \quad p_2=\text{anti}\log4\cdot3521\;\; 2\cdot25\times10^4\ \text{Nm}^{-2}$

Ans. In slow process final temperature $=20°$C

and pressure $=3\cdot3\times10^4\ \text{Nm}^{-2}$.

In sudden process final temperature $=-84\cdot2°$C

and pressure $=2\cdot25\times10^4\ \text{Nm}^{-2}$.

Ex. 3. *A certain volume of a gas (diatomic) expands isothermally at temperature* 20°C *until its volume is doubled and then adiabatically until its volume is again doubled. Find the final temperature of the gas.* ($\gamma=1\cdot4$). *There is* ·1 *mole of the gas. Also calculate work done in the two cases.* $R=8\cdot3\ J\ mol^{-1}\ K^{-1}$.

Sol. We require T-V relation to calculate the final temperature.

We have, $TV^{\gamma-1}=\text{const.} \quad \therefore \quad (273+20).V^{\gamma-1}=(273+t)(2V)^{\gamma-1}$

or $\quad 273+t=\dfrac{293}{2^{1\cdot4-1}}=\dfrac{293}{2^{\cdot4}}$

$\log(273+t)=\log293-{\cdot}4\log2=\log293-{\cdot}4\times{\cdot}3010$

$\qquad =2\cdot4669-{\cdot}1204$

or $\quad \log(273+t)=2\cdot3465$ or $273+t=\text{anti}\log2\cdot3465$

or $\quad 273+t=222\cdot1; \quad \therefore \ t=-50\cdot9°\text{C}.$ **Ans.**

(i) Work done in isothermal process

$$=RT\log_e\frac{V_2}{V_1}=\frac{8\cdot3}{10}\times293\log_e\frac{2V}{V}$$

$$= \cdot 83 \times 293 \times 2 \cdot 3 \log_{10} 2 \quad (\because \quad \log_e x = 2 \cdot 3 \log_{10} x)$$

$$= \cdot 83 \times 293 \times 2 \cdot 3 \times \cdot 3010 = 1 \cdot 684 \times 10^2 \text{ J.} \qquad \textbf{Ans.}$$

(ii) Work done in adiabatic process

$$= \frac{R(T-T')}{\gamma - 1} = \frac{\cdot 83(293 - 222 \cdot 1)}{1 \cdot 4 - 1}$$

$$= \frac{\cdot 83 \times 70 \cdot 9}{\cdot 4} = 1 \cdot 47 \times 10^2 \text{ J.} \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. A quantity of air at 10°C ($\gamma = 1 \cdot 4$) is compressed (a) slowly and (b) suddenly to one-third of its initial volume. Find the change in temperature in each case. **[Ans.** (a) no change, (b) 165·98° C]

2. A volume of gas at atmospheric pressure is compressed adiabatically to half of its original volume. Calculate the resulting pressure ($\gamma = 1 \cdot 4$). 1 atmosphere $= \cdot 76$ m of mercury. (**Ans.** 2 m of *Hg*)

3. A gas of $\gamma = 1 \cdot 4$ and initial temperature 0°C is suddenly compressed to $\frac{1}{5}$th of its original volume. Calculate the rise in temperature. (**Ans.** 246·5°C)

4. A volume of gas at 15°C expands adiabatically until its volume is doubled. Find the resultant temperature given that the ratio of specific heat capacity of the gas $= 1 \cdot 4$. (**Ans.** $-54 \cdot 7$°C)

5. A gas of given mass at a pressure of 10^5 Nm^{-2} expands isothermally until its volume is doubled and then adiabatically until its volume is again doubled. Find the final pressure of the gas. ($\gamma = 1 \cdot 4$) (**Ans.** $1 \cdot 89 \times 10^4$ Nm^{-2})

6. A gram mole of a gas at 27°C expands isothermally until its volume is doubled. Calculate the amount of work done. ($R = 8$ J mol^{-1}K^{-1})

$$\left[\textit{Hint}: \text{Work done in isothermal process} = RT \log_e \frac{V_2}{V_1} \right]$$

(**Ans.** $1 \cdot 66 \times 10^3$ J)

7. The tube of a motor car containing air at 27°C and 2 atmospheric pressure bursts suddenly. Find the temperature immediately after bursting. ($\gamma = 1 \cdot 4$)

(**Ans.** $-26 \cdot 9$)

8. A certain mass of a gas is taken at 0°C in a cylinder whose walls are perfect insulators. The gas is compressed (a) slowly, (b) suddenly till its pressore is increased 20 times the initial pressure ($\gamma = 1 \cdot 42$). Calculate final temperature in each case. (**Ans.** in each case final temperature is 389·2°C)

9. One cubic metre of argon at 27°C is adiabatically compressed so that the temperature is 127°C. Calculate the new volume of the gas ($\gamma = 5/3$).

(**Ans.** ·65 m^3)

10. A gas at constant pressure P_1, volume V_1 and temperature T_1 is suddenly

compressed to $\frac{V_1}{2}$ and then slowly expanded to V_1 again. Find the final temperature and pressure. (**Ans.** $2^{\gamma-1} P_1$ and $2^{\gamma-1} T_1$)

11. A cubic metre of dry air at N. T. P. is allowed to expand to 5 cubic metre (i) isothermally, (ii) adiabatically. Calculate in each case pressure, temperature and work done. ($\gamma=1{\cdot}4$ and 1 atm$=1{\cdot}013\times10^5$/Nm^{-2})

[**Ans.** (i) $\frac{1}{5}$ atm or $2{\cdot}03\times10^4$ Nm^{-2}, 0°C and $3{\cdot}6\times10^3$ J,
(ii) $1{\cdot}064\times10^4$ Nm^{-2}, $-129{\cdot}6$°C and 120×10^3 J]

12. A mole of a monatomic perfect gas is adiabatically compressed when its temperature rises from 27°C to 127°C. Calculate the work done.

[*Hint* : Work done$=\frac{R(T-T')}{\gamma-1}$; γ for monatomic gas$=\frac{5}{3}$] (**Ans.** 1246·5 J)

(B)

13. A piston can freely move inside a horizontal cylinder closed from both ends. Initially the piston separates the space inside of the cylinder into two equal parts each of volume V_0 in which an ideal gas is contained under the same pressure P_0 and at the same temperature. What work has to be done by an external agent in order to increase isothermally the volume of one part of the gas n times compared to that of the other by slowly moving the piston.

[*Hint* : Let d be the distance by which the piston has to be displaced. Then $W=\int_0 F_{agent}\times dx$ and $F_{agent}=(p_2-p_1)A$] $\left(\textbf{Ans. } p_0V_0 \log\frac{(n+1)^2}{4n}\right)$

14. A certain mass of a gas is compressed n times (in terms of volume), first adiabatically, and then isothermally. In both the cases the initial state of the gas is the same. Find the ratio of the respective works expended in each compression. $\left(\textbf{Ans. } \frac{W_S}{W_T}=\frac{n^{\gamma-1}-1}{(\gamma-1)\log n}\cdot\right)$

□□□

CHAPTER 8

CONDUCTION OF HEAT

1. Thermal Conductivity : Consider the flow of heat through a wall.

$$Q \text{ (heat passing through the wall)} \propto A \text{ (area of wall)}$$
$$\propto t \text{ (time)}$$
$$\propto (\theta_1 - \theta_2), \text{ difference in temperature}$$
$$\propto \frac{1}{x}$$

where x = thickness of wall.

$\therefore \quad Q = \dfrac{\lambda A(\theta_1 - \theta_2)t}{x}$ where λ is a constant of the material of the wall. It is called coefficient of thermal conductivity.

Unit of λ : $\lambda = \dfrac{Qx}{A(\theta_1 - \theta_2)t}$, $\quad \therefore \quad \lambda = \dfrac{\text{Jm}}{\text{m}^2\text{K}s} = \text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$

or Watt $\text{m}^{-1}\text{K}^{-1}$ ($\because$ watt = Js^{-1})

or Cal $\text{m}^{-1}\text{K}^{-1}\text{s}^{-1}$.

2. Thermal Resistance : We have, $Q = \dfrac{\lambda A(\theta_1 - \theta_2)}{x}$ as rate of flow of heat through the wall;

or $\quad Q = \dfrac{(\theta_1 - \theta_2)}{\left(\dfrac{x}{\lambda A}\right)}$. Comparing this equation with Ohm's law, I (current which is rate of flow of charge) $= \dfrac{V}{R}$, we have $\left(\dfrac{x}{\lambda A}\right)$ in place of R. If we take rate of flow of heat analogous to current and difference of temperature analogous to potential difference, then $\dfrac{x}{\lambda A}$ is analogous to resistance. This is why this quantity $\left(\dfrac{x}{\lambda A}\right)$ is termed as thermal resistance. Because of this similarity all the electrical laws, viz. Ohm's law, Kirchhoff's point rule and loop rule can be extended to combination of thermal conductors in the 'steady state'.

EXAMPLES

Ex. 1. *A vessel of surface area* 1 m^2 *and of copper sheet of thickness* 5 mm *is filled with melting ice and is immersed in water at* 100°C. *Calculale the rate at which ice melts. The conductivtiy of copper is* 302·4 Js^{-1}m^{-1}K^{-1} *and the latent heat capacity of fusion of ice is* 336×10^3 Jkg^{-4}.

Sol. Q (Rate of heat conduction) $= \dfrac{\lambda A(\theta_1-\theta_2)}{x}$.

Heat absorbed by ice $= mL$.

Heat conducted by sheet of copper = Heat absorbed by ice.

$$\therefore \quad \frac{\lambda A(\theta_1-\theta_2)}{x}=mL$$

$$\text{or} \quad m=\frac{\lambda A(\theta_1-\theta_2)}{xL}=\frac{302{\cdot}4\times1\times(100-0)}{5\times10^{-3}\times336\times10^{3}}=18\ \text{kgs}^{-1}. \qquad \textbf{Ans.}$$

Ex. 2. *Two flat metal plates are placed with their surfaces in contact and their outer surfaces are maintained at temperatures of* 100°C *and* 0°C *respectively. If the thickness of plates are* 2 cm *and* 1 cm *and their thermal conductivities are* 10 *cal* m^{-1}K^{-1}s^{-1} *and* 20 *cal* m^{-1}K^{-1}s^{-1} *respectively, find the temperature of the surfaces in contact.*

Sol. Let θ be the temperature of the surfaces in contact. In the steady state, rate of conduction at common surface is the same.

$$\therefore \quad \frac{10A(100-\theta)}{2\times10^{-2}}=\frac{20\times A(\theta-0)}{1\times10^{-2}}$$

or $(100-\theta)=4\theta$; or $5\theta=100$; or $\theta=20$°C. **Ans.**

Ex. 3. *Calculate the rate of increment of the thickness of ice layer on a lake when thickness of ice is* 20 cm *and the air temperature is* -40°C. *Thermal conductivity of ice* = 1·68 Js^{-1} m^{-1} K^{-1} *and density of ice* = 920 kgm^{-3} *and latent heat capacity of ice* = 336×10^3 Jkg^{-1} K^{-1}.

Sol. Let x be the thickness at time t. In the next short interval dt let thickness increase by dx. Then $\dfrac{dx}{dt}$ is the rate of increase of thickness x.

Heat conducted during $dt = \dfrac{\lambda A(\theta_1-\theta_2)dt}{x}$.

Heat given up by ice formed $=(A dx \rho)\, L$.

$$\therefore \qquad \frac{\lambda A(\theta_1-\theta_2)dt}{x}=A dx \rho L$$

or

$$\frac{dx}{dt}=\frac{\lambda(\theta_1-\theta_2)}{\rho L x}.$$

$\therefore$ Rate of increment of thickness at 20 cm thickness

$$=\frac{1{\cdot}68\{0-(-40)\}}{920\times336\times10^3\times20\times10^{-2}}=1{\cdot}086\times10^{-6}\ \mathrm{ms^{-1}}. \qquad \textbf{Ans.}$$

Ex. 4. *When the air temperature is* $-10°\mathrm{C}$ *the ice on a large shallow lake increases by* 1 cm *in* 12 *minutes. Find thermal conductivity of ice.*

(Density of ice $=900\ \mathrm{kgm^{-3}}$ and $L=330\times10^3\ \mathrm{Jkg^{-1}}$)

Sol. It is true that

Heat conducted in short interval dt = Heat given out by ice formed.

$$\therefore \qquad \frac{\lambda A(\theta_1-\theta_2)dt}{x}=(A dx \rho)L$$

where x is the thickness of ice layer. The rest have their usual meaning,

$$\therefore \qquad dt=\frac{\rho L}{\lambda(\theta_1-\theta_2)}x\, dx,$$

$$\therefore \int_0^t dt=\frac{\rho L}{\lambda(\theta_1-\theta_2)}\int_0^x x\, dx; \quad \text{or} \quad t=\frac{\rho L}{\lambda(\theta_1-\theta_2)}.\frac{x^2}{2}$$

or

$$\lambda=\frac{\rho L x^2}{2(\theta_1-\theta_2)t},$$

$$\therefore \quad \lambda=\frac{900\times330\times10^3\times(1\times10^{-2})^2}{2\{0-(-10)\}.12\times60}=2{\cdot}06\ \mathrm{Js^{-1}\,m^{-1}\,K^{-1}}. \qquad \textbf{Ans.}$$

Ex. 5. *Three slabs of thicknesses* d_1, d_2, d_3 *and thermal conductivities* λ_1, λ_2 *and* λ_3 *are placed in contact. Show that in the steady state the combination behaves as a single slab of material of conductivity* λ *given by* $\dfrac{d_1+d_2+d_3}{\lambda}=\dfrac{d_1}{\lambda_1}+\dfrac{d_2}{\lambda_2}+\dfrac{d_3}{\lambda_3}$.

Sol. Let us suppose that the temperature of faces are $\theta_1, \theta_2, \theta_3, \theta_4$ respectively. Then in the steady state we have

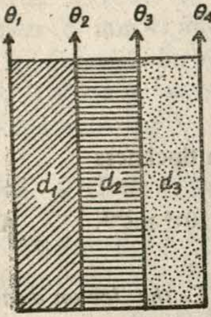

Fig. 8-1.

$$Q=\frac{\lambda_1 A(\theta_1-\theta_2)}{d_1}=\frac{\lambda_2 A(\theta_2-\theta_3)}{d_2}$$

$$=\frac{\lambda_3 A(\theta_3-\theta_4)}{d_3};$$

$$\therefore \quad \theta_1-\theta_2=\frac{Qd_1}{\lambda_1 A}; \quad \theta_2-\theta_3=\frac{Qd_2}{\lambda_2 A}$$

$$\theta_3-\theta_4=\frac{Qd_3}{\lambda_3 A}.$$

For the combination we have

$$Q=\frac{\lambda A(\theta_1-\theta_4)}{d_1+d_2+d_3} \quad \text{or} \quad \theta_1-\theta_4=\frac{Q(d_1+d_2+d_3)}{\lambda A}.$$

Now, $\theta_1-\theta_4=(\theta_1-\theta_2)+(\theta_2-\theta_3)+(\theta_3-\theta_4)$

$$\therefore \quad \frac{Q(d_1+d_2+d_3)}{\lambda A}=\frac{Qd_1}{\lambda_1 A}+\frac{Qd_2}{\lambda_2 A}+\frac{Qd_3}{\lambda_3 A}$$

or $$\frac{d_1+d_2+d_3}{\lambda}=\frac{d_1}{\lambda_1}+\frac{d_2}{\lambda_2}+\frac{d_3}{\lambda_3}.$$ **Proved.**

Ex. 6. *Two rods of material* x *and two of material* y *are connected as shown in the figure. All the rods are identical in length and cross-sectional area. If the end A is maintained at* 80°C *and the last point D is maintained at* 10°C, *calculate the temperature of the junctions B and C in the steady state. Thermal conductivity of* x *is double of that of* y.

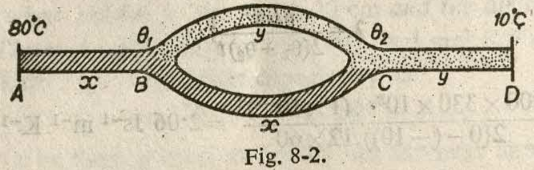

Fig. 8-2.

Sol. In the steady state there is no accumulation of heat at any point. Hence at B, the incoming heat current must be equal to the sum of outgoing heat currents. This reminds us of Kirchhoff's point rule in electricity. Suppose 2λ = thermal conductivity of x material and λ = thermal conductivity of y material. Applying point rule to point B we have

$$\frac{2\lambda A(80-\theta_1)}{L}=\frac{2\lambda A(\theta_1-\theta_2)}{L}+\frac{\lambda A(\theta_1-\theta_2)}{L}$$

or $$2(80-\theta_1)=2(\theta_1-\theta_2)+(\theta_1-\theta_2)$$

or $$160=5\theta_1-3\theta_2 \qquad \text{.. (i).}$$

Again applying point rule to point C we have

$$\frac{2\lambda A(\theta_1-\theta_2)}{L}+\frac{\lambda A(\theta_1-\theta_2)}{L}=\frac{\lambda A(\theta_2-10)}{L}$$

or $$2(\theta_1-\theta_2)+(\theta_1-\theta_2)=\theta_2-10$$

or $$3\theta_1-4\theta_2=-10.$$

Solving for θ_1 and θ_2 we have

$$\theta_1=60\tfrac{10}{11}=61°\text{C and } \theta_2=48{\cdot}3°\text{C}.$$ **Ans.**

Ex. 7. *Two rods identical in length and cross-sectional area but one of brass and the other of lead are given coating of wax and then fitted in the side holes of a bath. Boiling water is poured into the bath. In which rod wax will melt first and to what extent? Thermal conductivity of brass* = 110 *S. I. units and that of lead* = 35. *Sp. heat capacity of brass* = 370 $\text{Jkg}^{-1}\ \text{K}^{-1}$ *and that of lead* = 126. *Density of brass* = 8500 kgm^{-3} *and that of lead* = 11340 kgm^{-2}.

Sol. Thermometric conductivity i.e. rapidity with which a rod is heated is given by $h=\frac{\lambda}{\rho s}$.

∴ Thermometric conductivity of brass

$$=\frac{110}{8500\times 370}=3{\cdot}5\times 10^{-5}.$$

Thermometric conductivity of lead

$$=\frac{35}{11340\times 126}=2{\cdot}5\times 10^{-5}.$$

Therefore brass will get heated first. The length to which wax melts is proportional to the square root of thermal conductivity. (See theory of Ingen Hauz experiment.)

$$\therefore \quad \frac{l_1\text{ (of brass)}}{l_2\text{ (of lead)}}=\frac{\sqrt{\lambda_1}\text{ (brass)}}{\sqrt{\lambda_2}\text{ (lead)}}=\frac{\sqrt{110}}{\sqrt{35}}=\frac{1{\cdot}77}{1}.$$ **Ans.**

EXERCISES

(A)

1. An iron boiler 1·25 cm thick contains water at atmospheric pressure. The heated surface is 2·5 m^2 in area and the temperature of the underside is 120°C. If the thermal conductivity of iron is 20 cal $s^{-1}m^{-1}K^{-1}$ and the latent heat of evaporation of water $5{\cdot}36 \times 10^3$ cal kg^{-1} ($J=4{\cdot}2$ J cal^{-1}). find the mass of water evaporated per hour. (**Ans.** 537·3 kg per hour)

2. A slab consists of two parallel layers of different materials 4 and 2 cm thick and of thermal conductivities 54 and 36 cal $s^{-1}m^{-1}K^{-1}$ respectively. If the faces of the slab are at 100°C and 0°C, calculate the temperature of the surface opposite dividing the two materials. (**Ans.** 42·85°C)

3. Heat is conducted through a slab composed of parallel layer of two different materials of conductivities 134·4 and 58·8 S. I. units and of thicknesses 3·6 cm and 4·2 cm respectively. The temperature of the outer faces of the compound slab are 96°C and 8°C. Find (i) the temperature of interface, (ii) temperature gradient in each section of the slab.

$$\left[\textit{Hint}: \text{Temperature gradient} = \frac{\text{Difference in temperature}}{\text{Thickness}}.\right]$$

[**Ans.** (i) 72°C, (ii) 666·7°C per metre; (iii) 1523·8°C per metre]

4. A copper rod of length ·25 m and 10^{-4} m in area of cross-section is heated at one end by putting it in a liquid boiling at 125°C whereas the other end is kept cold by dipping it in ice-cold bath. Find (i) temperature gradient, (ii) the rate of heat transfer, (iii) the temperature at a point in the rod ·1 m from the higher temperature end. (λ of Copper$=92$ cal s^{-1} m^{-1} K^{-1})

[**Ans.** (i) 500°C m^{-1}, 4·6 cal s^{-1}, (iii) 75°C]

5. Calculate the thermal resistance of an aluminium rod of length 20 cm and area of cross-section 4 cm^2. The thermal conductivity of aluminium is 210 Js^{-1} m^{-1} K^{-1}.

$$\left[\textit{Hint}: \text{Thermal resistance} = \frac{x}{\lambda A}\right]$$

(**Ans.** 2·38 S. I. Units)

6. Show that if three slabs of thermal resistances R_1, R_2 and R_3 are placed in contact, the thermal resistance of the compound slab is given by $R=R_1+R_2+R_3$.

[*Hint* : See Ex. 5)

7. A hollow cube of metal with sides ·8 cm (internal) and ·5 cm thick is filled with ice at 0°C and is immersed in boiling water at 100°C. How many kg of ice will melt in one minute ? Thermal conductivity of metal$=252$ J $m^{-1}s^{-1}K^{-1}$ and the latent heat capacity of ice$=336 \times 10^3$ Jkg^{-1}. (**Ans.** 34·56 kg)

8. A boiler whose thickness is 3 cm is placed on a plate having an area 1 m^2. The temperature of the plate is 300°C and that of boiling water in the boiler is 100°C. How much water will evaporate per minute ? (Conductivity of steel$=63{\cdot}0$ Js^{-1} m^{-1} K^{-1} and sp. latent heat of vaporisation of water$=2251{\cdot}2 \times 10^3$ Jkg^{-1}.) (**Ans.** 11·2 kg)

9. A hollow glass sphere whose thickness is 2 mm and external radius is 10 cm is filled with ice and is placed in a bath containing boiling water at 100°C. Calculate the rate at which ice melts. Thermal conductivity of glass$=1{\cdot}1$ Js^{-1} m^{-1} K^{-1} and L of ice$=336\times10^3$ Jkg^{-1}. (Ans. ·02 kgs^{-1})

10. In Ingen Hauz's experiment there are two identical rods, identical in respect of length and cross-sectional area coated uniformly with wax. The material of one rod is copper whose thermal conductivity, sp. heat capacity and density are 385, 385 and 8930 S. I. units respectively and the material of the other is silver whose thermal conductivity, sp. heat capacity and density are 419, 235 and 10500 S.I. units respectively, Compare the lengths to which wax will melt in the two rods. In which one wax will melt first ?

[Ans. ·96 : 1; h (of copper)$=1{\cdot}12\times10^{-4}$ and h (silver)$=1{\cdot}70\times10^{-4}$; in silver]

(B)

11. Three rods of material x and three of material y are connected as shown in the figure 8-3. All the rods are identical in length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C, calculate the temperature of the junctions B, C and D. The thermal conductivity of x is 192 cal s^{-1} m^{-1} $°C^{-1}$ and 96 cal s^{-1} m^{-1} $°C^{-1}$. (I. I. T. 1973)

[*Hint* : See Ex. 6 and Fig. 8-3.)

(Ans. temperature of $B=30°C$; temperature of D=temperature of $C=20°C$)

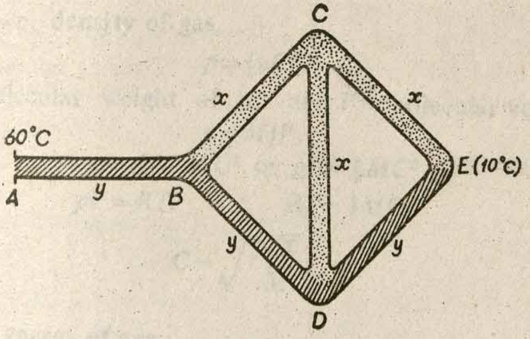

Fig. 8-3

12. A closed cubical box made of a perfectly insulating material has walls of thickness 8 cm and the only way for heat to enter or leave the box is through two solid, cylindrical, metallic plugs, each of cross-sectional area 12 cm² and length 8 cm in the opposite walls of the box. The outer surface of one plug is kept at 100°C while the outer surface of the other plug is maintained at 4°C. The thermal conductivity of the plug is 50 cal s^{-1} m^{-1} K^{-1}. A source of energy generating 36 cal s^{-1} is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface. (I. I. T. 1976) (Ans. 76°C)

13. A room is maintained at 20°C by a heater of resistance 20 ohms connected to 200 volt mains. The temperature is uniform throughout the room and heat is

transmitted through a glass window of area 1 m^2 and thickness 2 mm. Calculate the temperature outside. Thermal conductivity of glass is ·2 cal s^{-1} m^{-1} K^{-1} and mechanical equivalent of heat is 4·2 J cal^{-1}. (I. I. T. 1978) (Ans. 15·2°C)

14. Two chunks of metal of masses m_1 and m_2 and specific heat capacities c_1 and c_2 are interconnected by a rod of length l and cross-sectional area S and conductivity λ. The whole system is thermally insulated from the environment. At the moment $t=0$, the temperature difference between the chunks of metal equals θ_0. Assuming the heat capacity of the rod negligible, find the temperature difference between the chunks after time t.

[*Hint* : $\frac{dQ}{dt}=\frac{\lambda S\theta}{l}$; $dQ=-m_1c_1d\theta_1=m_2c_2d\theta_2$ and $\theta=\theta_1-\theta_2$)

$$\left[\textbf{Ans. } \theta=\theta_0 e^{-\alpha t} \text{ where } \alpha=\left(\frac{1}{m_1c_1}+\frac{1}{m_2c_2}\right)\frac{\lambda S}{l}\right]$$

□□□

CHAPTER 9

KINETIC THEORY OF GASES

1. According to this theory gases consist of innumerable particles called molecules in chaotic motion and temperature of gas is manifestation of the energy of the molecules. The pressure of the gas arises due to the impact of the molecules on the wall of the container. According to this theory pressure of a gas is given by

$$\boxed{p=\tfrac{1}{3}mnC^2}$$

where m = mass of each molecule, n = number of molecules per unit volume, C^2 is mean square velocity of molecules and C is root mean square velocity.

$$C^2=\frac{C_1^2+C_2^2+C_3^2+\ldots+C_n^2}{n}.$$

Since $mn=\rho$, density of gas,

$$\therefore \quad p=\tfrac{1}{3}\rho C^2.$$

If M = molecular weight of gas and V = molecular volume then

$$\rho=M/V.$$

$$\therefore \quad p=\tfrac{1}{3}M/V.C^2 \text{ or } pV=\tfrac{1}{3}MC^2$$

$$pV=RT. \qquad \therefore \quad RT=\tfrac{1}{3}MC^2.$$

$$\therefore \quad C=\sqrt{\frac{3RT}{M}}.$$

2. Kinetic energy of gas :

Kinetic energy per unit volume

$$=\tfrac{1}{2}mC_1^2+\tfrac{1}{2}mC_2^2+\tfrac{1}{2}mC_3^2+\ldots+\tfrac{1}{2}mC_n^2$$

$$=\tfrac{1}{2}mnC^2=\tfrac{1}{2}\rho C^2=\tfrac{3}{2}p.$$

$\therefore$ u (K.E. of a gas per unit volume) $=\tfrac{3}{2}p.$

$\therefore$ U (K.E. per mole of a gas) $=\tfrac{3}{2}pV$

$$=\tfrac{3}{2}RT \quad (\because \quad pV=RT).$$

3. Avogadro's Number : The number of molecules in a mole of a gas is called Avogadro's number.

$$N \text{ (Avogadro's number)} = 6{\cdot}022 \times 10^{23} \text{ gm mol}^{-1}$$

or $$6{\cdot}022 \times 10^{26} \text{ kg mol}^{-1}.$$

4. Boltzmann Constant (k) : The universal gas constant per molecule is called Boltzmann constant. It is denoted by k.

$$\therefore \quad k = \frac{R}{N}$$

or $$k = 1{\cdot}38 \times 10^{-26} \text{ JK}^{-1}.$$

$$\therefore \quad \text{K. E. per molecule} = \frac{\frac{3}{2} RT}{N} = \tfrac{3}{2} kT.$$

We have, $$pV = RT$$

$$\therefore \quad p = \frac{RT}{V} = \frac{NkT}{V}$$

or $$p = nkT \qquad \left(\because \; n = \frac{N}{V}\right).$$

EXAMPLES

Ex. 1. *Calculate the r. m. s. velocity of molecules of a gas whose density is* 1·4 *kgm*$^{-3}$ *at a pressure of* 76 *cm of mercury (sp. gr. of mercury* = 13·6 *and g* = 9·81 *ms*$^{-2}$).

Sol. We have $$p = \tfrac{1}{3}\rho C^2; \text{ or } C = \sqrt{\frac{3p}{\rho}}$$

$$\therefore \quad C = \sqrt{\frac{3 \times {\cdot}76 \times 13{\cdot}6 \times 1000 \times 9{\cdot}81}{1{\cdot}4}}$$

($\because$ density of water = 1000 kgm^{-3})

or $$C = 4{\cdot}66 \times 10^2 \text{ ms}^{-1}.$$ **Ans.**

Ex. 2. *Calculate the root mean square velocity of the molecules of hydrogen at* 0°*C and* 100°*C from the following : Density of hydrogen at normal pressure and temperature* = ·0896 *kgm*$^{-3}$ *and Density of mercury* = 13·6 × 10^3 *kgm*$^{-3}$.

Sol. We have, $$C = \sqrt{\frac{3p}{\rho}}.$$

$$\therefore \text{ At } 0°\text{C}, \quad C_0 = \sqrt{\frac{3 \times {\cdot}76 \times 13{\cdot}6 \times 10^3 \times 9{\cdot}81}{{\cdot}0896}}$$

$$= 1{\cdot}84 \times 10^3 \text{ ms}^{-1}.$$

For a perfect gas we have, $\frac{p}{\rho.T}=$a constant.

$$\frac{p_0}{\rho_0 T_0}=\frac{p}{\rho T}$$

$$\therefore \quad \frac{C}{C_0}=\sqrt{\frac{T}{T_0}} \text{ or } C=C_0\sqrt{\frac{T}{T_0}}$$

$$\therefore \quad C=1{\cdot}84\times 10^3\sqrt{\frac{273+100}{273}}=2{\cdot}18\times 10^3 \text{ ms}^{-1}.$$ **Ans.**

Ex. 3. *A certain mass of hydrogen occupies* 100 *c.c. at a pressure* 10^5 Nm^{-2} *at* 27°C. *What is the mass of hydrogen ? Molecular weight of hydrogen is* 2 *and* $R=8{\cdot}3$ $J\ mol^{-1}K^{-1}$.

Sol. Let there be x mole of hydrogen.

R is gas constant for a mole, therefore for x mole of gas, the constant is xR.

$PV=xRT$ is the equation for x mole of gas.

$$\therefore \quad 10^5\times(100\times 10^{-6})=x.8{\cdot}3\times(273+27) \quad (\because\ 1 \text{ c.c.}=10^{-6} \text{ m}^3)$$

$$\therefore \quad x=\frac{10^5\times 10^{-4}}{8{\cdot}3\times 300}=\frac{1}{83\times 3}.$$

$\because$ 1 mole hydrogen has mass 2 gm,

$\therefore$ x ,, ,, ,, ,, $2x$ gm.

$$\therefore \quad \text{Required mass}=\frac{2}{83\times 3}={\cdot}008 \text{ gm}=8\times 10^{-6} \text{ kg}.$$ **Ans.**

Ex. 4. *At what temperature will the root mean square velocity of molecules of a gas be double of that at* 0°C ?

Sol. We have, $C\propto\sqrt{T}$;

$$\therefore \quad \frac{C}{C_0}=\sqrt{\frac{T}{T_0}}.$$

It is required that C be $2C_0$.

$$\therefore \quad \frac{2C_0}{C_0}=\sqrt{\frac{T}{T_0}}$$

$$\text{or} \quad 4=\frac{T}{T_0}, \text{ or } T=4T_0=4\times 273=1092 \text{ K}$$

$$\therefore \quad t=1092-273=819°\text{C}.$$ **Ans.**

Ex 5. *The molecular weight of a gas is 2. Calculate the mean square velocity of its molecules at* 0°C *and* 100°C *given that* $R = \cdot 83$ $J\ mol^{-1}K^{-1}$.

Sol. We have, $pV = RT$ and $pV = \frac{1}{3}MC^2$

$$\therefore \quad \frac{1}{3}MC^2 = RT \quad \text{or} \quad C = \sqrt{\frac{3RT}{M}}.$$

Here $R = 8 \cdot 3 \text{ J mol}^{-1}\text{K}^{-1} = 8 \cdot 3 \times 1000 \text{ J kg mol}^{-1}\text{K}^{-1}$

$$M = 2 \text{ kg}.$$

$$\therefore \quad C = \sqrt{\frac{3 \times 8 \cdot 3 \times 1000 \times 273}{2}} = 1 \cdot 844 \times 10^3 \text{ ms}^{-1}.$$

At 100°C, $C = \sqrt{\frac{3 \times 8 \cdot 3 \times 1000 \times (273 + 100)}{2}}$

$$= \sqrt{\frac{3 \times 8 \cdot 3 \times 1000 \times 373}{2}} = 2 \cdot 155 \times 10^3 \text{ ms}^{-1}.$$ **Ans.**

EXERCISES

(A)

1. Calculate the root mean square velocity of a nitrogen molecule at N.T.P. if the density of hydrogen under the same condition is 9×10^{-2} kgm^{-3}.

(**Ans.** $4 \cdot 91 \times 10^2$ ms^{-1})

2. If the root mean square velocity of the molecules of hydrogen at N.T.P. is $1 \cdot 84$ kms^{-1}, calculate the root mean square velocity of oxygen molecules at N.T.P. Molecular weights of hydrogen and oxygen are 2 and 32 respectively.

[*Hint* : $PV = \frac{1}{3}MC^2$; $\therefore$ $C \propto \frac{1}{\sqrt{M}}$ at a constant pressure and temperature.)

(**Ans.** $\cdot 46$ kms^{-1})

3. Calculate the number of molecules per unit volume of a perfect gas at 27°C and at pressure 10 mm of mercury. Density of mercury $= 13 \cdot 6 \times 10^3$ kgm^{-3} and Boltzmann constant $= 1 \cdot 38 \times 10^{-23}$ J K^{-1}. (**Ans.** $3 \cdot 22 \times 10^{23}$)

(*Hint* : Use $p = nkT$)

4. Calculate the molecular kinetic energy of hydrogen at 100°C. Molecular weight of hydrogen is 2 and $R = 8 \cdot 3$ J mol^{-1} K^{-1}. Is it same for all gases ?

(**Ans.** $4643 \cdot 85$ J mol^{-1}; yes)

5. Calculate kinetic energy per unit volume of a gas at pressure 10 mm of *Hg*. (Density of $Hg = 13 \cdot 6 \times 10^3$ kgm and $g = 9 \cdot 8$ ms^{-2}) (**Ans.** $1999 \cdot 2$ joule)

[*Hint* : K.E. per unit vol. $= 3/2p$]

(B)

6. An electric bulb of 250 c.c. was sealed off during manufacture at a pressure 10^{-3} mm of *Hg* at 27°C. Compute the number of air molecules in the bulb. (Avogadro's number $=6\times 10^{23}$ mol^{-1} and $R=8{\cdot}3$ J mol^{-1}K^{-1}.)

[*Hint* : k (Boltzmann constant)$=\frac{R}{N}$. Use $p=nkT$.] **(Ans.** $8{\cdot}05\times 10^{15}$**)**

7. A vessel of water is put in a dry sealed room of volume 50 m^3 at a temperature 27°C. The saturated vapour pressure of water at 27°C is 40 mm of mercury. How much water will evaporate before the water is in equilibrium with its vapour ? (Relative density of mercury$=13{\cdot}6$; $g=9{\cdot}8$ ms^{-2} and $R=8{\cdot}3$ J mol$^{-1}{\cdot}$K^{-1})

[*Hint* : Considering vapour as a perfect gas, calculate mass of water vapour. Molecular weight of water vapour is 18. To calculate mass of vapour, see Ex. 3.) **(Ans.** 1·927 kg**)**

8. A lamp of volume 50 c.c. was sealed off during manufacture at a pressure ·1 newton per square metre at 27°C. Calculate the mass of the gas enclosed in the lamp. Molecular weight of the gas$=10$ and $R=8{\cdot}3$ J mol^{-1} K^{-1}.

(Ans. 2×10^{-11} kg**)**

[*Hint* : See Ex. 3]

9. A cylinder of length 42 cm is divided into chambers of equal volumes and each half contains a gas of equal mass at temperature 27°C. The separator is a frictionless piston of insulating material. Calculate the distance by which the piston will be displaced if the temperature of one half is increased to 57°C.

(Ans. 1 cm**)**

10. A column of mercury 10 cm long is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically ?

(I. I. T. 1978**)**

[*Hint* : Apply Boyle's Law.] **(Ans.** by 3 cm downward**)**

11. One gram-mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel. Calculate the number of collisions per second which the molecules make with one square metre of the vessel wall. **(I. I. T.** 1983**)**

(Ans. $2{\cdot}95\times 10^{27}$**)**

[*Hint* : $v=\frac{1}{4}\,n\,\bar{c}$ (a result of kinetic theory after rigorous calculation. Take $\bar{c}=c_{rms}$)]

12. A vessel containing one gram-mole of oxygen is enclosed in a thermally insulated vessel. The vessel is next moved with a constant speed v_0 and then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed v_0. **(I. I. T.** 1983**)** **(Ans.** 36·0 ms^{-1}**)**

[*Hint* : $\frac{1}{2}Mv_0^2=C_v\,\Delta T=\frac{R}{\gamma-1}\,\Delta T$]

13. A thermally insulated vessel containing an ideal gas ($M=4$) at a temperature $t=27$°C moves with velocity $v=100$ ms^{-1}. How much (in per cent) will the gas pressure change on a sudden stoppage of the vessel ?

$$\left(\text{Ans. } \frac{\Delta p}{p}=\frac{Mv^2}{3RT}=\cdot 54\%\right)$$

□□□

CHAPTER 1

WAVE-MOTION : INTERFERENCE AND BEATS

1. The equation of a progressive wave of amplitude a, frequency n and wavelength λ is

$$y=a\sin\left(\omega t-\frac{2\pi}{\lambda}x\right) \text{ where } \omega=2\pi n$$

or $$a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) \text{ where } T=\text{period of wave}$$

or $$a\sin\omega\left(t-\frac{x}{C}\right) \text{ where } C=\text{velocity of wave.}$$

$C=n\lambda$ where n = frequency in hertz (Hz)

and λ = wavelength in metre (m).

2. If two progressive waves of the same amplitude and frequency emerging from the same source meet at a point they may produce parmanent 'Silence' at that point or Maximum sound. This phenomenon is known as Interference.

Condition for 'Silence' :

Path difference $=(2s-1)\lambda/2$ where $s=1, 2, 3, \ldots$

Condition for maximum loudness :

Path difference $=s\lambda$ where $s=0, 1, 2, 3, \ldots$

3. When two progressive sound waves of the same amplitude but slightly differing in frequency meet together, loudness of resultant wave alternately becomes maximnm with waxing noise and minimnm with waning noise. This phenomenon of waxing and waning of loudness of resultant sound wave is known as Beats.

If n_1 = frequency of one wave and n_2 = frequency of the other wave then number of beats per sec. $=n_1\sim n_2$.

[*Note* : Frequency of a body decreases on being loaded and increases when it is filed.]

EXAMPLES

Ex. 1. *Show that the equation $y = a \sin (pt - kx)$ represents a progressive wave. Calculate the velocity of the wave.*

Sol. According to this equation y is the disturbance at a distance x at time t. Let y' be the disturbance after 1 sec at a distance $(x+p/k)$.

Then we must have,

$$
\begin{aligned}
y' &= a \sin \{p(t+1) - k(x+p/k)\} \\
&= a \sin\{pt+p-hx-p\} \\
&= a \sin(pt-kx) = y.
\end{aligned}
$$

Thus according to this equation disturbance which exists at a distance x now will occur at a distance $(x+p/k)$ one second after. Hence we say that this equation represents a progressive wave of velocity p/k.

Ex. 2. *In Quincke's experiment on interference of sound a tuning fork of frequency* 540 *Hz is used. Calculate the distance by which the sliding tube is to be drawn out to pass from one minimum to the next minimum.* (*Velocity of sound* = 350 ms^{-1})

Sol. Let x_1 be the length of path of one wave and x_2 be the length of path of the other wave. Then path difference of waves is $x_2 - x_1$. For minimum we have

$$x_2 - x_1 = (2n-1)\lambda/2$$

where n may be any integer. Let us draw out the tube by x. The path of the second wave is increased by $2x$. Hence path difference is now $(2x + x_2 - x_1)$. For next minimum we have

$$2x + x_2 - x_1 = (2n+1)\lambda/2$$

$$2x = \lambda \text{ or } x = \lambda/2.$$

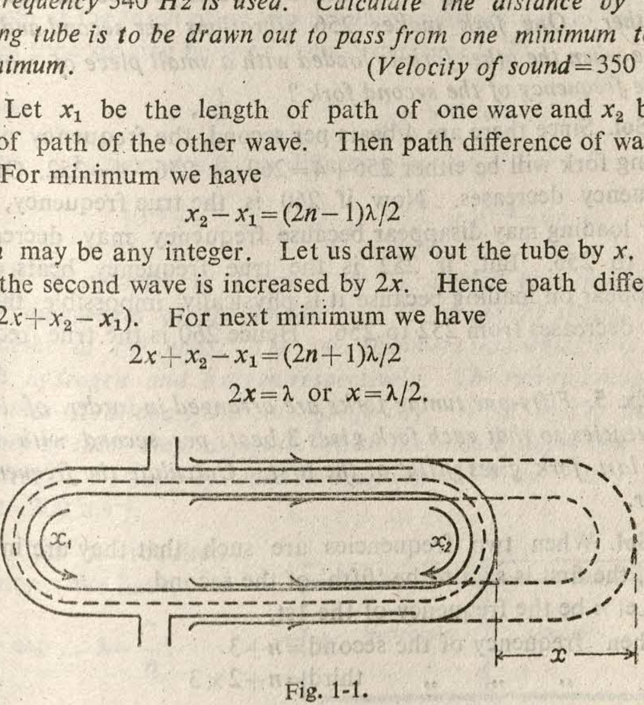

Fig. 1-1.

Now, $n\lambda = C$;

$$\therefore \qquad \lambda = \frac{C}{n} = \frac{350}{540} \text{ m}$$

$$\therefore \qquad x = \frac{350}{2 \times 540} \text{ m} = \cdot 324 \text{ m or } 32 \cdot 4 \text{ cm.} \qquad \textbf{Ans.}$$

Ex. 3. *Calculate the velocity of sound in a gas in which two waves of wavelength* 1 m *and* 1·01 m *produce* 20 *beats in* 6 *seconds.*

Sol. We have, $n\lambda = C$, $\therefore \; n_1 \; \frac{C}{1}$ and $n_2 = \frac{C}{1 \cdot 01}$.

Obviously, $n_1 > n_2$.

Number of beats per second $= \frac{20}{6} = \frac{10}{3}$, $\therefore \; n_1 - n_2 = \frac{10}{3}$

$$\text{or} \qquad \frac{C}{1} - \frac{C}{1 \cdot 01} = \frac{10}{3} \quad \text{or} \quad \frac{C \times \cdot 01}{1 \cdot 01} = \frac{10}{3}$$

$$\text{or} \qquad C = \frac{1010}{3} = 336 \cdot 67 \text{ ms}^{-1}. \qquad \textbf{Ans.}$$

Ex. 4. *Two tuning forks make* 4 *beats per second when sounded together. One fork makes* 256 *vibrations per second and the beats cease when the other fork is loaded with a small piece of wax. What is the frequency of the second fork* ?

Sol. Since there are 4 beats per second, the frequency of second tuning fork will be either $256 + 4 = 260$ or $256 - 4 = 252$. On loading frequency decreases. Now if 260 is the true frequency, the beats after loading may disappear because frequency may decrease from 260 to 256. But, if 252 is the true frequency beats can never disappear on loading because it is physically impossible that frequency decreases from 252 to 256. Hence 260 is the true frequency.

Ans. 260.

Ex. 5. *Fifty-one tuning forks are arranged in order of increasing frequencies so that each fork gives* 3 *beats per second with the next. The last fork gives 'fifth' of the first. Calculate the frequency of the latter.*

Sol. When two frequencies are such that they are in the ratio 3 : 2, the first is said to be 'fifth' of the second.

Let n be the frequency of the 1st,

then frequency of the second $= n + 3$

" " " third $= n + 2 \times 3$

. . . .

. . . .

" " " fifty-first $= n + 50 \times 3$.

It is given that, $\frac{n+50\times 3}{n}=\frac{3}{2}$

or $2n+300=3n$

or $n=300$. **Ans.**

Ex. 6. *A wave of frequency* 300 *Hz has a phase velocity of* 350 ms^{-1}. (a) *How far apart are two points* $\pi/6$ *out of phase* ? (b) *What is the phase difference between two displacements at a certain point at times* 10^{-2} s *apart* ?

Sol.. We have, $C=n\lambda$; $\therefore\ \lambda=\frac{C}{n}=\frac{350}{300}=\frac{7}{6}$ m.

According to the definition of wavelength, λ is the minimum distance between two particles which are out of phase by 2π.

$\therefore$ Phase difference $2\pi\equiv$ Path difference 7/6 m

$\therefore$,, ,, $\pi/6\equiv$,, ,, $\frac{7}{6\times 2\pi}\times\frac{\pi}{6}=\frac{7}{72}$

$=\cdot 097$ m. **Ans.**

Again at the same place two displacements are out of phase by 2π when time spacing between their occurrence is T.

Now, $n=\frac{1}{T}$; $\therefore\ T=\frac{1}{300}$ second.

Thus, $\frac{1}{300}$ second $\equiv$ Phase difference by 2π

$\therefore$ 10^{-2} s $\equiv$,, ,, ,, $\frac{2\pi}{\frac{1}{300}}\times 10^{-2}=6\pi$. **Ans.**

Ex. 7. *A cylinder of length* 1 m *is divided by a thin perfectly flexible diaphragm at the middle. It is closed by similar flexible diaphragms at the ends. The two chambers into which it is divided contain hydrogen and oxygen respectively. The two end diaphragms are set into vibrations of the same frequency. What is the minimum frequency of these diaphragms for which the middle diaphragm will be motionless ? Velocity of sound in hydrogen*$=1100$ ms^{-1} *and that in oxygen*$=300$ ms^{-1}.

Sol. Let $n=$ required frequency. We have,

$C=n\lambda$, $\therefore\ \lambda=\frac{C}{n}$.

$\therefore\ \lambda_H=\frac{1100}{n}$,

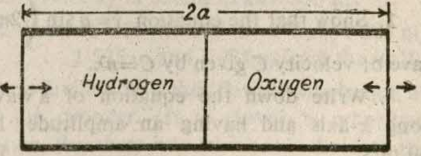

Fig. 1-2.

$$\lambda_O=\frac{300}{n}.$$

We know that, path diff. $\lambda \equiv$ phase diff. 2π

$$\therefore \quad ,, \quad ,, \quad a\equiv\frac{2\pi}{\lambda}a,$$

where $2a$ = length of cylinder.

$\therefore$ Phase diff. of waves reaching the middle diaphragm

$$=\frac{2\pi}{\lambda_O}a-\frac{2\pi}{\lambda_H}a=2\pi a\left(\frac{1}{\lambda_O}-\frac{1}{\lambda_H}\right)=2\pi a\left(\frac{n}{300}-\frac{n}{1100}\right)$$

$$=2\pi an\left(\frac{1}{300}-\frac{1}{1100}\right).$$

For the diaphragm to be motionless, it must be a seat of node. A point is a node when two waves meet at that point in antiphase, i.e. their phase difference at that point is π, 3π, 5π, 7π, ... Hence for minimum frequency we have

$$2\pi an\left(\frac{1}{300}-\frac{1}{1100}\right)=\pi.$$

Here, $2a=1$ m, or $a=\frac{1}{2}$ m.

$$\therefore \quad 2\pi.\frac{1}{2}n\left(\frac{11-3}{3300}\right)=\pi, \text{ or } n=412{\cdot}5 \text{ Hz}.$$ **Ans.**

EXERCISES

(A)

1. A plane progressive wave is $y={\cdot}02 \sin(400t-{\cdot}12x)$. Calculate the following—(i) amplitude of wave, (ii) frequency of wave, (iii) wavelength of wave, (iv) interval at which a compression follows a rarefaction, (v) velocity of the wave.

[*Hint* : Compare this given equation with the standard formula

$$y=a\sin\left(\omega t-\frac{2\pi}{\lambda}x\right).\Big]$$

[**Ans.** (i) ·02 m, (ii) 63·6 Hz, (iii) 52·4 m, (iv) ·008 second, (v) 3333·3 ms^{-1}]

2. Show that the equation $y=a\sin\left(2\pi nt-\frac{2\pi}{\lambda}x\right)$ represents a progressive wave of velocity C given by $C=n\lambda$.

3. Write down the equation of a wave travelling in the negative direction along x-axis and having an amplitude ·10 m, a frequency 550 Hz and a speed 330 m/s.

$$\left[\textbf{Ans. } y={\cdot}01\sin\left(1100\pi t+\frac{10\pi x}{3}\right)\right]$$

4. Sixty-four tuning forks are arranged in order of increasing frequency and any two successive forks give four beats per second when sounded together. If the last fork gives the octave of the first, calculate the frequency of the latter.

[*Hint* : When two frequencies are such that one is double of the other, higher one is said to be 'octave higher' of the first and the lower one is said to be 'octave lower' to the second.) (**Ans.** 252)

5. Fifty-six tuning forks are arranged in order of increasing frequencies so that each fork gives 4 beats per second with the next one. The last fork gives the octave of the first. Find the frequency of the first. (**Ans.** 220)

6. A fork of unknown frequency gives 4 beats per second when sounded with another of frequency 256. The fork is now loaded with a piece of wax and again 4 beats per second are heard. Calculate the frequency of the unkown fork.

(**Ans.** 260)

7. Calculate the velocity of sound in a gas in which the two waves of wavelength 50 cm and 50·5 cm produce 6 beats per second. (Ans. 303 ms^{-1})

8. The prongs of a tuning fork *A* originally in unison with a fork *B* of frequency 312 are filed and the forks produce 5 beats per second when sounded together. What is the pitch of *A* after filing ? (**Ans.** 317 Hz)

9. Two tuning forks *A* and *B* when sounded together produce 2 beats per second. *A* has frequency 340. It is now loaded and again sounded with *B*. It is found that number of beats is 4 per second. Can you say decidedly what is the frequency of *B* from this observation ?

(**Ans.** no; the unknown frequency may be 342 or 338)

10. Two tuning forks when sounded together produce 4 beats per second. The frequency of one is 256. The other one when loaded and sounded again with the first produces 8 beats per second. Calculate the frequency of the other.

(**Ans.** 252 or 260)

11. A wave of frequency 500 Hz has a phase velocity of 350 m/s. (a) How far apart are two points 60° out of phase ? (b) What is the phase difference between two displacements at a certain point at times 10^{-3} s apart ?

[**Ans.** (a) ·116 m, (b) π]

(B)

12. A sensitive microphone with its receiving surface turned towards a long vertical wall is placed at a distance of 2 m from the wall. A strong source of sound of 500 Hz is placed on the line perpendicular to the wall and passing through the position of microphone. Find the positions of the source when no sound will be heard in the microphone. (Velocity of sound in air=350 ms^{-1})

(**Ans.** at distance ·175 m, ·525 m, ·875 m, 1·225 m, 1·575 m and 1·925 m from the wall.)

13. A hollow cylinder of length 1 m is divided by a thin flexible diaphragm at distance one-third of the length of the cylinder. It is closed by other two flexible diaphragms at the ends. The two chambers into which it is divided, the longer one contains hydrogen and the other oxygen. The two end diaphragms are set into vibrations of the same frequency. What are the frequencies of the

end diaphragms for which the middle diaphragm will be motionless ? Velocity of sound in hydrogen $=1100\ \text{ms}^{-1}$ and that in oxygen $=300\ \text{ms}^{-1}$.

(Ans. 990 and its odd multiples)

14. Sound waves from a tuning fork *A* reach a point *P* by two separate paths *ABP* and *ACP*. When *ACP* is greater than *ABP* by 11·5 cm, there is silence at *P*. When the difference is 23 cm the sound becomes loudest at *P* and when 34·5 cm there is silence again and so on. Calculate the minimum frequency of the fork if the velocity of sound is taken to be 331·2 m/s. (Ans. 1440 Hz)

15. The following equations represent transverse waves : $z_1=A\cos(kx-\omega t)$, $z_2=A\cos(kx+\omega t)$, $z_3=A\cos(ky-\omega t)$. Identify the combination (*s*) of the waves which will produce (i) standing wave (*s*), (ii) a wave travelling in the direction making an angle of 45° with the positive *x* and positive *y* axes. In each case find the positions at which resultant intensity is always zero. (I. I. T. 1987)

[Ans. 1 and 2 combine to form stationary waves : $x=\frac{(2s+1)\pi}{2k}$. 1 and 3 combine to form progressive wave making an angle of 45° with *x* and *y* axis.

$$x=y\pm(2s+1)\frac{\pi}{k}.]$$

□□□

CHAPTER 2

VELOCITY OF SOUND

1. Velocity of Sound : Newton first showed mathematically that velocity (C) of a wave through a medium is given by

$$C=\sqrt{\frac{E}{D}}$$

where $E=$ bulk modulus in case of liquid and gas and Young's modulus in case of solids; $D=$ density of medium.

Next he thought specially for sound travelling through a gaseous medium that change in pressure and volume of layers of the medium takes place at constant temperature. Under this condition $E=P$, normal pressure of the medium. $\therefore\ C=\sqrt{\frac{P}{D}}$. This is known as Newton's formula for velocity of sound through a gaseous medium.

2. Laplace's formula : According to Newton's formula velocity of sound through air at N. T. P. should be only 280 ms^{-1} whereas the actual value is 332 ms^{-1}. Hence Newton's formula is incorrect. The correct formula was given by Laplace. Laplace argued that change in pressure and volume is so rapid that it cannot be isothermal, rather it is adiabatic. In adiabatic change, $E=\gamma P$ where γ is a constant depending on the atomicity of the molecules and is equal to the ratio of the specific heat capacity at constant pressure to the specific heat capacity at constant volume. For monoatomic gas $\gamma=1{\cdot}66$, $\gamma=1{\cdot}41$ for diatomic and $\gamma=1{\cdot}33$ for triatomic molecules.

$$C=\sqrt{\frac{\gamma P}{D}}.$$

This formula gives the correct value of the velocity of sound through air.

3. Effect of temperature and pressure on velocity of sound : Let C_1 be velocity of sound at pressure P_1 and temperature t_1°C and C_2 at pressure P_2 and temperature t_2°C.

$$\therefore\quad C_1=\sqrt{\frac{\gamma P_1}{D_1}} \text{ and } C_2=\sqrt{\frac{\gamma P_2}{D_2}};\quad \therefore\quad \frac{C_1}{C_2}=\sqrt{\frac{P_1}{D_1}\times\frac{D_2}{P_2}}.$$

For a perfect gas we have $PV=RT$ or $\frac{P}{DT}=$ a constant.

$$\therefore \quad \frac{P_1}{D_1T_1}=\frac{P_2}{D_2T_2}; \quad \therefore \quad \frac{C_1}{C_2}=\sqrt{\frac{T_1}{T_2}} \quad \text{or} \quad \frac{C_1}{\sqrt{T_1}}=\frac{C_2}{\sqrt{T_2}}.$$

Thus velocity of sound is independent of change of pressure but it depends on change of temperature.

Velocity of sound is proportional to square root of absolute temperature.

4. Effect of humidity on velocity of sound : The presence of water vapour lowers the density of air and hence the velocity of sound in moist air at a given temperature and pressure is greater than the velocity of sound in dry air at the same temperature and pressure.

$$C_m=\sqrt{\frac{\gamma P}{D_m}} \text{ and } C_d=\sqrt{\frac{\gamma P}{D_d}}$$

where m stands for 'moist air' and 'd' for 'dry air'.

$$\therefore \quad \frac{C_m}{C_d}=\sqrt{\frac{D_d}{D_m}}.$$

$D_m=$ mass of unit volume of moist air at pressure P and temperature t

$=$ mass of unit volume of water vapour at pressure f and temperature $t+$ mass of unit volume of dry air at pressure $(P-f)$ and temperature t where $f=$ vapour pressure

$=$ mass of f/P volume of water vapour at pressure P and temperature $t+$ mass of $\frac{P-f}{P}$ volume of dry air at pressure P and temperature t.

It is a fact that at the same temperature and pressure, the density of water vapour (saturated) bears a constant ratio with density of dry air and this constant ratio is $\frac{5}{8}$.

$$\therefore \quad D_m=\frac{f}{P}\times\frac{5}{8}D_d+\frac{P-f}{P}D_d=\frac{P-\cdot385f}{P}\times D_d$$

$$\therefore \quad \frac{C_m}{C_d}=\sqrt{\frac{P}{P-\cdot385f}}$$

or

$$C_m=C_d\sqrt{\frac{P}{P-\cdot385f}}.$$

EXAMPLES

Ex. 1. *Show that velocity of sound increases by* ·61 ms^{-1} *per* °C *rise in temperature. Velocity of sound at* 0°C *is* 332 ms^{-1}.

Sol. We know that velocity is proportional to the square root of absolute temperature.

$$\therefore \qquad \frac{C}{C_0}=\sqrt{\frac{T}{T_0}}$$

where T=absolute temperature corresponding to 0°C$=273+t$ and
T_0=absolute temperature corresponding to 0°C=273.

$$\therefore \quad \frac{C}{332}=\sqrt{\frac{273+t}{273}}, \text{ or } C=332\left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

$$\text{or} \quad C=332\left(1+\frac{1}{2}\cdot\frac{t}{273}\right) \qquad (\because \quad t<<273)$$

neglecting other terms in the expansion

$$\text{or} \quad C=332+\frac{332.t}{546}, \text{ or } C-332=\frac{332t}{546}$$

or Increase in velocity for t°C rise in temperature$=\dfrac{332t}{546}$.

Put $t=1$°C.

Then increase in velocity per °C rise in temperature

$$=\frac{332}{546}=\cdot 61 \text{ ms}^{-1}.$$ **Ans.**

Ex. 2. *If the velocity of sound in hydrogen at* 0°C *is* 1284 ms^{-1}, *what will be the velocity of sound in a mixture of two parts by volume of hydrogen to one part of oxygen?* (*Oxygen is* 16 *times heavier than hydrogen.*)

Sol. Suppose volume of hydrogen$=2V$ where V is any volume taken. Then volume of oxygen$=1V$.

Then volume of mixture$=3V$.

$$\therefore \qquad 3VD=2VD_H+1V\times 16D_H$$

where D=density of mixture; D_H=density of hydrogen.

$$\therefore \qquad D=6D_H.$$

$$\text{Now,} \qquad C=\sqrt{\frac{\gamma P}{D}} \quad \text{and} \quad C_H=\sqrt{\frac{\gamma P}{D_H}}$$

$$\therefore \quad \frac{C}{C_H}=\sqrt{\frac{D_H}{D}} \text{ or } C=1284\sqrt{\frac{1}{6}}=\frac{1284}{\sqrt{6}}$$

or $$C=\frac{1284\sqrt{6}}{6}=214\sqrt{6}=214\times 2{\cdot}45$$

or $$C=524{\cdot}3 \text{ ms}^{-1}.$$ **Ans.**

Ex. 3. *One cubic metre of water is taken in a strong cylinder and the pressure on it is increased from one atmosphere* (10^5 *newton per square metre*) *to two atmospheres. It is found that it has been compressed by only* 50 *c.c. Is this much sufficient for calculating velocity of sound in water ?*

Sol. $$\text{Volume strain}=\frac{\text{change in volume}}{\text{original volume}}=\frac{50\times 10^{-6}\text{ m}^3}{1\text{ m}^3}$$

$$=50\times 10^{-6},$$

Stress = increase in pressure $=10^5$ Nm^{-2}.

$$E \text{ (Bulk modulus)}=\frac{\text{stress}}{\text{strain}}=\frac{10^5}{50\times 10^{-6}}=2\times 10^9 \text{ Nm}^{-2}.$$

D (Density of water) $=1000$ kgm^{-3}.

We have, $$C=\sqrt{\frac{E}{D}}.$$

$$\therefore \quad \text{Velocity of sound in water}=\sqrt{\frac{2\times 10^9}{1000}}$$

$$=1{\cdot}414\times 10^3=1414 \text{ ms}^{-1}.$$ **Ans.**

Ex. 4. *Calculate the velocity of sound in moist air at* 27°C *and* ·7 m *of pressure. The dew point is* 15°C. *The saturated vapour pressure at* 15°C *is* 17 mm *of mercury and velocity of sound at* 0°C *in dry air is* 330 ms^{-1}.

Sol. $$\frac{\text{Velocity of sound in dry air at 27°C}}{\text{Velocity of sound in dry air at 0°C}}=\sqrt{\frac{273+27}{273}}.$$

$$\therefore \quad \text{Velocity of sound in dry air at 27°C}=330\sqrt{\frac{330}{273}} \text{ ms}^{-1}.$$

We know, vapour pressure 'f' at present = saturated vapour pressure at dew point = 17 mm of mercury = ·017 m of Hg.

We have, $$C_m=C_d\sqrt{\frac{P}{P-{\cdot}385f}}$$

$$\therefore \quad C_m = 330\sqrt{\frac{330}{273}}\sqrt{\frac{\cdot 7}{\cdot 7 - \cdot 385 \times \cdot 017}}$$

or $$C_m = 330\sqrt{\frac{300}{273}}\sqrt{\frac{\cdot 7}{\cdot 6935}}$$

or $$C_m = 348\cdot 7 \text{ ms}^{-1}.$$ **Ans.**

Ex. 5. *Calculate velocity of sound on a day when relative humidity is* 80% *and temperature and pressure are respectively* 25°C *and* ·68 m *of mercury.* (*Velocity of sound at S. T. P.* = 332 ms^{-1}. *Saturated vapour pressure at* 25° *is* ·026 m *of mercury.*)

Sol. Velocity in dry air at 25°C

$$= 332\sqrt{\frac{273+25}{273}} = 332\sqrt{\frac{298}{273}}.$$

We know, relative humidity

$$= \frac{\text{vapour pressure '}f\text{' at } 25°\text{C}}{\text{saturated vapour pressure at } 25°\text{C}}$$

$$\therefore \quad \frac{80}{100} = \frac{f}{\cdot 026} \quad \text{or} \quad f = \cdot 026 \times \cdot 8 = \cdot 0208 \text{ m of } Hg.$$

Now, $$C_m = C_d\sqrt{\frac{P}{P - \cdot 385 f}}$$

$$\therefore \quad C_m = 332\sqrt{\frac{298}{273}}\sqrt{\frac{\cdot 68}{\cdot 68 - \cdot 385 \times \cdot 0208}}$$

or $$C_m = 332\sqrt{\frac{298}{273}}\sqrt{\frac{\cdot 68}{\cdot 6720}} = 348\cdot 9 \text{ ms}^{-1}.$$ **Ans.**

EXERCISES

(A)

1. Find at what temperature the velocity of sound in air is double the velocity of sound in air at 0°C.

$\left(Hint: \text{Use } \frac{C}{C_0} = \sqrt{\frac{T}{T_0}} \right)$ (Ans. 819°C)

2. If the velocity of sound in air is 332 metre per second at N. T. P., find the velocity at 30°C and ·7 m of mercury pressure. (Ans. 349·8 ms^{-1})

3. A man sets his watch by the noon-whistle of a factory at a distance of 1·5 kilometre. How many seconds is his watch slower than the clock of the factory? (Velocity of sound in air is 332 ms^{-1}) **(Ans.** 4·52 seconds)

4. Calculate the velocity of sound in steel given Young's modulus of steel $=2\times10^{11}$ Nm^{-2} and density of steel=7800 kgm^{-3}. **(Ans.** 5064 ms^{-1})

5. The velocity of sound in hydrogen gas is 1270 ms^{-1} at 0°C and the frequency of a fork is 355 Hz. Find the distance travelled by sound in hydrogen at 0°C and 30°C in the time in which the fork completes 71 vibrations. **(Ans.** 254 m; 267·6 m)

6. A ship steams towards a hill in the sea and sounds its siren and echo is heard after 6 s. The siren is sounded again 3 minutes later after the first sounding and echo is heard after 4 s. If the velocity of ship is 6·87 kmph, calculate the velocity of sound in air. **(Ans.** 343·5 ms^{-1})

7. A man claps his hands in front of a wall and he hears the echo after 1·6 s. He walks 33 m nearer the wall and hears the echo 1·4 s after clapping. Find the velocity of sound in air. **(Ans.** 330 ms^{-1})

8. A man while beating a drum at the rate 5 per second walks towards a cliff till echo of beating disappears completely. He walks at the rate of 8 kilometres per hour. Calculate the distance of man from cliff in the beginning if he walked for 5 minutes. (Velocity of sound in air=350 ms^{-1}) **(Ans.** 702 m)

9. A man standing in front of a mountain at a certain distance beats a drum at regular intervals. He increases the beating rate and finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 m and finds that the echo is again not heard when the beating rate is 60 per minute. Calculate the velocity of sound and the distance between the mountain and the initial position of the man. **(Ans.** 360 ms^{-1}; 270 m)

(B)

10. Velocity of sound through an unknown gas at S.T.P. is 258·4 ms^{-1} and its density 1·977 kgm^{-3}. What is the atomicity of the molecules of the gas? **(Ans.** 3)

11. Calculate the velocity of sound in air saturated with moisture at 25°C and 745 mm of pressure. The saturation pressure at 25°C is 23·76 mm of mercury and the velocity of sound at 0°C in dry air is 332 m^{-1}.

[*Hint* : See Ex. 4.] **(Ans.** 348·9 ms^{-1})

12. Show that velocity of sound in a gas of $\gamma=1{\cdot}41$ is ·68 C where C is the root mean squar veleocity of its molecules.

[*Hint* : $C=\sqrt{\frac{\gamma P}{D}}$ and $P=\frac{1}{3}DC^2$ where C is root mean square velocity]

13. If the velocity of sound in dry air at 0°C is 332 ms^{-1}, find the temperature at which it will be 345 ms^{-1}, coefficient of volume expansion of air being ·00368/°C.

[*Hint* : Use $\frac{C}{C_0}=\sqrt{\frac{T}{T_0}}$. Now, $T_0=\frac{1}{\alpha}$ where α=coefficient of expansion.] **(Ans.** 21·7°C)

14. Calculate the velocity of sound in air on a day when temperature is 30°C, pressure ·74 m of mercury and relative humidity 60%. Velocity of sound at N. T. P. = 330 ms^{-1}. Saturated vapour pressure at 30°C = ·032 m of mercury.

[*Hint* : $\text{Relative Humidity} = \dfrac{\text{Vapour pressure at } 30°\text{C}}{\text{Saturated vapour pressure at } 30°\text{C}}$

$$C_m = C_a \sqrt{\frac{P}{P - {\cdot}385f}}\Bigg]$$

(Ans. 349·4 ms^{-1}**)**

□□□

CHAPTER 3

VIBRATION OF STRINGS

1. Velocity of Transverse Wave along a Stretched string : When a string stretched with a force T is set into transverse vibration, velocity of disturbance (wave) along the string is given by

$$C=\sqrt{\frac{T}{m}}$$

where m = mass per unit length of the string called linear density of the string.

2. Fundamental, Overtones and Harmonics : The disturbance (wave) travelling along the string in either direction is reflected from the fixed ends and so there is superposition of two identical waves in the string. As a result stationary waves are formed with definite nodes and antinodes. Since the two ends are fixed they are essentially the seats of *nodes*. The different stationary waves that may possibly form are shown in the Fig. 3-1. These are called modes of vibration of string. The simplest mode of vibration is called its *Fundamental* and the other modes are called its *Overtones*. When the frequencies of overtones are exact multiples of fundamental, they are specially called *Harmonics*. In the fundamental mode,

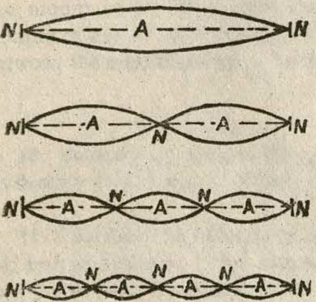

Fig. 3-1.

$$l=\lambda/2, \text{ or } \lambda=2l$$

$$n_1\lambda=C; \quad \therefore \quad n_1=\frac{C}{\lambda}$$

or

$$n_1=\frac{1}{2l}\sqrt{\frac{T}{m}}.$$

In the next mode, there are two segments (loops) and $l=\lambda$.

$$\therefore \quad n_2=\frac{C}{\lambda}=\frac{1}{l}\sqrt{\frac{T}{m}}=\frac{2}{2l}\sqrt{\frac{T}{m}}=2n_1.$$

In the third mode, there are three loops and $l = 3\lambda/2$.

$$\therefore \quad n_2 = \frac{C}{\lambda} = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n_1.$$

In general, $n_s = \dfrac{s}{2l}\sqrt{\dfrac{T}{m}}$ where s = number of loops.

3. **Melde's experiment** : In this experiment long string is tied to one prong of a tuning fork while the string stretched horizontally passes over a pulley and carries a pan at the other end. When the tuning fork is set to vibration, the string is thrown into forced vibration. On changing the length of the string its frequency changes and at one particular length it is thrown into resonant vibration when very big loops are formed.

There are two arrangements of the tuning fork : (a) Transverse arrangement—In this arrangement the tuning fork vibrates transversely (at right angles) to the string. In this arrangement resonance occurs when $n = N$—condition for resonance in transverse arrangement, n = frequency of string and N = frequency of tuning fork.

(b) Longitudinal arrangement—In this arrangement the tuning fork is arranged to vibrate along the length of the string. In this arrangement resonance occurs when $N = 2n$—condition for resonance in longitudinal arrangement.

EXAMPLES

Ex. 1. *A string whose* 200 *cm length weighs* 2 *gm is stretched by a weight of* 20 *gm. Calculate the velocity with which a transverse wave will travel along the string.*

Sol. We have $C = \sqrt{\dfrac{T}{m}}$.

Here $T = 20 \times 10^{-3} \times 9{\cdot}8$ newton

$$m = \frac{2 \times 10^{-3}}{200 \times 10^{-2}}\ \text{kgm}^{-1} = 10^{-3}\ \text{kgm}^{-1}$$

$$\therefore \quad C = \sqrt{\frac{20 \times 10^{-3} \times 9{\cdot}8}{10^{-3}}} = 16\ \text{ms}^{-1}.$$ **Ans.**

Ex. 2. *A note from a tuning fork is in unison with the note of* 25 *cm length of a stretched wire. If the length is changed to* 25·5 *cm tension remaining unchanged,* 3 *beats per second are heard. Find the frequency of the fork.*

Sol. Let N be the frequency of fork and n and n' be the frequencies of wire corresponding to the two lengths given.

$$\text{Then} \quad n=\frac{1}{2\times 25\times 10^{-2}}\sqrt{\frac{T}{m}} \text{ and } n'=\frac{1}{2\times 25{\cdot}5\times 10^{-2}}\sqrt{\frac{T}{m}}.$$

Now $n=N$ (because the tuning fork was in unison with string in the beginning).

$N-n'=3$ (because now tuning fork gives three beats with the string).

$$\therefore \qquad N=\frac{1}{2\times 25\times 10^{-2}}\sqrt{\frac{T}{m}}$$

$$\text{and} \qquad N-3=\frac{1}{2\times 25{\cdot}5\times 10^{-2}}\sqrt{\frac{T}{m}}.$$

$$\therefore \qquad \frac{N}{N-3}=\frac{25{\cdot}5}{25}, \text{ or } 25N=25{\cdot}5N-76{\cdot}5; \; N=153. \qquad \textbf{Ans.}$$

Ex. 3. *A tuning fork gives* 15 *beats per second when sounded with a sonometer wire of length* 20 *cm and* 20 *beats per second with that of length of* 25 *cm. Calculate the frequency of the fork.*

(Tension and mass per unit length are 1·25 kg and $\cdot 025\times 10^{-3}$ kg respectively and $g=9{\cdot}8$ ms^{-2}.)

$$\textbf{Sol.} \quad \text{We have, } n=\frac{1}{2\times 20\times 10^{-2}}\sqrt{\frac{1{\cdot}25\times 9{\cdot}8}{\cdot 025\times 10^{-3}}}=175.$$

Since string gives 15 beats with tuning fork,

$\therefore$ N (frequency of fork) $=175\pm 15=190$ or 160.

When length is changed to 25 cm,

$$n'=\frac{1}{2\times 25\times 10^{-2}}\sqrt{\frac{1{\cdot}15\times 9{\cdot}8}{\cdot 025\times 10^{-3}}}=140.$$

Now the same tuning fork produces 20 beats with the string.

Obviously this is possible when **N = 160**. **Ans.**

Ex. 4. *A certain fork is found to give two beats per second when sounded with a stretched string vibrating transversely under a tension either* 10·2 *or* 9·9 *kg. Calculate the frequency of the fork.*

Sol. Let n = frequency of string when tension is 10·2 kg,

n' = ,, ,, ,, ,, ,, is 9·9 kg.

$$\therefore \quad n=\frac{1}{2l}\sqrt{\frac{10{\cdot}2\times g}{m}} \text{ and } n'=\frac{1}{2l}\sqrt{\frac{9{\cdot}9\times g}{m}}.$$

$$\therefore \quad \frac{n}{n'}=\sqrt{\frac{10{\cdot}2}{9{\cdot}9}}. \qquad \therefore \quad n>n'.$$

The tuning fork gives 2 beats with both n and n'.

$$\therefore \quad n-N=2, \quad \text{or} \quad n=N+2$$

$$\text{and} \quad N-n'=2. \quad \text{or} \quad n'=N-2.$$

$$\therefore \quad \frac{n}{n'}=\frac{N+2}{N-2};$$

$$\therefore \quad \frac{N+2}{N-2}=\sqrt{\frac{10{\cdot}2}{9{\cdot}9}}=1{\cdot}015$$

$$\text{or} \quad N+2=1{\cdot}015N-2{\cdot}030$$

$$\text{or} \quad N=\frac{4{\cdot}03}{{\cdot}015}=\frac{4030}{15}=268{\cdot}7.$$ **Ans.**

Ex. 5. *In a transverse arrangement of a Melde's experiment with the string vibrates in 3 loops when the tension is 200 gm. Calculate the tension required to make the string vibrate in two loops in the longitudinal arrangement.*

Sol. In transverse arrangement we have

$$N=n=\frac{3}{2l}\sqrt{\frac{200\times 10^{-3}g}{m}}.$$

In longitudinal arrangement we have

$$N=2n=2.\frac{2}{2l}\sqrt{\frac{T}{m}}.$$

$$\therefore \quad \frac{3}{2l}\sqrt{\frac{200\times 10^{-3}\times g}{m}}=\frac{4}{2l}\sqrt{\frac{T}{m}}$$

$$\text{or} \quad 9\times 200\times 10^{-3}g=16T, \quad \text{or} \quad T=\frac{9\times 200\times 10^{-3}}{16}\, g \text{ newton}$$

$$\text{or} \quad T=\frac{1800}{16}\times 10^{-3}\text{ kg}=112{\cdot}5\times 10^{-3}\text{ kg}.$$ **Ans.**

Ex. 6. *A sonometer wire is stretched by a solid mass M. It produces a fundamental note of certain frequency in tune with a tuning fork when its length is* 70 *cm. When the mass is immersed in water it is found that the length has to be changed by* 5 *cm in order to bring it again in tune with the same tuning fork. Calculate relative density of the material of hanging mass.*

Sol. We have, $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$; $\therefore$ $l\propto\sqrt{T}$ when n and m remain constant; or $l^2\propto T$.

$\therefore$ Mg (weight of mass in air) $=kl^2$

$M'g$ (weight of mass in water) $=kl'^2$.

$$\text{Relative density}=\frac{\text{weight in air}}{\text{loss in wt. in water}}=\frac{Mg}{Mg-M'g}=\frac{l^2}{l^2-l'^2}$$

$$\text{Relative density of mass}=\frac{70^2}{70^2-65^2}=7{\cdot}27. \qquad \textbf{Ans.}$$

$\therefore$ Density of mass $=7{\cdot}27\times1000=7270$ kg m^{-3} **Ans.**

($\because$ Density of water $=1000$ kg m^{-3}).

Ex. 7. *In Melde's experiment, a string passing over a smooth pulley carries a stone at one end, while its other end is attached to a vibrating tuning fork and the string shows* 8 *loops. When the stone is immersed in water* 10 *loops are formed. Calculate the specific gravity of the stone.*

Sol. Let us suppose that Melde's experiment is in transverse arrangement.

$$\therefore \qquad N=n=\frac{8}{2l}\sqrt{\frac{Mg}{m}},$$

where $M=$ mass of stone in air when the stone is immersed in water;

$$N=n'=\frac{10}{2l}\sqrt{\frac{M'g}{m}},$$

$M'=$ mass of the same stone in water.

$$\therefore \qquad \frac{8}{2l}\sqrt{\frac{Mg}{m}}=\frac{10}{2l}\sqrt{\frac{M'g}{m}};$$

$$\text{or} \qquad 16M=25M', \text{ or } \frac{M'}{M}=\frac{16}{25}.$$

Now, $$\text{Sp. gr.}=\frac{M}{M-M'}=\frac{1}{1-\frac{M'}{M}}=\frac{1}{1-\frac{16}{25}}=\frac{25}{9}$$

or Sp. gr. $=2{\cdot}78$. **Ans.**

EXERCISES

(A)

1. Calculate the frequency of a note emitted by a wire 20 cm in length when stretched by a weight 8 kg, if 2 m of the wire is found to weigh 4 gm. Also calculate the velocity of transverse waves along the string. **(Ans.** 495, 198 m/s)

2. A stretched sonometer wire gives 2 beats per second with a tuning fork when its length is 14·3 cm and also when its length is 14·5 cm. What is the frequency of the tuning fork ? **(Ans.** 288)

3. Two wires of a sonometer are in unison. When one of the wires is increased by 1%, three beats are heard in 2 seconds. Calculate the original frequency of each wire. **(Ans.** 151·5 Hz)

4. The length of the stretched wire of a sonometer is 110 cm. How would you place two bridges in order to divide into three segments whose fundamental frequencies will be in the ratio 1 : 2 : 3 ?

(Ans. $l_1=60$ cm; $l_2=30$ cm and $l_3=20$ cm)

5. A wire 50 cm long vibrates 100 times per second. If the length is shortened to 30 cm and the stretching force is quadrupled, what will be the frequency ? **(Ans.** 333·33)

6. A stone hangs in air from a wire which is stretched over a sonometer. The bridges of the sonometer are 40 cm apart when the wire is in unison with a tuning fork. When the stone is immersed in water the length is changed to 32 cm to re-establish unison with the wire. Calculate the specific gravity of the stone. (Ans. 2·77)

7. Two tuning forks when sounded together give 4 beats per second. One is in unison with a length of 96 cm of a sonometer wire under a certain tension and the other with 97 cm of the same wire under the same tension. Find the frequencies of the forks. **(Ans.** 388 and 384)

8. The frequency of a note emitted by silver wire of length 25 cm is 256 when the tension is 10 kg. Calculate the radius of the wire. (Density of silver $=10{\cdot}5\times10^3$ kgm^{-3}) **(Ans.** $4{\cdot}26\times10^{-4}$ m)

9. When the stretching force of a wire is increased by 2·5 kg, the frequency of the note emitted is changed in the ratio 2/3. Calculate the original stretching force. **(Ans.** 2 kg)

10. A fork and a monochord string of length 100 cm give 4 beats per second. The string is made shorter without any change of tension until it is in unison with the fork. If its length is now 99 cm, what is the frequency of the fork ? **(Ans.** 400)

11. Two tuning forks *A* and *B* produce 5 beats per second when sounded together. *A* is in unison with 40 cm of a sonometer wire under a constant tension and *B* is in unison with the same wire of length 40·5 cm under the same tension. Calculate the frequencies of the forks. (**Ans.** 405 Hz, 400 Hz)

12. A string vibrates with frequency 320 Hz. It is divided into three parts by using two bridges in the ratio 1 : 2 : 3. Calculate the frequency of each part. (**Ans.** 1920, 960 and 640 Hz)

13. In Melde's experiment the string vibrates in 4 loops when a 50 cm weight is placed on the pan of weight 15 gm. How much weight must be added or removed to make the string vibrate in 6 loops ? (**Ans.** 36 gm to be removed)

14. In Melde's experiment the string is stretched by an iron weight and the string vibrates in 5 segments. When an electromagnet is switched on under iron weight, the string vibrates in 4 segments. Compare the pull of the magnet with that of the earth. (**Ans.** 9 : 16)

15. In transverse arrangement the string vibrates in three loops when the tension is 200 gm. Calculate the tension required to make the string vibrate in 2 loops in the longitudinal arrangement. (**Ans.** ·45 kg)

16. Ordinary cotton thread 200 cm of which weighs 1 gm is used in Melde's experiment. It is attached at one end to a vibrator of frequency 100 Hz and at the other to a pan weighing 6 gm. What length of the string will vibrate in 4 loops in the longitudinal arrangement if 10 gm weight is put on the pan ? (**Ans.** ·708 m)

17. In Melde's experiment a string is found to vibrate in 8 loops and the total weight by which it is stretched is 20 gm. What length of it will vibrate in the same arrangement in 6 loops when weight is reduced to 15 gm ?
(**Ans.** to reduce the length of the string by 57·8% of its previous length)

18. A vibrator makes 150 cm of a string vibrate in 6 loops in the longitudinal arrangement when it is stretched by 15 gm. The entire length of the string is then weighed and is found to weigh 500 mgm. What is the frequency of the vibrator ? What is the distance between two nodes ? (**Ans.** 84 Hz; 25×10^{-2} m)

19. A string in Melde's experiment in transverse arrangement vibrates in 4 loops when it is stretched by a load of 100 gm. What additional load will be needed to make the same string vibrate in 1 loop in longitudinal arrangement ?
(**Ans.** to increase load by 300×10^{-3} kg or 300 gm)

(B)

20. A bridge is placed under the string of a monochord at a point near the middle and it is found that the two parts produce 3 beats per second when the stretching force is 8 kg. If the load be then increased to 12 kg, determine the rate of beating of the two parts of the string. (**Ans.** 3·67)

21. A wire of density 9000 kg m^{-3} is stretched between two clamps 1 m apart when it suffers an increase in length by ·05 cm. Calculate the frequency of the fundamental transverse vibration of the string. (*Y* of the material of wire $=9\times10^{10}$ N/m^2)

[*Hint* : Calculate tension of the wire from formula, Young's modulus $=\frac{\text{stress}}{\text{strain}}$.]

(**Ans.** 35·4 Hz)

22. With what force a wire of radius 1 mm must be stretched so that velocity of transverse wave along it may be same as that of longitudinal wave ? Give answer in kg wt. (Young's modulus of material of wire$=2\times10^{11}$ Nm^{-2}.) Is it physically possible ?

$$\left[\textit{Hint}: C_{longitudinal}=\sqrt{\frac{Y}{D}}\ ;\ C_{transverse}=\sqrt{\frac{T}{M}}\right]$$

(**Ans.** $6{\cdot}4\times10^4$ kg; impossible)

23. Show that if $n_1, n_2, n_3, n_4,$.........are the fundamental frequencies of the segments into which a string is divided by placing a number of bridges below it, the frequency of the string is given by

$$\frac{1}{n}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}+\ldots\ldots$$

24. A metal wire of diameter 1 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire, vibrating with its fundamental frequency, and a vibrating tuning fork together produce 5 beats per second. The tension in the wire is then reduced to 81N. When the two are excited beats are heard again at the same rate. Calculate (i) the frequency of the fork; (ii) the density of the material of the wire. (**Ans.** 95 Hz; 12730·8 kgm^{-3})

25. An aluminium wire of length ·6 m and of cross-sectional area 10^{-6} m^2 is connected to a steel wire of same cross-sectional area and length ·866 m. The compound wire is loaded with 10 kg. Find the lowest frequency of excitation for which the joint in the wire is a node. Also find the number of nodes, excluding the two at the ends of the wire. The density of aluminium is 2600 kgm^{-3}, and that of steel is 7800 kgm^{-3}. (**Ans.** 323 Hz; 6)

26. Two wires of radii r and $2r$ respectively are welded together end to end. This combination is used as a sonometer wire and is kept under tension T. The welded point is midway between the two bridges. What would be the ratio of the number of the loops formed in the wires such that the joint is a node when stationary vibrations are set up in the wires ? (**I. I. T. 1976**)

(**Ans.** 1 : 2)

□□□

CHAPTER 4

VIBRATION OF AIR COLUMNS

(Organ Pipes and Kundt's Tube)

1. Vibration of Air Column : When a vibrating fork is held over the mouth of a tube, the air inside of it is thrown into 'forced vibration'. On changing the length of the air column in it by a movable piston or by immersing the tube in water and lowering and raising the tube, the natural frequency of the air column changes and at one particular length it is thrown into resonant vibration when suddenly very loud sound is heard. Due to reflection of the wave sent down the tube by the fork, stationary waves are also formed in the tube with definite nodes and antinodes. The fixed end is essentially a seat of a 'node' and the open end is that of an antinode. Therefore, in the fundamental mode of vibration we have only a node at the bottom and an antinode at the top. Therefore l (length of the tube) $=\lambda/4$. This, however, requires a correction because antinode is not formed exactly at the mouth of the tube and it is a little above the mouth. By how much it is above that of course depends on the diameter of the tube. *This distance by which antinode is away from the open end is called end-correction.* Let this be e.

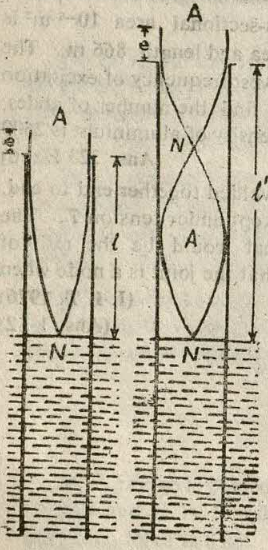

Fig. 4-1.

Then,

$$l+e=\lambda/4.$$

The first resonance occurs at this length. The second resonance occurs when the air column is in the next mode of vibration.

Now, $l'+e=3\lambda/4.$

$\therefore\ l'-l=\lambda/2$; or $\lambda=2(l'-l)$.

$$C=n\lambda.$$

$$\therefore\quad C=2n(l'-l).$$

According to Laplace the end-correction is : $e=\cdot 6r$ where r is the radius of the tube.

2. Organ pipes : Organ pipes are musical instruments in the form of pipe of wood or metal and hence called Organ pipes. In these instruments musical notes are emitted by vibration of air in the pipe.

(i) Closed Organ pipe : When one end is open and the other is closed, it is called closed organ pipe.

In the fundamental mode of vibration of a closed organ pipe,

$l=\lambda/4$ when end-correction is not taken into account;

or $\lambda=4l$.

We have, $n\lambda=C$;

$\therefore \quad n_1=\frac{C}{4l}$.

When end-correction is considered, $n=\frac{C}{4(l+e)}$.

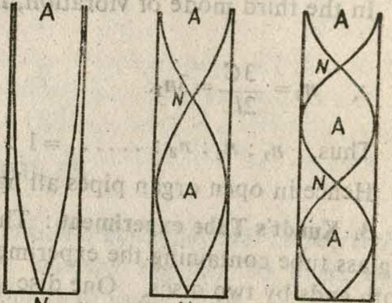

Fig. 4-2.

In the next mode of vibration,

$$\frac{3\lambda}{4}=l; \text{ or } \lambda=\frac{4l}{3}; \quad \therefore \quad n_2=\frac{C}{\lambda}=\frac{3C}{4l}; \quad \therefore \quad n_2=3n.$$

In the third mode of vibration, $\frac{5\lambda}{4}=l$; or $\lambda=\frac{4l}{5}$;

$$\therefore \quad n_3=\frac{C}{\lambda}=\frac{5C}{4l}; \quad \therefore \quad n_3=5n.$$

Thus, $n_1 : n_2 : n_3 : \ldots\ldots = 1 : 3 : 5 : \ldots\ldots$

(ii) Open Organ pipe : A pipe which is open at both ends is called open organ pipe. Both the ends are seats of antinodes.

In the fundamental mode of vibration,

$l=\frac{\lambda}{2}$: or $\lambda=2l$

(neglecting end-correction);

$\therefore \quad n_1=\frac{C}{2l}$.

When end-correction is considered, $n_1=\frac{C}{2(l+2e)}$.

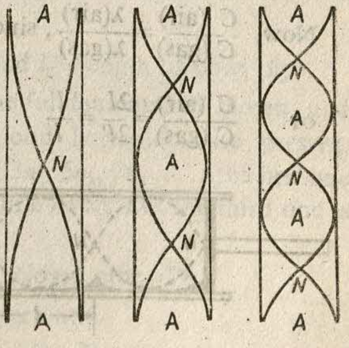

Fig. 4-3.

In the next mode of vibration, $\frac{2\lambda}{2}=l$, or $\lambda=l$;

$$\therefore \quad n_2=\frac{C}{l}, \text{ or } n_2=\frac{2C}{2l}=2n_1.$$

In the third mode of vibration, $l=\frac{3\lambda}{2}$, or $\lambda=\frac{2l}{3}$;

$$\therefore \quad n_3=\frac{3C}{2l}=3n_1.$$

Thus, $n_1 : n_2 : n_3 : \ldots\ldots = 1 : 2 : 3 : \ldots\ldots$

Hence in open organ pipes all harmonics are present.

3. **Kundt's Tube experiment** : The experimental set up consists of a glass tube containing the experimental gas or liquid and closed at both ends by two discs. One disc is attached to a rod (called sounding rod) clamped rigidly at its centre and the other disc can be slided inside the tube. The rod is set to longitudinal vibrations by rubbing it with a resined cloth along its length. The air in the tube is thrown into forced vibration by the vibrating rod. Also stationary waves are formed in the tube with definite nodes and antinodes.

On sliding the movable disc, the frequency of air or gas or liquid, whatever it is, changes and at one particular length it is thrown into resonant vibration when the lycopodium powder already sprinkled into the tube collects in heaps at the nodes. Thus the positions of nodes are marked clearly by lycopodium powder. The distance between nodes is noted.

Let l = distance between nodes in air,
l' = ,, ,, ,, in experimental gas.

Now, $\frac{C\text{ (air)}}{C\text{ (gas)}}=\frac{\lambda\text{(air)}}{\lambda\text{(gas)}}$, since n is the same for both the media

or $\frac{C\text{ (air)}}{C\text{ (gas)}}=\frac{2l}{2l'}=\frac{l'}{l}$ $\left(\because \frac{\lambda}{2}=l, \text{ or } \lambda=2l\right)$

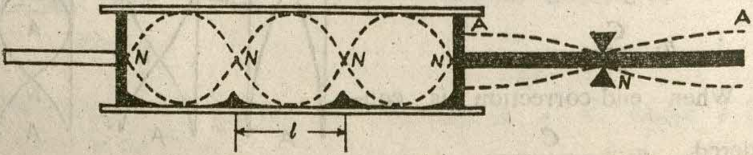

Fig. 4-4.

or $C\,(\text{gas}) = C\,(\text{air}).\dfrac{l'}{l}.$

If L = length of the rod then $L = \dfrac{\lambda\,(\text{solid})}{2}$

or $\lambda\,(\text{solid}) = 2L.$

$$\therefore \quad \frac{C\,(\text{solid})}{C\,(\text{air})} = \frac{\lambda\,(\text{solid})}{\lambda\,(\text{air})} = \frac{2L}{2l} = \frac{L}{l}.$$

$$\therefore \quad C\,(\text{solid}) = C\,(\text{air}) \times \frac{L}{l}.$$

EXAMPLES

Ex. 1. *The lengths of open and closed organ pipes are respectively* 160 cm *and* 75 cm. *When both are sounded together* 70 *beats are counted in* 10 *seconds. Find the velocity of sound in air.*

Sol. We have, $n = \dfrac{C}{2l}$ for open organ pipe and $n = \dfrac{C}{4l}$ for closed organ pipe.

$$\therefore \quad n_1 = \frac{C}{2 \times 1{\cdot}60} = \frac{C}{3{\cdot}2} \text{ and } n_2 = \frac{C}{4 \times {\cdot}75} = \frac{C}{3{\cdot}0}.$$

Obviously $n_2 > n_1$; $\quad \therefore\ n_2 - n_1 = \dfrac{70}{10}$

or $\dfrac{C}{3} - \dfrac{C}{3{\cdot}2} = 7$; or $C = \dfrac{7 \times 3 \times 3{\cdot}2}{{\cdot}2} = 336 \text{ ms}^{-1}$. **Ans.**

Ex. 2. *The third overtone of a closed pipe is found to be in unison with the first overtone of an open pipe. Find the ratio of the lengths of the pipes.*

Sol. Let l_1 = length of closed pipe and l_2 = length of open pipe.

Let n_1 and n_2 be the frequency of the fundamental of closed and open pipe respectively. In closed pipe odd harmonics are present. Hence harmonics in closed pipe are $n_1, 3n_1, 5n_1, 7n_1 \ldots$; the first one is called fundamental; the second, first overtone; the third one is called second overtone and so on.

$\therefore$ Frequency of third overtone of closed pipe $= 7n_1$.

In open pipe all the harmonics are present.

Hence harmonics in open pipe are $n_2, 2n_2, 3n_2, 4n_2, \ldots$

$\therefore$ Frequency of first overtone of pipe $= 2n_2$.

According to question, $7n_1 = 2n_2$.

But $n_1 = \dfrac{C}{4l_1}$ and $n_2 = \dfrac{C}{2l_2}$.

$$\therefore \quad 7\frac{C}{4l_1} = 2.\frac{C}{2l_2}; \quad \therefore \quad \frac{l_1}{l_2} = \frac{7}{4}. \qquad \textbf{Ans.}$$

Ex. 3. *The shortest length of a resonance tube which resonates with a tuning fork of frequency* 256 *Hz is* 32 cm. *The corresponding length for a fork of frequency* 384 *Hz is* 20·8 cm. *Calculate the end-correction and velocity of sound in air.*

Sol. We have in a closed pipe, $n = \dfrac{C}{4(l+e)}$ where e is the end-correction.

$$\therefore \quad 256 = \frac{C}{4(32+e)\times 10^{-2}} \text{ and } 384 = \frac{C}{4(20{\cdot}8+e)\times 10^{-2}}.$$

Dividing, we have

$$\frac{256}{384} = \frac{20{\cdot}8+e}{32+e}. \quad \text{Hence } e = 1{\cdot}6 \text{ cm.}$$

$$\therefore \quad C = 256 \times 4 \times (32+1{\cdot}6) \times 10^{-2} = 344 \text{ ms}^{-1}. \qquad \textbf{Ans.}$$

Ex. 4. *In a Kundt's tube experiment it was found that the distance between nodes was* 6·6 cm *when the sounding rod used was of total length* 80 cm. *Calculate the velocity of sound in the rod if velocity of sound in air is* 330 ms^{-1}.

Sol. We have, $\dfrac{C\text{ (solid)}}{C\text{ (air)}} = \dfrac{L}{l}$.

$$\therefore \quad C\text{ (solid)} = \frac{80\times 10^{-2}}{6{\cdot}6\times 10^{-2}} \times 330; \text{ or } C\text{ (solid)} = 4000 \text{ ms}^{-1}. \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. A closed pipe 28 cm long is filled with a gas and is in resonance with a tuning fork. If a 42 cm long open pipe filled with air, is in unison with the same fork, find the velocity of sound in the gas. (Velocity of sound in air at that temperature = 350 ms^{-1}) (**Ans.** 466·7 ms^{-1})

2. A tuning fork is held a little above the open end of a resonance tube. Resonance is heard when the length of air columns are respectively 24 cm and 74·1 cm. Calculate end-correction and diameter of the tube.

(**Ans.** $1{\cdot}05\times10^{-2}$ m or 1·05 cm, Diameter$=3{\cdot}5\times10^{-2}$ m)

3. Two open pipes are sounded together, each produces notes consisting of the fundamental and also two upper harmonics. Fundamental note of one is 256 and that of the other is 170. Would there be any beats produced, if so, how many ? (**Ans.** 2 beats per second)

4. Two open organ pipes 100 and 101 cm long give 17 beats in 10 seconds when each is sounding its fundamental. Calculate the frequencies of the fundamental and the velocity of sound in air.

(**Ans.** 171·7 Hz; 170 Hz; 343·4 ms^{-1})

5. The frequency of an organ pipe at 40°C is 256. What will be its frequency at 20°C ? (**Ans.** 247·3)

6. Two organ pipes give 5 beats per second when sounded together in air at a temperature 20°C. How many beats would be produced by them at 40°C ? (Velocity of sound at 0°C$=332$ ms^{-1}) (**Ans.** 5·168)

7. Two tuning forks *A* and *B* give 18 beats in 2 seconds. *A* resonates with a closed column of air 15 cm long and *B* with an open column 30·5 cm long. Calculate their frequencies. (**Ans.** 549 Hz and 540 Hz)

8. Calculate the least mass of air that will resonate with a tuning fork of frequency 340 Hz in a closed tube of diameter 4 cm at 30°C. (Velocity of sound in air at 30°C$=350$ ms^{-1} and density of air at 30°C$=1{\cdot}3$ kg per cubic metre.) Neglect end-correction. (**Ans.** $4{\cdot}2\times10^{-4}$ kg or ·42 gm)

9. A tuning fork of frequency 440 Hz is to be mounted on a wooden box with one end open to reinforce its sound. What would be the length of the sounding box ? (Velocity of sound in air$=332$ ms^{-1}.)

(**Ans.** $18{\cdot}9\times10^{-2}$ m or 18·9 cm)

10 The first overtone of an open pipe and the fundamental tone of a closed pipe give 5 beats per second when sounded together. If the length of the closed pipe is 25 cm, what are the possible lengths of the open pipe ? (Velocity of sound in air is 340 ms^{-1}) (**Ans.** $98{\cdot}5\times10^{-2}$ m or $101{\cdot}5\times10^{-2}$ m)

11. An open organ pipe is emitting a note of 200 Hz. What will be the effect on the frequency of the note emitted when one of its ends is suddenly closed ?

(**Ans.** When suddenly closed, the open pipe becomes a closed pipe; frequencies emitted are now 100, 300, 500,.........)

12. Two organ pipes, open at both ends, are sounded together and six beats per second are produced. The length of the shorter pipe is 60 cm. Find the length of the other. (Velocity of sound in air$=330$ ms^{-1})

(**Ans.** 61·34 cm or ·6134 m)

13. Calculate the length of a narrow tube closed at one end, which will resonate with a tuning fork of frequency 510 Hz if the velocity of sound is 340 ms^{-1}.

(**Ans.** ·167 m)

14. A fork of frequency 512 Hz is found to produce resonance in the air column, first when the length of the air column is 16·5 cm and again when it is

50·5 cm. Find the velocity of sound, wavelength of wave emitted by the fork and end-correction. Also calculate radius of the tube.

(**Ans.** 348·16 ms^{-1}; ·68 m; $\cdot5\times10^{-2}$ m; $\cdot83\times10^{-2}$ m)

15. A pipe of length 1·5 m closed at one end is filled with a gas and it resonates at 30°C in its fundamental with a tuning fork. Another pipe of the same length but open at both ends is filled with air and it resonates in its fundamental with the same tuning fork. Calculate the velocity of sound at 0°C in the gas, given that the velocity of sound in air is 360 ms^{-1} at 30°C.

(**Ans.** 683·4 ms^{-1})

16. A tuning fork having a frequency of 340 Hz is vibrated just above a cylindrical tube 1·20 m long. Water is slowly poured in it. At what lengths of the air column resonance will take place ? (Velocity of sound in air at room temperature=340 ms^{-1})

(**Ans.** ·25 m and ·75 m)

17. In a Kundt's tube experiment a brass rod 120 cm long was used which had a fundamental frequency of 1400 Hz. The gas in the tube was CO_2 at 20°C and the distance between nodal heaps of powder was found to be 9 cm. Calculate the velocity of sound in brass and in CO_2 at 0°C.

[**Ans.** $C\,(CO_2)=243\cdot3$ ms^{-1} at 0°C; C (Brass)=3243·3 ms^{-1} at 0°C]

18. In a Kundt's dust tube experiment a brass rod was found to resonate with 10 cm of a sonometer wire stretched by 6 kg. One hundred centimetre of the wire was found to weigh 653 mgm. This rod produced nodal heaps at 12 cm apart in air. Calculate velocity of sound in air.

(**Ans.** 360 ms^{-1})

19. The sounding rod of a dust tube apparatus is made of brass and is 160 cm long. The distance between adjacent nodes in the wave tube was 11·35 cm. Calculate Young's modulus of rod assuming that velocity of sound in air at room temperature is 350 ms^{-1} and density of brass 9000 kgm^{-3}.

(**Ans.** $2\cdot2\times10^{11}$ Nm^{-2})

(B)

20. A tuning fork of frequency 270 resonates with air column of length 31·2 cm. When the tube is raised it again resonates when the length is 96·3 cm. Calculate velocity of sound in moist air in the tube. Reduce this velocity to S.T.P. if the pressure and temperature at the time of the experiment are ·7 m of *Hg* and 30°C respectively. Saturated vapour pressure at 30°C=32 mm of mercury.

$$\left[\textit{Hint}: C_m=2n(l_2-l_1) \text{ and } C_m=C_d\sqrt{\left(\frac{P}{P-\cdot385\,f}\right)} \text{ and } \frac{C}{C_0}=\sqrt{\frac{T}{T_0}}.\right]$$

(**Ans.** $C_m=351\cdot6$ ms^{-1}; C_d at S.T.P.=330·6 ms^{-1})

21. Find the number of possible natural frequencies of an air column in a pipe whose frequencies lie below 1250 Hz. The length of the pipe is 85 cm. The speed of sound is 340 ms^{-1}. Consider the two cases :

(a) the pipe is closed at one end, (b) the pipe is opened at both ends.

(**Ans.** 6, 6)

□□□

CHAPTER 5

INTENSITY AND PITCH OF MUSICAL NOTES

1. Intensity : The rate at which flow of energy through unit area of a plane perpendicular to the direction of wave propagation takes place is called Intensity of wave.

Intensity of wave $=\frac{1}{2}\rho a^2\omega^2 C$ watts per square metre (Wm^{-2}) where ρ = density of medium, a = amplitude of wave and $\frac{\omega}{2\pi}$ = frequency of wave. One arbitrary unit of intensity is Bel. If I is the absolute intensity in watt per square metre then it is said that the absolute intensity level of the note is $\log_{10} 1000I$ bel. One-tenth of a bel is called a decibel (db).

A standard level is the intensity level of a wave of frequency 1000 Hz having root mean square pressure $2\times 10^{-5}\ Nm^{-2}$ corresponding to the absolute intensity of 10^{-12} watt per square metre (Wm^{-2}) in air. Thus the intensity level of a sound wave of absolute intensity I (Wm^{-2}) relative to the standard level $=\log 1000I - \log\ 1000\times 10^{-12}$

$$=12+\log I.$$

The intensity falls off as the square of the distance if there is no absorption of energy by the medium. That is, $I=\frac{P}{r^2}$ where P is the energy emitted by the source in every second through unit solid angle. This is called inverse square law in sound. If there is absorption by the medium, then $I=I_0 e^{-\mu x}$, where μ is a constant called the absorption coefficient of the medium. The distance within which the intensity is reduced to half of the initial value is called 'half-thickness' of the medium for sound.

2. Pitch : The degree of shrillness of a note is called its pitch. It is measured by frequency of wave as perceived by observer.

3. Siren or Toothed wheel : These are instruments to measure pitch of a note. The pitch of a note emitted by a siren or toothed wheel is '$m\times n$' where m = number of holes or teeth and n = number of revolutions per second.

4. Interval : The ratio of the frequencies of two notes is called interval between them. All intervals are not pleasant to the ear.

Only the simple ones are pleasant and they are used in musical scale. Certain common consonant intervals with their names are given below :

1 : 1 Unison 3 : 2 Fifth 5 : 4 Major third 5 : 3 Major sixth
1 : 2 Octave 4 : 3 Fourth 6 : 5 Minor third 8 : 5 Minor sixth

EXAMPLES

Ex. 1. *The disc of a siren revolves* 600 *times in one minute and it is in unison with a tuning fork of frequency* 480 *Hz. Calculate the number of holes in the disc.*

Sol. Frequency of note emitted by siren $= mn = m \times \frac{600}{60} = 10$ m. Since siren is in unison with tuning fork,

$$10m = 480, \text{ or } m = 48. \qquad \textbf{Ans.}$$

Ex. 2. *A tuning fork of frequency* 512 *Hz causes air particles to vibrate with amplitude* 2×10^{-6} m. *What energy current density does it produce in air ? Velocity of sound in air* $= 340$ m/s. *Density of air* $= 1 \cdot 3$ kgm^{-3}.

Sol. Intensity of wave is the energy current density,

$$I = \tfrac{1}{2}\rho a^2\omega^2 C = \tfrac{1}{2} \times 1 \cdot 3 \times (2 \times 10^{-6})^2 . (2\pi . 512)^2 . 340$$
$$= 1 \cdot 3 \times 2 \times 4\pi^2 \times 34 \times 512^2 \times 10^{-11}$$
$$= 9 \cdot 15 \times 10^{-8} \text{ watt per square metre.} \qquad \textbf{Ans.}$$

Ex. 3. *Calculate the intensity of the wave* $y = \cdot 002 \sin (4000t - 10x)$. *Density of medium* $= 1 \cdot 29$ kgm^{-3}.

Sol. We have, $I = \frac{1}{2}\rho a^2\omega^2 C = \frac{1}{2}\rho a^2\omega^2 n\lambda = \frac{1}{4\pi}\rho a^2\omega^2\lambda \quad (\because \omega = 2\pi n)$.

Here, $a = \cdot 002$ m, $\omega = 4000$ and $\frac{2\pi}{\lambda} = 10$ or $\lambda = \frac{2\pi}{10}$.

$$\therefore \quad I = \frac{1}{4\pi} . 1 \cdot 29 (\cdot 002)^2 (4000)^3 \frac{2\pi}{10} = 165 \cdot 1 \times 10^2 \text{ watt m}^{-2}. \qquad \textbf{Ans.}$$

$\therefore$ Absolute intensity level $= \log 1000 \times 165 \cdot 1 \times 10^2 = 7 \cdot 22$ bels and intensity level relative to the standard

$$= 12 + \log 165 \cdot 1 \times 10^2 = 12 + 4 \cdot 22 = 16 \cdot 22 \text{ bels.} \qquad \textbf{Ans.}$$

Ex. 4. *A smoked plate falls vertically under gravity and a tuning fork traces wave on it. I was found from the trace that there* *were 10 waves in the beginning and these 10 waves are spaced over 4·2 cm. Calculate the frequency of tuning fork.*

Sol. Obviously time of fall of plate through 4·2 cm is equal to time in which the fork makes 10 oscillations. We have,

$$s=ut+\tfrac{1}{2}gt^2=\tfrac{1}{2}gt^2 \quad (\because \ u=0)$$

$$t=\sqrt{\frac{2s}{g}}=\sqrt{\frac{2\times 4{\cdot}2\times 10^{-2}}{9{\cdot}8}}\ s;$$

$$\therefore \ \text{frequency}=\frac{10}{t}=10\sqrt{\frac{9{\cdot}8}{2\times 4{\cdot}2\times 10^{-2}}}=108 \text{ Hz.} \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. A siren with 20 holes in its disc gives 5 beats per second with a closed pipe sounding its fundamental note when the disc makes 2400 revolutions per minute. What is the effective length of the pipe ? (velocity of sound in air $=360$ ms^{-1}) **(Ans.** $11{\cdot}18\times10^{-2}$ m or $11{\cdot}32\times10^{-2}$ m**)**

2. A siren having a ring of 200 holes is making 132 revolutions per minute and found to emit a note which is an octave higher than that of a tuning fork. Find the frequency of the latter. **(Ans. 220)**

3. A note of frequency 300 Hz has an intensity 1 microwatt per square metre. What is the amplitude of the air vibration caused by this sound ? (Density of air $=1{\cdot}293$ kgm^{-3} and velocity of sound in air $=332$ ms^{-1}.) **(Ans.** $3{\cdot}62\times10^{-8}$ m**)**

4. The disc of a siren has 100 holes and rotates at the rate of 1000 revolutions per minute. Find the frequency and wavelength of the note emitted by the siren. (Velocity of sound in air $=350$ m/s.) **(Ans. 1666·67 Hz; ·21 m)**

5. A smoked plate falls vertically under gravity. A tuning fork traces wave on it. It is found that the lengths of two consecutive group of 10 waves are 5·143 and 6·64 cm respectively. What is the frequency of the fork ? **(Ans. 256)**

6. A pointer attached to a vibrating fork presses lightly a smoked cylinder. The cylinder is suddenly rotated through one complete revolution in ·75 second. The pointer traces 192 waves on the cylinder. Calculate the frequency of the tuning fork. **(Ans. 256)**

7. The minimum intensity of audibility of a source is 10^{-12} watt/m^2. If a frequency of the note is 1000 Hz, calculate the amplitude of vibration of air particles. Density of air $=1{\cdot}293$ kgm^{-3} and velocity of sound $=340$ ms^{-1}.

$$\left[\textit{Hint}: \ a=\sqrt{\frac{I}{2\pi^2 n^2 \rho C}}\right]$$ **(Ans.** $1{\cdot}07\times10^{-11}$ m**)**

8. Calculate the intensity of a note of frequency 1000 Hz if the amplitude of vibration is 10^{-9} cm. Density of air $=1{\cdot}3$ kgm^{-3} and velocity of sound $=340$ ms^{-1}. **(Ans.** $\cdot87\times10^{-12}$ watt/m^2**)**

9. Calculate the amplitude of vibration of air particles when a wave of intensity 10^{-3} watt/m^2 is produced by a tuning fork of frequency 540 Hz. Velocity of sound in air $= 340$ ms^{-1} and Density of air $= 1{\cdot}3$ kgm^{-3}.

(**Ans.** $6{\cdot}27 \times 10^{-7}$ m)

(B)

10. S_1 and S_2 are two loud speakers having same frequency of 165 Hz and acoustic output $1{\cdot}2 \times 10^{-3}$ and $1{\cdot}8 \times 10^{-3}$ watt respectively. They vibrate in the same phase. P is a point at a distance 4 m from S_1 and 3 m from S_2.

(a) How are the phases of the two waves arriving at P related ?

(b) What is the intensity at P if S_1 is turned off (S_2 on) ?

(c) What is the intensity of sound at P if S_2 is turned off ?

(d) What is the intensity at P with S_1 and S_2 on ?

[*Hint* : For phase relationship see Ex. 6 of chapter I.
Intensity is inversely proportional to square of the distance.]

[**Ans.** (a) π, (b) $\cdot 2 \times 10^{-3}$ watt, (c) $\cdot 075 \times 10^{-3}$ watt, (d) $\cdot 125 \times 10^{-3}$ watt]

11. The intensity of a sound wave 20 m away from the sound source is $3n$W/m^2. Find the intensity of the wave 32 m away from the source, if the half-thickness for sound of this frequency is 120 m. (**Ans.** $1{\cdot}1\ n$W/m^2)

12. Calculate the interval between two sound waves of frequencies 1000 Hz and 400 Hz. Express the interval on logarithmic scale and in centioctaves.

[*Hint* : absolute interval $= n_1/n_2$; interval on log scale $= \log n_1/n_2$, interval in centioctaves $= \dfrac{\log n_1/n_2}{\log 2} \times 100$.] (**Ans.** $2{\cdot}5$, $0{\cdot}3979$, $132{\cdot}2$)

□□□

CHAPTER 6

DOPPLER'S EFFECT

1. Doppler's effect : Whenever there is a relative motion between an observer and the source, the pitch of the note emitted by the source appears to be changed to the observer. The phenomenon of apparent change in pitch of a note emitted by a source due to relative motion between observer and source is called Doppler's effect.

If V = velocity of wave, V_s = velocity of the source and V_o = velocity of the observer then apparent pitch (n') is given by

$$n' = \left(\frac{V - V_o}{V - V_s}\right) \times n \text{ hertz (Hz)}$$

where n = actual frequency of the source. In this formula all are supposed to move in the same direction.

Note : In tackling problems with this general formula, take the direction from *source to observer* as positive direction. If V_o and V_s are also in this direction, take them as positive and if opposite take them as negative.

2. Mach Number : The ratio of the speed of the source to the speed of the sound wave is called the 'Mach number'. The speed of supersonic jet planes is expressed in terms of Mach number.

EXAMPLES

Ex. 1. *A person standing on one side of a road blows a whistle which has a frequency of* 600 *Hz. A cyclist passes at* 16 *kilometres per hour. What does the frequency of the whistle appear to be to the cyclist before and after passing* ?

(Velocity of sound in air = 350 ms^{-1}.)

Sol. We have in general, $n' = \frac{V - V_o}{V - V_s} \times n.$

Case I.

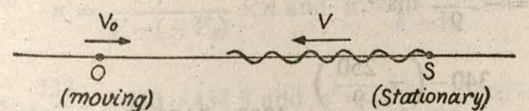

Fig. 6-1.

Here $V = 350\ ms^{-1}$, $V_s = 0$,

$$V_o = -16 \text{ km.p.h.} = -\frac{16\times 1000}{60\times 60} \text{ ms}^{-1} = -\frac{40}{9} \text{ ms}^{-1}.$$

$$\therefore \quad n = \frac{350-\left(-\frac{40}{9}\right)}{350-0}\times 600 = \frac{350+\frac{40}{9}}{350}\times 600 = 608 \text{ Hz.} \qquad \textbf{Ans.}$$

Case II.

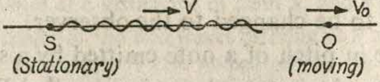

Fig. 6-2.

Now, $V = 350 \text{ ms}^{-1}, V_s = 0, V_o = +\frac{40}{9} \text{ ms}^{-1}$

$$\therefore \quad n = \frac{350-\frac{40}{9}}{350}\times 600 = 592 \text{ Hz.} \qquad \textbf{Ans.}$$

Ex. 2. *Two express trains moving at* 100 *kilometres per hour are meeting each other while one of them is whistling continuously. If the frequency of the note is* 100 *Hz, find the apparent pitch to an observer in the other train* (a) *before the trains meet, and* (b) *after they have passed each other.* (*Velocity of sound in air* = 340 m/s.)

Sol. (a) We have in general, $n' = \frac{V-V_o}{V-V_s}\times n.$

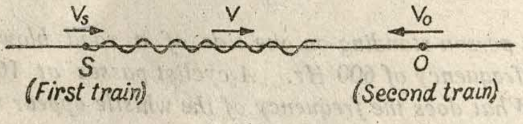

Fig. 6-3.

Here, $V = 340 \text{ ms}^{-1}$, $V_s = +100 \text{ km.p.h.} = \frac{100\times 1000}{60\times 60} = \frac{250}{9} \text{ ms}^{-1}$,

$$V_o = -\frac{250}{9} \text{ ms}^{-1}.$$

$$\therefore \quad n' = \frac{340-\left(-\frac{250}{9}\right)}{340-\frac{250}{9}}\times 1000 = 1178 \text{ Hz.} \qquad \textbf{Ans.}$$

(b) After they have passed each other :

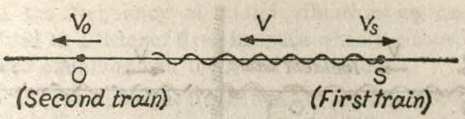

Fig. 6-4.

Now, $V_s = -\frac{250}{9}$ ms^{-1} and $V_o = +\frac{250}{9}$ ms^{-1}

$$\therefore \quad n' = \frac{340 - \frac{250}{9}}{340 - \left(-\frac{250}{9}\right)} \times 1000 = 894 \text{ Hz.} \qquad \textbf{Ans.}$$

Ex. 3. *A tuning fork of frequency* 440 *Hz approaches a wall with a velocity of* 4 ms^{-1}. *What will be the number of beats between the direct and reflected sound if the velocity of sound is equal to* 332 ms^{-1} *?* (Similar I.I.T. 1981)

Sol. Case I. When the tuning fork is moving away from the observer :

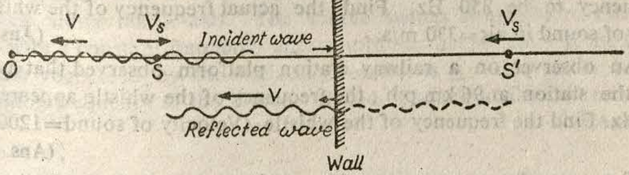

Fig. 6-5.

The observer is receiving waves from two moving sources (i) one direct from the source, and (ii) the other from the image source.

Let n' = appt. frequency of direct wave,

n'' = ,, ,, ,, of reflected wave.

$$n' = \frac{V}{V - (-V_s)} \times n \text{ and } n'' = \frac{V}{V - V_s} \times n$$

$$\therefore \quad n' = \frac{332}{332+4} \times 440 = 434{\cdot}8 \text{ and } n'' = \frac{332}{332-4} \times 440 = 445{\cdot}4.$$

Hence number of beats/sec. = 445·4 − 434·8 = 10·6. **Ans.**

Case II. When the tuning fork is moving towards the observer :

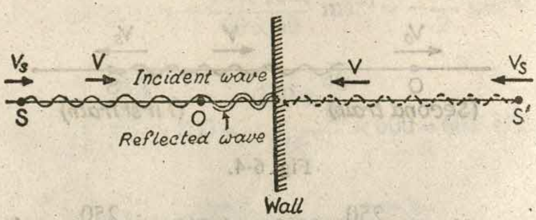

Fig. 6-6.

In this case both the sources and the images are approaching the observer with the same speed. Hence the frequency of the direct and reflected wave will be same. Hence no beat will be formed. **Ans.**

EXERCISES

(A)

1. A train approaches a stationary observer at a speed of 75 kilometres per hour sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle to the observer ? Velocity of sound$=332$ ms^{-1}.

(**Ans.** 1067 Hz)

2. A train is whistling while approaching a tunnel at a speed of 36 km p.h. The driver hears the echo of the whistle reflected from the tunnel and estimates its frequency to be 850 Hz. Find the actual frequency of the whistle. The velocity of sound in air$=330$ m/s. (**Ans.** 800 Hz)

3. An observer on a railway station platform observed that as the train passed the station at 96 km.p.h., the frequency of the whistle appeared to drop by 400 Hz. Find the frequency of the whistle. (Velocity of sound$=1200$ km p.h.)

(**Ans.** 2484 Hz)

4. Two aeroplanes are approaching each other, and their velocities are 160 and 240 kilometres per hour. The frequency note emitted by the first as heard by the passengers in the other is 1000 Hz. Calculate the true frequency of the note as heard by its own passengers. (Velocity of sound$=1200$ km.p.h.)

(**Ans.** 772·2 Hz)

5. A man is standing beside a railway line listening to the whistle of a passing train. The whistle which has a frequency of 1000 Hz suffers an apparent change of 100 Hz. What is the speed of the train ? Velocity of sound $=333{\cdot}3$ ms^{-1}. (**Ans.** 16·7 ms^{-1})

6. Show that Doppler's effect is greater when the source approaches the observer than when the observer approaches the source with the same speed.

7. The wavelength of spectral line coming from a star is changed by the motion of the star from 6000A° to 6001A°. Find the velocity of the star with respect to the earth. (Velocity of light$=3\times10^8$ m/s.)

(**Ans.** 5×10^4 ms^{-1}—receding)

8. A supersonic jet plane of Mach 1·5 is sighted overhead. How far will it be after 10 s ? (Velocity of sound = 340 ms^{-1}) (**Ans.** 5·1 km)

9. Show that the frequency of sound vibrations emitted by a source will appear to be doubled to a listener fixed in space when the source approaches the listener with a speed equalling half the speed of sound.

10. The ratio of the apparent frequencies of the horn of a car when approching and receding a stationary observer is 11 : 9. What is the speed of the car, if the velocity of sound in air is 300 m/s ? (**Ans.** 30 m/s)

(B)

11. Can you go fast enough to have red light appear green ? λ (red) = 6200 A and λ (green) = 5400 A. (**Ans.** Speed needed = $4{\cdot}4 \times 10^7$ m/s)

12. A bullet is fired with a speed of 670 ms^{-1}. Find the angle made by the 'shock' wave with the line of motion of the bullet. What is the mach number of the wave ? (Velocity of sound in air = 335 ms^{-1}.) (**Ans.** 30°; 2)

$$\left[\textit{Hint} : \sin\theta = \frac{V}{V_s}. \right]$$

13. A sonometer wire under tension of 64 N vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and has a mass of one gram. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s. (IIT 1983) (**Ans.** 0·75 m/s)

14. A receiver and a source of sonic oscillations of frequency $\nu_0 = 2000$ Hz are located along the *X*-axis. The source swings simple harmonically along that axis with a constant frequency and amplitude $a = 50$ cm. If the frequency band-width registered by the stationary receiver is $\Delta\nu = 200$ Hz, find the value of oscillation frequency of the source. Speed of sound in air = 340 ms^{-1}. (**Ans.** 5·4 Hz)

□□□

LIGHT

CHAPTER 1

REFRACTION AT PLANE AND SPHERICAL SURFACES

1. Refraction : When a ray of light passes from one medium to another, it bends towards or away from the normal according as the second medium is denser or rarer than the first medium. This phenomenon of bending of path of light is known as Refraction.

Laws of refraction : (i) The incident ray, the refracted ray and the normal to the refracting surface at the point of incidence lie in the same plane.

(ii) If a ray makes an angle θ_1 with the normal in the first medium and θ_2 in the second medium then $\frac{\sin\theta_1}{\sin\theta_2}$ is a constant for the two media and given colour of light. This is called index of refraction of the second medium with respect to the first. This is written as ${}_1\mu_2$.

Absolute index of refraction : The index of refraction of a medium with respect to vacuum or air is called absolute index of refraction. It is written as μ_1, μ_2, μ_3, .. It can be shown from the principle of reversibility of path of light that ${}_1\mu_2=\frac{\mu_2}{\mu_1}$.

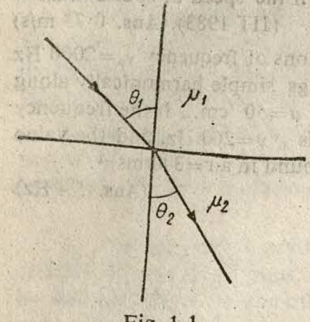

Fig. 1-1.

$$\therefore \frac{\sin\theta_1}{\sin\theta_2}=\frac{\mu_2}{\mu_1} \quad \text{or} \quad \mu_1 \sin\theta_1=\mu_2 \sin\theta_2.$$

This is the most general formula for refraction at a surface.

2. Total Internal Reflection and Critical Angle :

(i) Critical angle : When light passes from denser medium to rarer medium, angle of refraction is always greater than angle of incidence. Hence angle of refraction reaches 90° before angle of incidence can reach it. This particular angle of incidence when ray of light is refracted in the rarer medium at 90° i.e. ray grazes over the surface is called critical angle.

From general refraction formula $\mu_1\sin\theta_1 = \mu_2\sin\theta_2$, we have

$$\mu_1\sin 90^\circ = \mu_2\sin C$$

or $\sin C = \frac{\mu_1}{\mu_2} = \frac{1}{\frac{\mu_2}{\mu_1}} = \frac{1}{\mu}$ when $\mu = \frac{\mu_2}{\mu_1}$ where μ is the refractive index of the second medium relative to the first medium.

(ii) Total internal reflection : When ray of light is incident at an angle greater than critical angle, it is reflected back into the same medium instead of being refracted into the second medium. This is known as total internal reflection.

(iii) **Deviation and lateral displacement** : Deviation, in general, is the angle through which a ray is turned after reflection or refraction. At a plane surface due to refraction deviation is $(i-r)$ where i = angle of incidence and r = angle of refraction. When a ray of light passes through a parallel sided slab the deviation at the second surface is exactly equal and opposite to the deviation at the first surface. Hence a ray is not deviated in a parallel sided slab, rather it is shifted sideways. This is called lateral displacement.

3. Sign conventions :

Old convention : (i) All distances are proposed to be taken from the pole or optical centre as the case may be (pole in case of surfaces and optical centre in case of lenses).

(ii) Distances from pole or optical centre towards the object are to be treated as positive and away from the object as negative.

(iii) The medium in which light travels first before reflection or refraction is the first medium.

Co-ordinate convention : (i) Same as the old convention.

(ii) Distances from the pole or optical centre along the direction of light are to be treated as positive and against the direction of light as negative.

(iii) The medium in which light travels before refraction or reflection is to be treated as the first one and the other medium as the second one.

New or Modern convention : (i) Same as in the above two conventions.

(ii) Distances of real objects from pole or optical centre are to be taken positive and that of virtual objects as negative.

(iii) The signs of image distance, focal lengths, radii of curvature are to be fixed up as positive or negative according as the image, focus and centre of curvature lie on the *R*-side and *V*-side (here *R* is for real and *V* is for virtual). For refracting surfaces and thin lenses the side opposite to the incident is the *R*-side and the side from which the incident light comes is the *V*-side.

Incident light **V-side** **Medium I**	**R-side** **Medium II**

Convention for *V*-side and *R*-side for refracting surfaces and thin lenses

In case of mirrors the conventions for *R*-side and *V*-side is just the opposite.

Incident light $\longrightarrow$ **R-side**	**V-side**

Convention for *R*-side and *V*-side for mirrors

(iv) The medium in which light travels before refraction is taken as the first medium and the other as the second medium.

Note. As the old convention has become obsolete we use only coordinate and new conventions.

4. Refraction at spherical surfaces : When an object is placed at a distance u from a surface (curved or plane) in a medium of index of refraction μ_1, the relation between object and image distance (v) in old covention is given by

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r} \qquad \text{(in co-ordinate convention)}$$

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}. \qquad \text{(in modern convention)}$$

This is the most general formula for refraction at a surface curved or plane because *a plane surface may also be treated as a curved surface of infinite radius.*

Magnifications : (i) Lateral magnification (m)

$$=\frac{\text{sideways length of image}}{\text{sideways length of object}}=\frac{v}{u}\cdot\frac{\mu_1}{\mu_2}.$$

(ii) Longitudinal magnification

$$=\frac{\text{length of image along axis}}{\text{length of object along axis}}=\frac{\mu_2}{\mu_1}.m^2.$$

Thin lens as a medium bounded by curved surfaces : There will be bending of rays twice before final image is formed. So we will apply refraction formula twice, first to the refraction at first surface and then at the second surface. Always bear in mind in applying the above formula that the first medium is always that one in which a light travels.

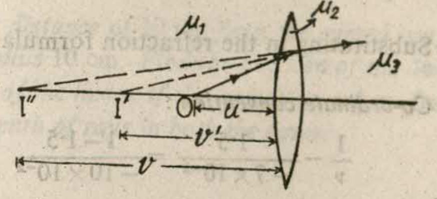

Fig. 1-2.

$$\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r_1}+\frac{\mu_3-\mu_2}{r_2} \quad \text{(in co-ordinate convention)}$$

$$\frac{\mu_3}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r_1}-\frac{\mu_3-\mu_2}{r_2} \quad \text{(new convention).}$$

5. **Focal length and Power of a lens** : When the object is at infinity, the image distance from the optical centre in case of thin lenses is called the second principal focal length. Ordinarily by focal length of a lens we mean this focal length. When the image of an object is formed at infinity, the distance of the object is called the first principal focal length of the lens. *The power of a lens is its capacity to impart curvature to a plane wave-front incident on it and is given by the reciprocal of its focal length.*

When a lens is bounded by the same medium, namely, air, $\mu_1=1$, $\mu_3=1$ and $\mu_2=\mu$ (index of refraction of the material of the lens).

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \text{(co-ordinate convention and new convention)}$$

$$\text{Power}=\frac{1}{f} \text{ dioptre (D).} \quad \text{(new and coordinate convention)}$$

EXAMPLES

Ex. 1. *A glass sphere of* 10×10^{-2} m *radius has a small black particle at a distance* 3×10^{-2} m *from its centre. The particle is viewed along a diameter from the side on which it lies. How far from the surface will it appear to be if index of refraction of glass is* 1·5 ?

Sol. The medium in which the object lies is always the first medium. Hence, here, $\mu_1 = 1{\cdot}5$, $\mu_2 = 1$,

$u = (10 \times 10^{-2} - 3 \times 10^{-2}) = 7 \times 10^{-2}$ m, $r = 10 \times 10^{-2}$ m.

Substituting in the refraction formula, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{r}$.

Co-ordinate convention :

$$\frac{1}{v} - \frac{1{\cdot}5}{-7 \times 10^{-2}} = \frac{1 - 1{\cdot}5}{-10 \times 10^{-2}} \text{ or } v = -{\cdot}0609 \text{ m.}$$

Since v is negative, the image is on the side of the object.

Ans. The image is formed at a distance ·0609 m on the side of the object.

New convention : $\dfrac{1}{v} + \dfrac{1{\cdot}5}{7 \times 10^{-2}} = \dfrac{1 - 1{\cdot}5}{-10 \times 10^{-2}}$ or $v = -{\cdot}0609$ m.

Since v is negative, the image is on the V-side, that is, on the side of the object.

Ex. 2. *A thin plano convex lens acts like a concave mirror of* 28×10^{-2} m *focal length when its plane surface is silvered. Calculate the radius of curvature of its curved surface if refractive index of material of lens is* 1·5.

Sol. The lens acts like a concave mirror of focal length ·28 m means that an object coincides with its image when it is at a distance $2 \times {\cdot}28 = {\cdot}56$ m from the lens because it is the property of a concave mirror that it forms the image of an object coincident with the object itself when it is at its centre of curvature.

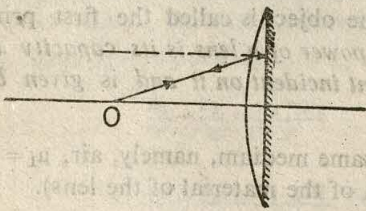

Fig. 1-3.

Consider refraction at spherical surface. The ray diverging from the object (O) must refract at curved surface in such a way that it falls normally on the plane surface. Hence the image of the object formed by the curved surface is at infinity.

So we have for curved surface,

$u = {\cdot}56$ m, $v = \infty$, $\mu_1 = 1$ and $\mu_2 = 1{\cdot}5$ (numerically).

Co-ordinate convention :

$$\frac{1{\cdot}5}{\infty} - \frac{1}{-{\cdot}56} = \frac{1{\cdot}5 - 1}{r} \text{ or } r = +{\cdot}28 \text{ m.}$$

Hence radius of curvature of the curved surface is ·28 m. **Ans.**

New convention : $\frac{1{\cdot}5}{\infty}+\frac{1}{{\cdot}56}=\frac{1{\cdot}5-1}{r}$ or $r=+{\cdot}28$ m. **Ans.**

Ex. 3. *An object is at a distance of* 10 cm *from the curved surface of a glass hemisphere of radius* 10 cm. *Find the position of the image. Also calculate the distance of the image if the object is taken further away by* 15 cm. *Trace the path of rays in both the cases.*

Sol. Case I.

Considering successive refraction at the two surfaces we have,

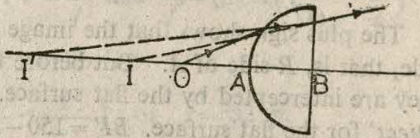

Fig. 1-4.

Co-ordinate convention :

$$\frac{1{\cdot}5}{v'}-\frac{1}{-10}=\frac{1{\cdot}5-1}{+10} \text{ or } v'=-30 \text{ cm.}$$

I' serves as object for the second surface.

$$\therefore \quad \frac{1}{v}-\frac{1{\cdot}5}{-40}=\frac{1-1{\cdot}5}{\infty} \text{ or } v=-26{\cdot}67 \text{ cm.}$$

Since v is −ve, the final image (I) is at a distance 26·67 cm from the flat surface towards the object.

New convention : $\frac{1{\cdot}5}{v'}+\frac{1}{10}=\frac{1{\cdot}5-1}{+10}$ or $v'=-30$ cm.

I' serves as object for the second surface.

$$\therefore \quad \frac{1}{v}+\frac{1{\cdot}5}{40}=\frac{1-1{\cdot}5}{\infty} \text{ or } v=-26{\cdot}67 \text{ cm.}$$

Case II.

Co-ordinate convention : $\frac{1{\cdot}5}{v'}-\frac{1}{-25}=\frac{1{\cdot}5-1}{+10}$ or $v'=+150$ cm.

The plus sign shows that the image I' is on the other side of A. But before the rays can converge to I', they are intercepted by the second surface. So I' serves as the '*virtual object*' for the second surface. $BI'=150-10=140$ cm.

$$\therefore \quad \frac{1}{v}-\frac{1{\cdot}5}{+140}=\frac{1-1{\cdot}5}{\infty} \text{ or } v=+93{\cdot}3 \text{ cm.}$$

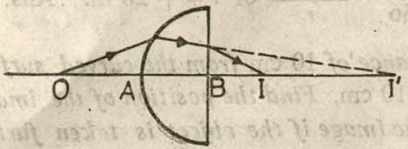

Fig. 1-5.

Since v is positive, the final image is at a distance 93·3 cm on the other side of the flat surface. **Ans.**

New convention :

$$\frac{1\cdot5}{v'}+\frac{1}{25}=\frac{1\cdot5-1}{+10}$$

or $v=+150$ cm.

The plus sign shows that the image I' is real and is on the other side, that is, R-side of A. But before the rays can converge to I', they are intercepted by the flat surface. So it serves as the *'virtual object'* for the flat surface. $BI'=150-10=140$ cm.

$$\therefore \quad \frac{1}{v}+\frac{1\cdot5}{-140}=\frac{1-1\cdot5}{\infty} \text{ or } v=+93\cdot3 \text{ cm.}$$

The plus sign shows that the image is real and is on the other side, that is, R-side of the flat surface at a distance 93·3 cm. **Ans.**

Ex. 4. *A concave mirror of radius of curvature* 30 cm *is placed on a table with its reflecting surface upward. A little liquid is poured on the surface of the mirror. Now a pin is held above it and is so adjusted that it coincides with its own image. The distance of the pin in this position is* 23 cm. *Calculate index of refraction of the liquid.*

Sol. Note that the liquid and the concave mirror form a liquid plano convex lens, curved surface of which is silvered. The ray OA bends along AB by refraction at plane surface (r ∞). The ray is returned along BA by the lower surface and along AO by the upper surface and the final image is formed at that very place where the object lies.

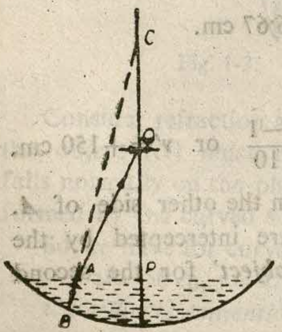

Fig. 1-6.

On producing BA it must pass through centre of curvature 'C' of the mirror.

Thus for plane surface PO = object distance = 23 cm (given) and PC = 30 cm (neglecting the thickness of the liquid). Applying refraction formula, *Co-ordinate convention* : $\dfrac{\mu}{-30}-\dfrac{1}{-23}=\dfrac{\mu-1}{\infty}$ or $\mu=\dfrac{30}{23}=1{\cdot}31$. **Ans.**

New convention : $\dfrac{\mu}{-30}+\dfrac{1}{23}=\dfrac{\mu-1}{\infty}$ or $\mu=1{\cdot}31$. **Ans.**

Ex. 5. *An equiconvex lens has power* $+2D$ *in air. What will be its power in water ? (Index of refraction of material of lens*$=1{\cdot}52$ *and Index of refraction of water*$=1{\cdot}34$.)

Sol. *Co-ordinate convention* : $P=\dfrac{1}{f}$ and $\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right)$.

Here $\quad P=+2D,\ r_1=+r$ (say), $r_2=-r$.

$$\therefore \qquad 2=(1{\cdot}52-1)\left(\frac{1}{r}-\frac{1}{-r}\right) \quad \text{or} \quad r={\cdot}52 \text{ m}.$$

In the above formula μ is the refractive index of the material of the lens relative to the surrounding medium. When the lens is immersed in water, μ is to be replaced by ${}_{w}\mu_{g}$. By the law of intermediate media for refractive indices,

$${}_{w}\mu_{g}={}_{w}\mu_{a}\times{}_{a}\mu_{g}=\frac{1}{{}_{a}\mu_{w}}\times{}_{a}\mu_{g}=\frac{1}{1{\cdot}34}\times1{\cdot}52=1{\cdot}134.$$

$$\therefore\ P\text{ (power in water)}=(1{\cdot}134-1)\left(\frac{1}{{\cdot}52}-\frac{1}{-{\cdot}52}\right)=+{\cdot}52\text{ D.} \quad \textbf{Ans.}$$

New convention : We have $P=\dfrac{1}{f}$ and $\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right)$.

Here $\quad P\ \ +2D,\ r_1=+r$ (say) and $r_2=-r$.

$$\therefore \qquad 2=(1{\cdot}52-1)\left(\frac{1}{r}+\frac{1}{r}\right) \quad \text{or} \quad r={\cdot}52 \text{ m}.$$

In the above formula μ is the refractive index of the material of the lens relative to the surrounding medium, that is, air.

When the lens is immersed in water, μ is to be replaced by ${}_{w}\mu_{g}$. By the law of intermediate media for refractive indices,

$${}_{w}\mu_{g}={}_{w}\mu_{a}\times{}_{a}\mu_{g}\quad\frac{1}{{}_{a}\mu_{w}}\times{}_{a}\mu_{g}=\frac{1}{1{\cdot}34}\times1{\cdot}52=1{\cdot}134.$$

$$\therefore\ P\text{ (power in water)}=(1{\cdot}134-1)\left(\frac{1}{{\cdot}52}+\frac{1}{{\cdot}52}\right)=+{\cdot}52\text{ D.} \quad \textbf{Ans.}$$

Ex. 6. *Three liquids of indices of refraction* μ_1, μ_2 *and* μ_3 *and thicknesses* d_1, d_2 *and* d_3 *respectively are placed one above the other,* μ_1 *being at the bottom and* μ_3 *on top. A small bright object is lying at the bottom of the first liquid. Find the apparent position of object when viewed normally from top.*

Sol. *Co-ordinate convention* : Applying refraction formula to the first surface (+ upward and −ve downward) $\frac{\mu_2}{v'}-\frac{\mu_1}{-d_1}=\frac{\mu_2-\mu_1}{\infty}$

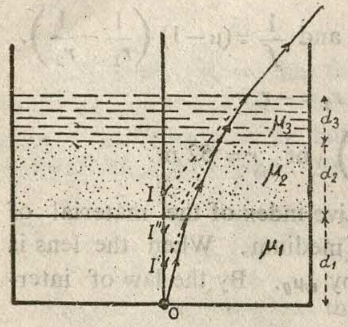

Fig. 1-7

(infinity because the surface is plane), or $v'=-\frac{\mu_2}{\mu_1}d_1$. The negative sign shows that it is on the side of the object. The image I' serves as object for the second surface. Object distance for the second surface $=d_2+\frac{\mu_2}{\mu_1}d_1$.

Applying refraction formula to the second surface,

$$\frac{\mu_3}{v''}-\frac{\mu_2}{-\left(d_2+\frac{\mu_2}{\mu_1}d_1\right)}=\frac{\mu_3-\mu_2}{\infty} \quad \text{or} \quad v''=-\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right)\mu_3.$$

The minus sign shows that I'' is on the side of I'.

I'' serves as object for the third surface. Object distance

$$=d_3+\mu_3\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right)=\mu_3\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\frac{d_3}{\mu_3}\right).$$

Applying refraction formula to the third surface,

$$\frac{1}{v}-\frac{\mu_3}{\mu_3\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\frac{d_3}{\mu_3}\right)}=\frac{1-\mu_3}{\infty} \quad \text{or} \quad v=-\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\frac{d_3}{\mu_3}\right). \qquad \textbf{Ans.}$$

New convention : Applying refraction formula to the first surface,

$$\frac{\mu_2}{v'}+\frac{\mu_1}{d_1}=\frac{\mu_2-\mu_1}{\infty} \quad (r=\infty \text{ because the surface is flat})$$

or $v'=-\frac{\mu_2}{\mu_1}d_1$. The negative sign shows that the image I' is virtual and it is on the side of the object. I' serves as object for the second surface. Object distance $= d_2+\frac{\mu_2}{\mu_1}d_1=\mu_2\left(\frac{d_2}{\mu_2}+\frac{d_1}{\mu_1}\right)$.

Applying refraction formula to the second surface,

$$\frac{\mu_3}{v''}+\frac{\mu_2}{\mu_2\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right)}=\frac{\mu_3-\mu_2}{\infty} \quad \text{or} \quad v''=-\mu_3\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right).$$

I'' serves as object for the third surface. Object distance

$$=d_3+\mu_3\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right)=\mu_3\left(\frac{d_3}{\mu_3}+\frac{d_2}{\mu_2}+\frac{d_1}{\mu_1}\right).$$

Applying refraction formula to the third surface,

$$\frac{1}{v}+\frac{\mu_3}{\mu_3\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\frac{d_3}{\mu_3}\right)}=\frac{1-\mu_3}{\infty} \quad \text{or} \quad v=-\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\frac{d_3}{\mu_3}\right). \quad \textbf{Ans.}$$

Ex. 7. *A hemispherical paper weight contains a small artificial flower of size* 2 cm *on its axis of symmetry at a distance* 3 cm *from the flat surface. What is the size of the flower as it appears to an observer when he looks at it along the axis of symmetry* (a) *from top,* (b) *from bottom* ? (*Radius of the hemisphere is* 10 cm. *Index of refraction of glass* = 1·5.)

Sol. *Co-ordinate convention* : (a) Applying refraction formula to the spherical surface (–ve downward and +ve upward),

$$\frac{1}{v}-\frac{1\cdot5}{-7}=\frac{1-1\cdot5}{-10} \quad \text{or} \quad v=-\frac{140}{23}\text{ cm.}$$

$$\text{Magnification}=\frac{v}{u}\cdot\frac{\mu_1}{\mu_2}=\frac{\text{size of image}}{\text{size of object}}$$

$$\therefore \quad \frac{\frac{140}{23}}{7}\cdot\frac{1\cdot5}{1}=\frac{I}{2}$$

or $I=2\cdot61$ cm. **Ans.**

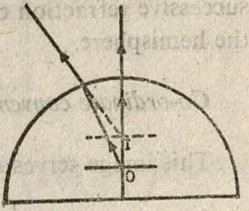

Fig. 1-8

(b) When viewed from bottom refraction takes place at the plane surface ($r=\infty$) with +ve downward and –ve upward.

$$\frac{1}{v}-\frac{1\cdot5}{-3}=\frac{1-1\cdot5}{\infty} \quad \text{or} \quad v=2\text{ cm.}$$

Now $\frac{2}{3} \cdot \frac{1{\cdot}5}{1} = \frac{I}{2}$ or $I = 2$ cm. **Ans.**

New convention : (a) Applying refraction formula to the spherical surface, $\frac{1}{v} + \frac{1{\cdot}5}{7} = \frac{1-1{\cdot}5}{-10}$ or $v = -\frac{140}{23}$.

Magnification $= \frac{v}{u} \cdot \frac{\mu_1}{\mu_2} = \frac{\text{size of image}}{\text{size of object}}$.

$$\therefore \quad \frac{\frac{140}{23}}{7} \cdot \frac{1{\cdot}5}{1} = \frac{I}{2} \quad \text{or} \quad I = 2{\cdot}61 \text{ cm.}$$ **Ans.**

(b) When viewed from bottom refraction takes place at the plane surface ($r = \infty$).

$$\frac{1}{v} + \frac{1{\cdot}5}{3} = \frac{1-1{\cdot}5}{\infty} \quad \text{or} \quad v = -2 \text{ cm.}$$

Now, $\frac{2}{3} \cdot \frac{1{\cdot}5}{1} = \frac{I}{2}$ or $I = 2$ cm. **Ans.**

Ex. 8. *A rod of length* 2·5 cm *is held longitudinally along the axis of symmetry of a glass hemisphere of radius* 10 cm *with its nearer end at a distance of* 5 cm *from the curved surface. Find the length of image of the scale.* (μ *of glass* = 1·5)

Sol. We will find the distances of the images of far and near end of the rod. The difference will give the length of the image. Consider successive refraction of rays from the nearer end at the surfaces of the hemisphere.

Co-ordinate convention : $\frac{1{\cdot}5}{v'} - \frac{1}{-5} = \frac{1{\cdot}5-1}{+10}$ or $v' = -10$ cm.

This image serves as object for the second flat surface.

$$\therefore \quad \frac{1}{v} - \frac{1{\cdot}5}{-20} \; \frac{1-1{\cdot}5}{\infty} \quad \text{or} \quad v = -13{\cdot}3 \text{ cm.}$$

For the far end, $\frac{1{\cdot}5}{v'} - \frac{1}{-7{\cdot}5} = \frac{1{\cdot}5-1}{+10}$ or $v' = -18$ cm

and $\frac{1}{v} - \frac{1{\cdot}5}{-28} = \frac{1-1{\cdot}5}{\infty}$ or $v = -18{\cdot}7$ cm.

$\therefore$ Length of the image $= 18{\cdot}7 - 13{\cdot}3 = 5{\cdot}4$ cm. **Ans.**

New convention : $\frac{1{\cdot}5}{v'}+\frac{1}{5}=\frac{1{\cdot}5-1}{+10}$ or $v'=-10$ cm.

This image serves as object for the flat surface.

$\therefore\ \frac{1}{v}+\frac{1{\cdot}5}{20}=\frac{1-1{\cdot}5}{\infty}$ or $v=-13{\cdot}3$ cm.

For the far end, $\frac{1{\cdot}5}{v'}+\frac{1}{7{\cdot}5}=\frac{1{\cdot}5-1}{+10}$ or $v'=-18$ cm

and $\frac{1}{v}+\frac{1{\cdot}5}{28}=\frac{1-1{\cdot}5}{\infty}$ or $v=-18{\cdot}7$ cm.

$\therefore$ Length of the image $=18{\cdot}7-13{\cdot}3=5{\cdot}4$ cm. **Ans.**

Ex. 9. *A glass slab of thickness* 10 cm *is placed at a distance* 5 cm *in front of a convex lens of focal length* 10 cm. *An object is placed at a distance* 5 cm *from the lens on the other side (opposite to the slab) of it. Find the position of the final image.* (μ *of glass* $=1{\cdot}5$)

Sol. *Co-ordinate convention* : $\frac{1}{v'}-\frac{1}{-5}=\frac{1}{+10}$ or $v'=-10$ cm.

Thus had there been no slab, the image would have been formed at a distance 10 cm on the same side of the lens as the object. The slab shifts the image by $\left(10-\frac{10}{1{\cdot}5}\right)$ towards the lens (Fig. 1-9).

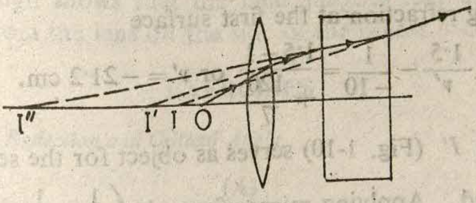

Fig. 1-9

$\therefore$ The final image is at a distance $=10-\left(10-\frac{10}{1{\cdot}5}\right)=6{\cdot}67$ cm.

Ans.

New convention : $\frac{1}{v'}+\frac{1}{5}=\frac{1}{+10}$ or $v'=-10$ cm. Thus had there been no slab, a virtual image would have been formed at a distance 10 cm on the side of the lens as the object. The slab shifts the image by $\left(10-\frac{10}{1{\cdot}5}\right)$ towards the lens (Fig. 1.9.)

$\therefore$ The final image is at a distance $= 10 - \left(10 - \dfrac{10}{1{\cdot}5}\right) = 6{\cdot}67$ cm.

Ans.

Ex. 10. *A pin is placed* 10 cm *in front of a convex lens of focal length* 20 cm *made of glass of index of refraction* 1·5. *The surface of the lens further away from the pin is silvered and has a radius of curvature* 24 cm. *Determine the position of the final image. Is the image real ar virtual* ?

Sol. *Co-ordinate convention* : We have, $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$.

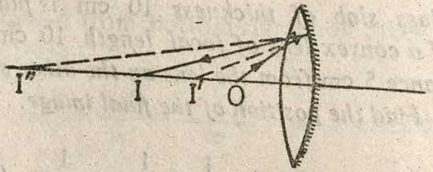

Fig. 1-10

$$\frac{1}{+20} = (1{\cdot}5 - 1)\left(\frac{1}{+r} - \frac{1}{-24}\right)$$

or $$r = \frac{120}{7} \text{ cm.}$$

Considering refraction at the first surface

$$\frac{1{\cdot}5}{v'} - \frac{1}{-10} = \frac{1{\cdot}5 - 1}{+\dfrac{120}{7}} \text{ or } v' = -21{\cdot}2 \text{ cm.}$$

The image I' (Fig. 1-10) serves as object for the second surface which is silvered. Applying mirror formula $\left(\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}\right)$ we have

$$\frac{1}{v''} + \frac{1}{-21{\cdot}2} = \frac{1}{-12} \text{ or } v'' = -27{\cdot}7 \text{ cm.}$$

Before rays can converge to I'' they are intercepted by the first surface again. So I'' serves as *virtual object* with reversal of direction of rays and so reversal of frame of reference as well (–ve rightward and +ve leftward).

$$\frac{1}{v} - \frac{1{\cdot}5}{+27{\cdot}7} = \frac{1 - 1{\cdot}5}{-\dfrac{120}{7}} \text{ or } v = +12 \text{ cm.}$$

The plus sign shows that the final image is real and is at a distance 12 cm from the lens on the side of the object. **Ans.**

New convention : We have $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$;

$$\therefore \quad \frac{1}{20}=(1{\cdot}5-1)\left(\frac{1}{r}-\frac{1}{-24}\right) \text{ or } r=\frac{120}{7} \text{ cm.}$$

Considering refraction at the first surface,

$$\frac{1{\cdot}5}{v'}+\frac{1}{10}=\frac{1{\cdot}5-1}{+\frac{120}{7}} \text{ or } v'=-21{\cdot}2 \text{ cm.}$$

The image I' (virtual) serves as object for the second surface which is silvered. Applying mirror formula $\left(\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\right)$ we have

$$\frac{1}{v''}+\frac{1}{21{\cdot}2}=\frac{1}{12} \text{ or } v''=+27{\cdot}7 \text{ cm.}$$

Before the rays can converge to I'' they are intercepted by the first surface again. So I'' serves as '*virtual object*' with reversal of direction of rays.

$$\frac{1}{v}+\frac{1{\cdot}5}{-27{\cdot}7}=\frac{1-1{\cdot}5}{-\frac{120}{7}} \text{ or } v=+12 \text{ cm.}$$

The plus sign shows that the final image is real and is at a distance 12 cm from the lens on the side of the object. **Ans.**

EXERCISES

Total Internal Reflection and Critical Angle

(A)

1. The index of refraction of water and castor oil are 1·3 and 1·5 respectively. Find the critical angle if a ray of light passes from oil to water. **(Ans. 60°)**

2. A ray of light falls at angle of 57° on the surface of water. The reflected and refracted ray are found to be at right angles to each other. What is the index of refraction of water ? **(Ans. $\mu=1{\cdot}54$)**

3. A glass cube has a spot at its centre. What parts of the cube face must be covered to make it invisible, no matter what the direction of viewing ? The edge of the cube is 10 cm and index of refraction of glass is 1·5.

(Ans. cover the centre of each face with a circle of radius ·0447 cm)

4. A man's eye is at a depth h below the calm surface of water. Prove that the surface of water appears to the man like a plane mirror with a circular hole

in it through which objects situated outside can be seen. Prove also that the radius of the hole$=\dfrac{h}{\sqrt{\mu^2-1}}$, where μ=index of refraction of water.

5. A beam of light falls upon a parallel sided slab of thickness 6 cm. The angle of incidence is 60°. Find the lateral displacement of the beam which passes through this slab. $\mu=1{\cdot}5$.

(Ans. 3·1 cm)

(B)

6. Show that, for small angle of incidence, the lateral displacement of a ray through a glass plate of thickness t is $x=\left(\dfrac{\mu-1}{\mu}\right)t\theta$, where μ is the index of refraction of glass and θ is angle of incidence in radian.

7. A point source of light is placed at a depth h below the surface of a large deep lake. Show that the fraction of the light energy that escapes directly from the water surface is independent of h and is given by

$$f=\left(\frac{\mu-\sqrt{\mu^2-1}}{2\mu}\right).$$

[*Hint*: Light escapes through the cone of semi-vertical angle equal to critical angle. Solid angle of a cone of semi-vertical angle $\theta=2\pi(1-\cos\theta)$.

8. A liquid drop sticking to one of the vertical faces of a glass cube of index of refraction μ become visible from the adjacent vertical face when the inclination of the line of sight is greater than φ from that surface. Calculate the index of refraction of the liquid in terms of μ and φ. (Ans. $\sqrt{\mu^2-\cos^2\phi}$)

9. A man standing on the edge of a swimming pool looks at a stone lying on the bottom. The depth of the swimming pool is equal to h. At what distance from the surface of water is the image of the stone formed if the line of vision makes an angle θ with the normal to the surface ?

$\left(\text{Ans. } \dfrac{\mu^2 h\cos^3\theta}{(\mu^2-\sin^2\theta)^{3/2}}\right)$

10. Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 ($n_1<n_2$) at an angle of incidence θ as shown in the figure. The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab $DEFG$ of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back into medium II. Consider separately the cases : (i) $n_3<n_1$ and (ii) $n_3>n_1$.

(I.I.T. 1986)

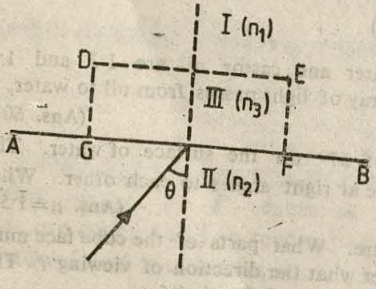

Fig. 1-11

Refraction at spherical surfaces

(A)

1. A glass sphere of radius 10 cm has a small particle enclosed in it at a distance of 5 cm from its centre. Where will it be if it is viewed along the diameter through the particle from the side opposite to it? ($\mu=1{\cdot}5$)

(Ans. 4 cm when viewed from the nearer surface and 20 cm when viewed from the distant surface.)

2. The two surfaces of a double convex lens are of radii 10 cm and 30 cm. Find the focal length of the lens in water, μ of water$=\frac{4}{3}$ and μ of material of lens$=3/2$. [Ans. $f={\cdot}6$ m (convergent)]

3. When viewed normally through the flat surface the apparent thickness of a plano-convex lens appears to be 2 cm. If the radius of curvature of the spherical surface is 10 cm and the actual thickness of the lens is 3 cm, what will be the apparent thickness of the lens when viewed from the curved surface?

(Ans. 2·22 cm)

4. A thin plano-convex lens acts like a concave mirror of radius of curvature 20 cm when its plane surface is silvered. Find the radius of curvature of the curved surface if index of refraction of its material is 1·5. (Ans. 10×10^{-2} m)

5. A mark on the surface of a glass sphere is viewed from a position diametrically opposite to it. If the diameter of the sphere is 10 cm and μ of glass is 1·5, find the position of the image.

(Ans. Virtual, 20 cm from the refracting surface)

6. A pin placed in front of a horizontal concave mirror, coincides with its own image when its distance is 30 cm from it. It is then filled with a liquid and it is found that the pin has now to be shifted through 10 cm towards the mirror to make it coincide with its image again. Calculate index of refraction of liquid.

(Ans. $\mu=1{\cdot}5$)

7. A small artificial flower is embedded at the centre of a glass sphere of radius 5 cm. If μ for glass is 1·5, find the apparent position and relative size of the image.

$\left[\textit{Hint}: m=\frac{\mu_1}{\mu_2}\cdot\frac{v}{u}\right]$ (Ans. 5 cm, 1·5 times enlarged)

8. A vessel of depth $2d$ is half filled with a liquid of index of refraction μ_1 and the other half by liquid of index of refraction μ_2. Find the apparent depth of the vessel when viewed normally $\left[\text{Ans. }\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)d\right]$

9. A hemisphere of glass ($\mu=1{\cdot}5$) of 7·5 cm radius serves as a lens. Find the position of the image when an object is placed 10 cm in front of the (a) plane surface, (b) curved surface.

[*Hint*: Do not neglect thickness of lens.]

[Ans. (a) final image is at infinity, (b) 35 cm from the plane surface]

10. A glass hemisphere of 10 cm radius is silvered over its curved surface. There is an air bubble in the glass 3 cm from the plane surface and along the axis. Find the positions of the images of this bubble seen by an observer looking along the axis into the flat surface of the hemisphere. ($\mu=1\cdot5$)

(**Ans.** one image is at a distance 2 cm from the flat surface and the other at a distance 5 cm from the same surface)

11. A small object is placed at a distance 60 cm from the surface of a sphere of radius 10 cm ($\mu=1\cdot5$) along a diameter. Where will the final image be formed by refraction in both surfaces?

(**Ans.** 9·09 cm beyond the far side of the sphere)

12. If the index of refraction from air to glass is 3/2 and that from air to water is 4/3, find the ratio of the focal lengths of a glass lens in water and in air.

(**Ans.** 4 : 1)

13. A rod of length 10 cm is held in air along the axis of a spherical surface separating air from water, the convex of the surface being towards air with the nearer end at a distance 5 cm from the surface. Calculate the length of the image formed. (μ of water$=4/3$ and radius of curvature of the surface$=10$ cm)

(**Ans.** image will be 32 cm long)

14. An object is placed at a distance 5 cm from the curved surface of a thin plano-convex lens, the plane surface of which is silvered. Find the nature and final position of the image formed. (Radius of curvature of curved surface$=10$ cm and $\mu=1\cdot5$.)

(**Ans.** image is virtual; 10 cm on the other side)

15. There is a large concave mirror immersed in a large tank of water at a depth 20 cm. It is held horizontally and its reflecting surface is turned upwards. Where is the image of the sun when it is just overhead? Radius of curvature of the mirror$=60$ cm and index of refraction of water of tank$=1\cdot54$. Where is the image if the tank is filled with water up to 40 cm?

(**Ans.** 6·5 cm above the surface of water; 6·5 cm inside the water)

16. A glass slab of thickness 5 cm is placed in front of a convex lens of focal length 15 cm at a distance 10 cm from the lens. An object is placed at a distance 5 cm from the near surface of the slab on the other side. Calculate the distance of final image from the lens. ($\mu=1\cdot5$)

(**Ans.** 82·5 cm on the other side of the lens)

17. Rays converging to a point *A* are intercepted by a parallel sided slab of thickness 10 cm before they converge to *A* from a distance 5 cm from *A*. If $\mu=1\cdot5$, then where will the rays be converged now?

(**Ans.** 8·33 cm away from the slab)

18. Two watch glasses of radii of curvature 10 cm and 30 cm are cemented at the edges to form an air-convex lens. What is its focal length in air and water? Is it convergent or divergent in water? μ of water$=4/3$.

[**Ans.** Infinity; 30 cm (divergent)]

19. Find the radii of curvature of a convexo-concave convergent lens made of glass with a refraction index of $\mu=1\cdot5$ having a focal length of $f=24$ cm. One of the radii of curvature is double the other.

[**Ans.** 6 cm; 12 cm)

(B)

20. A thick convex lens is bounded by surfaces of radii 5 cm and 8 cm and has thickness 1·5 cm at its centre. An object is placed at a distance 5 cm from it. Find image distance from the first surface. (μ of glass$=1\cdot5$)
[*Hint*: See Ex. 4)

(**Ans.** 35·2 cm on the same side, image is virtual)

21. A plano-convex lens of focal length 15 cm is placed with its curved surface on a plane mirror. The space between mirror and lens is filled with certain liquid. If a pin held in front of the lens coincides with its image when it is at a distance 30 cm from the lens, calculate index of refraction of liquid assuming index of refraction of material of lens = 1·5.

[*Hint*: Note there is a combination of two lenses : (i) glass lens, (ii) a liquid lens. So use $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$ and $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$.] (Ans. 1·25)

22. A cross '+' 2 cm by 2 cm is placed at the centre of a glass sphere of radius 10 cm. How long will the two appear when viewed along the prolongation of one of the arms of the cross ? (μ of glass = 1·5)

(Ans. transverse arm 3 cm, longitudinal arm 3·01 cm)

23. Calculate the focal length of an equi-convex lens bounded by surfaces of radius of curvature 20 cm when it is surrounded by air on one side and by a liquid on the other. Will it be convergent or divergent if the liquid be of index of refraction (i) 1·3, (ii) 1·6 ? Index of refraction of material of lens = 1·5.

(Ans. 37·14 cm, convergent; 80 cm, convergent)

24. The convex surface of a thin concavo-convex lens of glass of refractive index 1·5 has a radius of curvature of 20 cm. The concave surface has a radius of 60 cm. The convex side is silvered and placed on a horizontal surface.

(i) Where should a pin be placed on the optic axis such that its image is formed at the same place ?

(ii) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin. (I. I. T. 1981)

[*Hint* : A ray returns along its own path when it falls normally on the reflecting surface.] (Ans. 15 cm; 1·15 cm towards the lens)

25. A concave spherical mirror with a radius of curvature of 20 cm is filled with water $\left(\mu=\frac{4}{3}\right)$. What is the power of this system ?

[*Hint* : P_1 (power of the mirror) $=\frac{2}{R}$; P_2 (power of the plano-convex lens containing water) $=\frac{\mu-1}{R}$. P (power of the system) $=P_1+2P_2$, since the light passes through the water twice.] (Ans. 13·3 D)

26. A large bubble of radius 10 cm is formed in water. Where is the image of a distant object formed inside the water by the paraxial rays ? Will it act as a divergent lens or a convergent one ? μ of water $=\frac{4}{3}$.

(Ans. 15 cm from the centre on the side of the object; divergent)

27. A beam of light is incident on a spherical drop of water at angle 60°. Find the total deviation of the ray after a single reflection from the internal surface. μ of water$=\frac{4}{3}$. (Ans. 138°)

[*Hint* : $D=180°+2i-4r$]

28. The concave surface of a thin concavo-convex lens of glass of refractive index 1·5 has a radius of curvature of 50 cm. The convex surface has a radius of curvature of 10 cm. The concave side is silvered and placed on horizontal surface. Where should a pin be placed on the optic axis such that its image is formed at the same place ? If the object is placed at a distance of 60 cm, where is the final image formed ? (Ans. 50 cm, 23 cm)

□□□

CHAPTER 2

LENSES AND PRISMS

1. Lenses : The object distance u, image distance v and focal length f of a mirror are, in general, related by

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\text{ (\textit{new convention} and \textit{co-ordinate convention}).}$$

The object distance u, image distance v and focal length of a lens are, in general, related by

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\text{ (\textit{co-ordinate convention})}$$

and $$-\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\text{ (\textit{in new convention}).}$$

$$\text{Lateral Magnification}=\frac{\text{size of the image (lateral)}}{\text{size of the object (lateral)}}=\frac{v}{u}.$$

$$\text{Longitudinal magnification}=\frac{\text{size of the image along the axis}}{\text{size of the object along the axis}}.$$

Note : The focal length of a concave mirror *is positive in new convention* and *negative in co-ordinate convention* but in case of convex mirror it is just opposite. The fecal length of a convex lens is *positive in the co-ordinate and new convention* but in case of concave lens it is just opposite.

2. Refraction through a prism : When a ray of light passes through a prism, it is invariably bent towards the base of the prism.

In the figure PQ is a ray incident at an angle i and is refracted at an angle r such that $\dfrac{\sin i}{\sin r}=\mu$. It is incident on the adjacent face at an angle r' and finally emerges at an angle i' along RS such that $\mu=\dfrac{\sin i'}{\sin r'}$. The angle by which the ray is turned towards the base by the prism is called angle of deviation. In the figure this angle is $\angle EOR$. We denote deviation by D. Form the simple geometry of the figure.

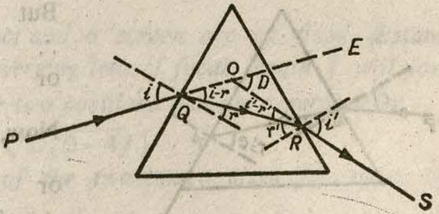

Fig. 2.1.

$$\pi-(r+r')+A=\pi$$

or $$r+r'=A \qquad \therefore \quad (1)$$

and $$D=(i-r)+(i'-r')$$

or $$D=(i+i')-(r+r')$$

or $$D=i+i'-A$$

or $$i+i'=A+D \qquad .. \quad (2).$$

Equations (1) and (2) govern the refraction of a ray through a prism.

3. Minimum deviation (D_m) : The angle of deviation depends on angle of incidence in a peculiar way. When angle of incidence is small, deviation is large. As i increases, D decreases, attains a minimum value and then increases with increase of i. The minimum value of D so attained is called minimum deviation.

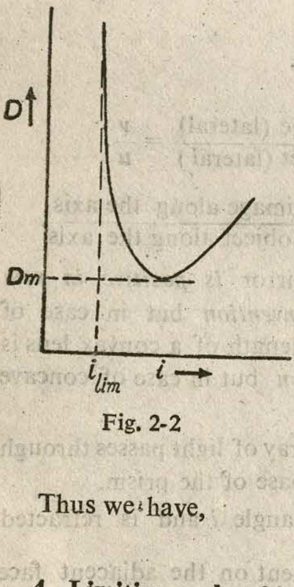

Fig. 2-2

Minimum diviation occurs when $i=i'$ and $r=r'$.

$$\therefore \quad r=\frac{A}{2} \quad \text{and} \quad i=\frac{A+D_m}{2}.$$

Thus we have, $$\mu=\frac{\sin\frac{A+D_m}{2}}{\sin\frac{A}{2}} \qquad .. \quad (3).$$

4. Limiting angle of incidence and Maximum deviation (D_{max}) :

There will be no emergent ray if r' is greater or equal to C, the critical angle of the material of the prism, that is, $r' \geqslant C$.

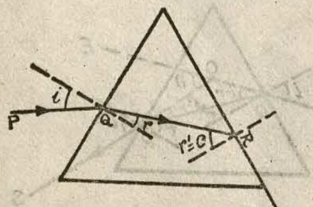

Fig. 2-3

But $r+r'=A$. (always)

$\therefore$ $r \leqslant A-C$

or $\sin r \leqslant \sin (A-C)$.

Now, $\mu=\frac{\sin i}{\sin r}$; (always)

or $\sin i=\mu \sin r$.

$\therefore$ The condition for no emergent ray is

$$\sin i \leqslant \mu \sin (A-C)$$

or $\quad \sin i \leqslant \mu(\sin A.\ \cos C - \cos A.\ \sin C)$.

We know, $\quad \mu = \dfrac{i}{\sin C}$.

$$\therefore \quad \sin i \leqslant \mu\left(\sin A \frac{\sqrt{\mu^2-1}}{\mu} - \cos A \frac{1}{\mu}\right)$$

or $\quad i \leqslant \sin^{-1} (\sqrt{{}^2\mu-1}\ \sin A - \cos A)$.

$\therefore \quad i_{lim} = \sin^{-1} (\sqrt{\mu^2-1}\ \sin A - \cos A) \quad .. \ (4).$

This gives the limiting value of the angle of incidence. If angle of incidence is less than this limiting value there will be no emergent ray.

When the emergent ray grazes along the surface, the deviation is maximum. This occurs when $i = i_{lim}$. In the Fig. 2-4, $\angle EOR = D_{max}$. In the triangle AQO, $\angle AQO = 90° - i_{lim}$.

$\therefore \quad 90 - i_{lim} + A + D_{max} = 180°$

or $\quad D_{max} = 90° + i_{lim} - A \quad .. \ (5)$.

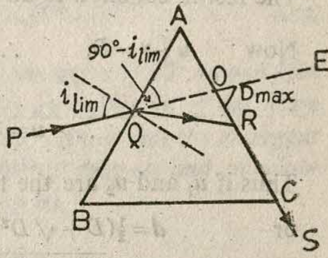

Fig. 2-4

5. Refraction through thin prism : When prism is thin and ray is almost normally incident i, i', r, r' and A are all small.

$$\therefore \quad \mu = \frac{\sin i}{\sin r} = \frac{i}{r}, \text{ or } i = \mu r, \quad \text{and } \mu = \frac{\sin i'}{\sin r'} = \frac{i'}{r'}, \text{ or } i' = \mu r'.$$

$$\therefore \quad i + i' = (\mu r + r').$$

But $i + i' = A + D$ is always true whether prism is thick or thin.

So also $\quad r + r' = A.$

$\therefore \quad A + D = \mu A$

or $\quad D = (\mu - 1)\ A \quad .. \ (6).$

EXAMPLES

Ex. 1. *A luminous object and a screen are at fixed distance D apart.* (*a*) *Show that a converging lens of focal length f will form a real image on the screen for two positions that are separated by*

$$d = \sqrt{D(D-4f)}.$$

(*b*) *Show that the ratio of the two image sizes for these two positions is*

$$\left(\frac{D-d}{D+d}\right)^2.$$

Sol. *Old convention* : For a converging lens forming real image we have $\frac{1}{-v}-\frac{1}{u}=\frac{1}{-f}$ or $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

Co-ordinate convention : For a converging lens forming real image,

$$\frac{1}{v}-\frac{1}{-u}=\frac{1}{f} \text{ or } \frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

New convention : For a converging lens forming real image,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

The rest is common to all the methods.

Now $u+v=D$. $\therefore \frac{1}{D-u}+\frac{1}{u}=\frac{1}{f}$ or $u^2-Du+Df=0$.

$$\therefore \quad u=\frac{D\pm\sqrt{D^2-4Df}}{2}.$$

Thus if u_1 and u_2 are the two values of u, then $d=u_1-u_2$

or $\quad d=\frac{1}{2}(D+\sqrt{D^2-4Df})-\frac{1}{2}(D-\sqrt{D^2-4Df})$
$\quad =\sqrt{D^2-4Df}$

or $\quad d=\sqrt{D(D-4f)}$. Proved.

(b) Eliminating u from (1) we obtain a similar quadratic in v, that is, $v^2-Dv+Df=0$.

$$\therefore \quad v_2, v_1=\frac{D\pm\sqrt{D^2-4Df}}{2}.$$

Since $u_1+v_1=D=u_2+v_2$, the value of v_1 corresponding to $u_1=\frac{1}{2}(D+\sqrt{D^2-4Df})$ must be $\frac{1}{2}(D-\sqrt{D^2-4Df})$ and that corresponding to $u_2=\frac{1}{2}(D-\sqrt{D^2-4Df})$ is $\frac{1}{2}(D+\sqrt{D^2-4Df})$.

Now, $\frac{\text{size of image in the first position}}{\text{size of image in the second position}}=\frac{m_1}{m_2}=\frac{v_1}{u_1}\times\frac{u_2}{v_1}$.

$$\therefore \; R \text{ (the required ratio)}=\frac{\frac{1}{2}(D-\sqrt{D^2-4Df})\times\frac{1}{2}(D\sqrt{(D^2-4Df)}}{\frac{1}{2}(D+\sqrt{D^2-4Df})\times\frac{1}{2}(D+\sqrt{D^2-4Df})}$$

$$=\frac{(D-\sqrt{D^2-4Df})^2}{(D+\sqrt{D^2-4Df})^2}=\left(\frac{D-d}{D+d}\right)^2. \qquad \textbf{Proved}$$

Ex. 2. *The refracting angle of a prism is* 60° *and the index of refraction is* $\sqrt{\frac{7}{3}}$. *What is the limiting angle of incidence of a ray that will be transmitted through the prism and what is the maximum deviation produced by it ?*

Sol. We have, $\sin i_{lim} = \sqrt{\mu^2 - 1}\,(\sin A - \cos A)$.

Here $A = 60^\circ$, $\mu = \sqrt{\frac{7}{3}}$.

$$\therefore \quad \sin i_{lim} = \left(\sqrt{\frac{7}{3} - 1}\ \sin 60^\circ - \cos 60^\circ\right)$$

$$= \left(\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2}\right) = \frac{1}{2}$$

$$= \sin 30^\circ.$$

$$\therefore \quad i_{lim} = 30^\circ. \qquad \textbf{Ans.}$$

Further, we have $\quad D_{max} = 90^\circ + i_{lim} - A;$

$$\therefore \quad D_{max} = 90^\circ + 30^\circ - 60^\circ = 60^\circ. \qquad \textbf{Ans.}$$

Ex. 3. *A ray of light is incident at an angle of* 60° *on one face of a prism which has angle of* 30°. *The ray emerging out of the prism makes an angle of* 30° *with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism.*

Sol. In prism refraction we have always

$$r + r' = A,$$

$$i + i' = A + D.$$

Here $i = 60^\circ$, $A = 30^\circ$ and $D = 30^\circ$.

$\therefore \quad 60^\circ + i' = 30^\circ + 30^\circ \quad$ or $\quad i' = 0.$

Hence emergent ray is perpendicular to the face through which it emerges.

$$\because \quad i' - 0,\ r' = 0 \qquad \left(\because \quad \mu = \frac{\sin i'}{\sin r'}\right)$$

$$\therefore \quad r + 0 = 30^\circ.$$

Now, $$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 30^\circ} = 1{\cdot}732. \qquad \textbf{Ans.}$$

Ex. 4. *A glass prism of angle* 60° *and index of refraction* 1·5 *is immersed in a liquid of refractive index* 1·33. *What is the angle of minimum deviation produced by the prism inside the liquid ?*

Sol. We have, $$\mu = \frac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}}.$$

Since the prism is immersed in a liquid we must use index of refraction of glass relative to the liquid in place of μ.

$$\therefore \quad \text{We have} \quad {}_l\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}.$$

By law of intermediate media,

${}_l\mu_g = {}_l\mu_a \times {}_a\mu_g$ [air (a) being the intermediate medium]

$$= \frac{1}{{}_a\mu_l} \times {}_a\mu_g = \frac{1\cdot5}{1\cdot33}.$$

$$\therefore \quad \frac{1\cdot5}{1\cdot33} = \frac{\sin\frac{60^\circ+\delta_m}{2}}{\sin\frac{60^\circ}{2}}; \text{ or } \sin\frac{60^\circ+\delta_m}{2} = \frac{1\cdot5}{1\cdot33} \times \frac{1}{2} = \frac{1\cdot5}{2\cdot66}$$

$$\text{or} \quad \sin\frac{60^\circ+\delta_m}{2} = \cdot5639 = \sin\ 34^\circ20'; \ \therefore \ \frac{60^\circ+\delta_m}{2} = 34^\circ20'$$

or $60^\circ+\delta_m = 68^\circ40'$, or $\delta_m = 8^\circ40'$. **Ans.**

Ex. 5. *A prism of refracting angle* 30° *is made of material of refraction* 1·5. *Find the emergent angle when a ray* (i) *falls normally on one face,* (ii) *almost grazes on that surface.*

Sol. Critical angle of the material of the prism is given by

$$\sin C = \frac{1}{1\cdot5}, \text{ or } C = \sin^{-1}(\cdot6667) = 41^\circ48'.$$

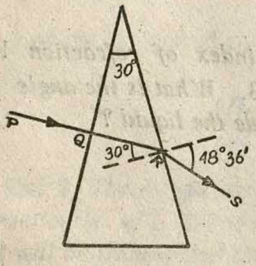

Fig. 2-4

(i) When $i=0$, $r=0$.

The ray will, therefore, fall on the adjacent face at a glancing angle 60° or at an angle of incidence 30°. Thus $r'=30^\circ$.

$$\mu = \frac{\sin i'}{\sin r'}, \text{ or } 1\cdot5 = \frac{\sin i'}{\sin 30^\circ}$$

or $\sin i' = \cdot75$, or $i' = 48^\circ36'$.

Thus ray will emerge at 48°36′ towards the base. **Ans.**

(ii) When $i=90°$ (i.e. ray is almost grazing), $r=C=41°48'$.

Complementary to $r=90°-41°48'$
$=48°12'$.

$\therefore$ Glancing angle of an adjacent face
$=48°12'+30°=78°12'$.

$\therefore$ Angle of incidence on adjacent face $=90°-78°12'=11°48'$.

Thus $r'=11°48'$.

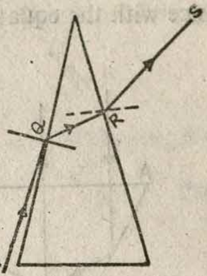

Fig. 2-5

Now, $$\mu=\frac{\sin i'}{\sin r'}$$

or $\sin i'=1{\cdot}5.\ \sin 11°48'=1{\cdot}5\times{\cdot}2045$

or $\sin i'={\cdot}3068$, or $i'=17°52'$.

Thus ray will emerge at 17°52′ away from the base. **Ans.**

Ex. 6. *A thin prism of angle* 2° 30′ *and of index of refraction* 1·58 *for sodium light is held at a distance* ·2 m *from a slit illuminated by sodium light. What is the distance between the slit and its image formed by the prism ?*

Sol. We have in a thin prism,

$D=(\mu-1)A$;

$\therefore\ D=(1{\cdot}58-1)\times 2°30'$

$$=1{\cdot}45°=1{\cdot}45\times\frac{\pi}{180}\text{ radian.}$$

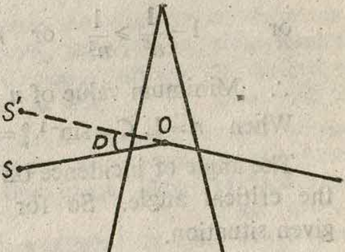

Fig. 2-6

Clearly from figure,

$SS'=SO\times D$.

('SO' is the distance of prism from S because its thickness is negligible.)

$$\therefore\ SS'={\cdot}20\times 1{\cdot}45\times\frac{\pi}{180}={\cdot}005\text{ m or }{\cdot}5\text{ cm.}$$ **Ans.**

Ex. 7. *The index of refraction of an equilateral prism is* $\sqrt{2}$. *If the angle of incidence of a ray of light on one of the faces of prism is* 45°, *calculate the angle of emergence and the deviation of the ray.*

Sol. We have $\sqrt{2}=\dfrac{\sin 45°}{\sin r}$ or $\sin r=\frac{1}{2}$, or $r=30°$.

We know that refraction through a prism takes place in accordance with the equations $r+r'=A$ and $i+i'=A+D$.

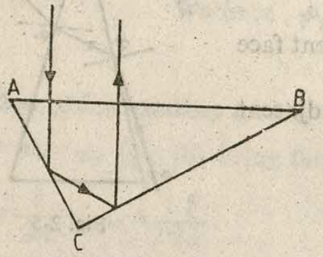

Fig. 2-7

Ex. 8. *A right prism is to be made by selecting a proper material and angle A and B ($B \leqslant A$), as shown in the figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two reflections.* (i) *What should be the minimum refractive index n for this to be possible ?* (ii) *For $n=\frac{5}{3}$ is it possible to achieve this with the angle B equal to* 30° *?*

(I.I.T. 1987)

Sol. When $B=C$, critical angle, it is possible to achieve the above situation. This means $\sin B=\sin C=1/n$.

Since $A+B=90°$, $\sin A=\sin(90°-B)=\cos B=\sqrt{1-\frac{1}{n^2}}$.

Since $A \geqslant B$, $\sin A \geqslant \sin B$ or $\sin^2 A \geqslant \sin^2 B$

or $1-\frac{1}{n^2} \geqslant \frac{1}{n^2}$ or $n \geqslant \sqrt{2}$.

$\therefore$ Minimum value of $n=\sqrt{2}=1{\cdot}414$. **Ans.**

When $n=\frac{5}{3}$, $C=\sin^{-1}\frac{3}{5}=37°$.

The angle of incidence on the face CB is 30° which is less than the critical angle. So for $n=\frac{5}{3}$ it is not possible to achieve the given situation.

$\therefore$ Here, $r'=60°-30°=30°$.

Again, $\mu=\frac{\sin i'}{\sin r'}$; $\therefore$ $\sqrt{2}=\frac{\sin i'}{\sin 30°}$, or $\sin i'=\frac{1}{\sqrt{2}}$

or $i'=45°$.

Now, from the formula $i+i'=A+D$, we have

$$45°+45°=60°+D$$

or

$$D=30°.$$ **Ans.**

Ex. 9. *Show that if the angle of a prism is twice the critical angle of glass of which it is made there will be no emergent ray.*

Sol. See the article and deduce the formula,

$$\sin i=\mu \sin (A-C).$$

If A is fixed, that fixes up the limiting value of i. Conversely if we put maximum possible value of i we will get limiting value of A. The maximum value of i is 90°.

∴ For limiting value of A, we have

$$\sin 90° = \mu \sin (A-C), \text{ or } \frac{1}{\mu} = \sin (A-C), \text{ or } \sin C = \sin (A-C)$$

or $$C = A - C, \text{ or } A \quad 2C.$$ **Proved.**

EXERCISES

Lenses :

1. Show that when the distance between object and screen is greater than four times the focal length of a convex lens, there are two positions of the lens where it forms the real image of the object on the screen. Prove the following :

(i) $f = \dfrac{D^2 - x^2}{4D}$; (ii) $f = \dfrac{x}{m_1 - m_2}$;

(iii) $S = \sqrt{S_1 S_2}$ where D=distance between object and screen, x=displacement of lens; m_1, m_2 are the magnifications in the two positions and S_1 and S_2 are the sizes of the images in the two positions.

2. Light is converging at a point A and a convex lens of focal length 20 cm is placed at P in the path of the beam where AP=30 cm. The beam now converges to Q. Calculate the distance AQ. (**Ans.** 12 cm)

3. A convex lens on the optical bench forms the image of an object on a screen, and the magnification is 2. Keeping the object and screen fixed, the image is again formed on the screen when the lens is shifted by 25 cm and the magnification is ·5. Find the focal length of the lens. (**Ans.** 16·67 cm)

4. By adjusting the distance of a convex lens the image of a lamp is obtained on a wall. By moving the lens 25 cm nearer the wall, the image is again formed on the wall which is this time $\frac{1}{4}$ of the previous image. Find the focal length of the lens. (**Ans.** 16·67 cm)

5. The distance between two point sources of light is 24 cm. Where should a convergent lens with a focal length 9 cm be placed between them of obtain the images of both sources at the same point ?

(**Ans.** 6 cm from one source and 18 cm from the other)

6. The height of a candle flame is 5 cm. A lens produces an image of this flame 15 cm high on a screen. The candle is then moved over a distance 1·5 cm away from the lens, and a sharp image of the flame 10 cm high is obtained after shifting the screen. Determine the focal length of the lens. (**Ans.** 9 cm)

Prisms :

(A)

1. The refracting angle of a prism is 60° and the refractive index for sodium light is known to be 1·5. What would be the angle of minimum deviation for the prism for sodium light ? (**Ans.** 37°10′)

2. A ray of light traverses a prism of refractive index 1·6 and just undergoes total reflection at the second face. If the refracting angle of the prism is 60°, what is the angle of incidence at the first face ?
(Ans. 35°35′)

3. A ray enters perpendicularly one face of an equilateral prism of index of refraction 1·5. From which face does it emerge ? Find the angle of deviation of the rays.
(Ans. from the third face; deviation=60°)

4. A glass prism of angle 72° and index of refraction 1·66 is immersed in a liquid of refractive index 1·33. What is the angle of minimum deviation for a parallel beam of light passing through the prism ?
(Ans. 22°24′)

[*Hint* : ${}_l\mu_g={}_l\mu_a \times {}_a\mu_g$.]

5. Find the minimum and maximum angles of deviation for a light ray passing through a glass prism ($\mu=1·5$) with refracting angle of 60°.

(Ans. $D_m=37°$; $D_{max}=58°$)

6. The refractive index of the material of a prism of refracting angle 45° is 1·6. What should be the minimum angle of incidence of a ray on this prism so that no total internal reflection takes as the ray comes out of the prism ?

(Ans. 10°)

7. A ray of light enters into a prism at angle of 45° with the normal to one face. Calculate the angle at which it emerges from the other face. Angle of the prism=60° and index of refraction of material of prism=1·55. (Ans. 57°12′)

8. Find the least value of the prism made of glass of index of refraction 1·58 so that no rays incident on one of the faces can emerge from the other.

(Ans. 78°32′)

9. Find the limiting angle of the prism in which there will be emergent ray corresponding to all rays irrespective of their angle of incidence. Index of refraction of material of prism=1·53.
(Ans. 40°49′)

10. A prism of refracting angle 30° is made of material of index of refraction 1·62 for sodium light. Find the range of the angle of emergence at which rays entering at one of the faces containing this angle will emerge from the other.

[*Hint* : See Ex. 5.) (Ans. from 0 to 54°6′ towards the base and from zero to 13°13′ away from the base)

11. A biprism (two thin prisms joined base to base) of acute angle 2°A is placed at a distance 30 cm from a source. What is the distance between the images formed by the two-halves of the biprism ? (μ of material of biprism =1·635)

(Ans. 1·33 cm)

12. Find the minimum value of the refracting angle of a prism made of glass of refractive index $\frac{7}{4}$ so that no rays incident on one of the faces containing this angle can emerge from the other.
(Ans. 69°42′)

13. Show that if the refracting angle of a prism is less than critical angle, rays incident from the base side will bend towards the base when angle of incidence lies between zero and certain maximum. If the angle of the prism be 30° and the index of refraction of the material of the prism be 1·62, what is this limiting angle of incidence ?
(Ans. 54°)

(B)

14. A prism of refracting angle 60° produces the minimum deviation of 37° in air. Find the minimum deviation produced by that prism in water ($\mu=\frac{4}{3}$). **(Ans. 8°20′)**

15. A ray of light is passing through a prism of refractive index $\sqrt{3}$, in such a way that the angle of incidence is twice the angle of emergence and the latter is equal to the angle of the prism. Prove that the angle of the prism is 60°.

16. A given monochromatic light ray, initially in air strikes a 90° prism at an angle θ_1 and is refracted to such an extent that it just grazes the right-hand prism surface after it emerges into air. Calculate the index of refraction, relative to air, of the prism for this wavelength in terms of the angle of incidence θ_1 which gives rise to this situation. Also calculate the upper bound for the index of refraction of the prism. **(Ans. $\sqrt{1+\sin^2\theta_1}$, $\sqrt{2}$)**

17. An equilateral prism ($\mu=1{\cdot}5$) is immersed in a liquid. What must be the maximum refractive index of the liquid so that a ray entering normally through one of the faces may be totally internally reflected ?

[*Hint* : There will be no emergent ray if $60 \geqslant C$ or $\sin 60 \geqslant \sin C$.
Now ${}_a\mu_g \sin C = {}_a\mu_l \sin 90°$) **(Ans. 1·299)**

18. A ray of light enters into an equilateral glass prism in such a way that it falls on the adjacent face at critical angle and finally emerges from the third face. The total deviation of the ray is 108°. Calculate the refractive index of the material of the prism. **(Ans. 1·45)**

19. A parallel beam of light falls normally upon the first face of a prism of small angle. The portion of the beam which is refracted at second face is deviated through an angle 1°35′ and the portion which is reflected at the second surface and emerges again at the first surface makes angle of 8°9′ with the incident beam. Calculate the angle of the prism and index of refraction of glass. **(Ans. 2°29′ ; 1·64)**

20. Light from a source of monochromatic light is allowed to fall on the mat surface of a prism. On looking at either of the remaining faces through a telescope set for parallel rays it is found that there is a distinct direction beyond which there is comparatively less illumination. Explain this. If this direction makes an angle α with the normal to the face then show that

$$\mu=\sqrt{1+\left(\frac{\sin\alpha+\cos A}{\sin A}\right)^2}.$$

21. Three right-angled prisms of refractive indices μ_1, μ_2 and μ_3 are placed in contact with their bases opposite. If a ray passes through the block of prisms so formed without being deviated, show that $\mu_1^2+\mu_3^2-\mu_2^2=1$.

[*Hint* : Consider successive deviations at the four surfaces making use of the formula $r+r'=A$.

The condition for no deviation, $i+i'=90°$.]

□□□

CHAPTER 3

COMBINATION OF THIN LENSES AND MIRRORS

1. Combination of lenses : (a) *In contact* :

When two thin lenses of focal lengths f_1 and f_2 are placed in contact, the focal length of the combination is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_1}. \text{ (in all conventions)}$$

(b) *Separated by a distance* :

When two thin lenses of focal lengths f_1 and f_2 are placed co-axially x distance apart, the focal length of combination is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}, \text{ or } F=\frac{f_1 f_2}{\triangle} \text{ where } \triangle=f_1+f_1-x.$$

(*in co-ordinate convention and new convention*)

Note : Remember that focal lengths of convergent lenses are +ve and that of divergent lenses −ve in co-ordinate convention and new convention.

2. Power of a lens : Power of a lens is the reciprocal of its focal length in metre.

P (power of a lens) $=\frac{1}{f}$ dioptre (D). (*co-ordinate convention and new convention*)

3. Principal planes :

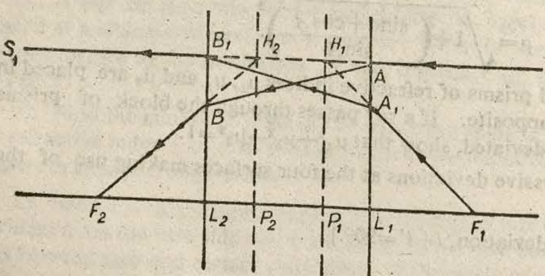

Fig. 3-1

Let a ray SA parallel to the principal axis of a system of lenses (here only two) emerge along BF_2 after refraction through two lens system. Let the incident ray SA and emergent ray

BF_2 meet at H_2. A plane H_2P_2 perpendicular to the principal axis is called second principal plane, P_2 second principal point and F_2 second principal focus. The second principal plane is in front of (front means on the side of object) of second lens by a distance β given by

$$\beta = \frac{x}{\triangle} f_2.$$

Suppose that in the object space F_1 is such a point of the principal axis that the rays diverging from it emerge parallel to the principal axis after refraction through the lens system. Such a point is called first principal focus. Let the incident ray F_1A_1 meet the emergent ray B_1S_1 at H_1. Imagine a plane H_1P_1 through H_1 perpendicular to the principal axis. This pane is called the first principal plane and P_1 is called the first principal point. The first principal plane is behind the first lens at a distance α given by

$$\alpha = \frac{x}{\triangle} f_1.$$

Use of principal planes : If the object distance is considered from the first principal plane and image distance from second principal plane, then we can use thin lens formula namely $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for a lens system as well. If U = object distance from first principal plane, V = image distance from second principal plane, then $\frac{1}{V} - \frac{1}{U} = \frac{1}{F}$ holds good for a lens system.

EXAMPLES

Ex. 1. *Two thin convex lenses of focal lengths* 20×10^{-2} m *and* 5×10^{-2} m *are placed coaxially at* 10×10^{-2} m *apart. Calculate the power of such a combination.*

Sol. *Coordinate convention* : We have for two lenses separated by a distance

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{x}{f_1 f_2}.$$

$$\therefore \quad \frac{1}{F} = \frac{1}{(+\cdot 2)} + \frac{1}{(+\cdot 05)} - \frac{\cdot 1}{(+\cdot 2)(+\cdot 05)} = +15 \text{ m}^{-1}.$$

$$\therefore \; P \text{ (power)} = \frac{1}{F} = +15 \text{ D}.$$

Ans.

New convention : We have for two lenses separated by a distance

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}.$$

$$\therefore \quad \frac{1}{F}=\frac{1}{+\cdot 2}+\frac{1}{+\cdot 05}-\frac{\cdot 1}{(+\cdot 2)(\cdot 05)}=+15 \text{ m}^{-1}.$$

$$\therefore \quad P \text{ (power)}=\frac{1}{F}=15 \text{ D}.$$

Ans.

Ex. 2. *A convex lens and a concave lens each of focal length* ·1 m *each are held co-axially* ·03 m *apart. Find the position of the image if an object is placed at a distance* ·15 m *in front of convex lens.*

Sol. *Co-ordinate convention* : Applying lens formula to the first lens,

$$\frac{1}{v'}-\frac{1}{-\cdot 15}=\frac{1}{+\cdot 1} \text{ or } v'=+\cdot 30 \text{ m}.$$

The plus sign shows that the image is formed on the other side of the lens (Fig. 3-2). But before the rays can converge to I', they are intercepted by the concave lens. So I' serves as the '*virtual object*' for the concave lens. Distance of $I'=\cdot 30-\cdot 03=\cdot 27$ m. The object distance for concave lens is +ve because I' lies on the +ve side.

$$\therefore \quad \frac{1}{v}-\frac{1}{+\cdot 27}=\frac{1}{-\cdot 1}, \text{ or } v=-\cdot 16 \text{ m}.$$

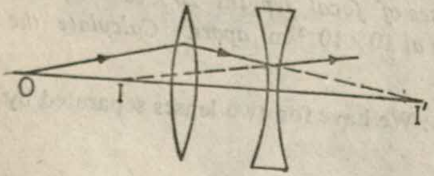

Fig. 3-2

The negative sign shows that the final image is formed at a distance 16 cm from the concave lens on the side of the object. **Ans.**

New convention : Applying lens formula to the first lens,

$$\frac{1}{v'}+\frac{1}{\cdot 15}=\frac{1}{\cdot 1}$$

or $v'=+\cdot 30$ m.

The plus sign shows that the image is real and is on the *R*-side of the lens. But before the rays can converge to I', they are intercepted by the concave lens. So I' serves as '*virtual object*' for the concave lens. Distance of $I' = {\cdot}30 - {\cdot}03 = {\cdot}27$ m.

$$\frac{1}{v} + \frac{1}{-{\cdot}27} = \frac{1}{-{\cdot}1} \quad \text{or} \quad v = -{\cdot}16 \text{ m.}$$

The negative sign shows that the image is virtual and is at a distance 16 cm from the concave lens on the side of the object. **Ans.**

Ex. 3. *Two plano concave lenses of glass of refractive index* 1·5 *have radii of curvature of* 20 cm *and* 30 cm *respectively. They are placed in contact with curved faces towards each other and the space between them is filled with a liquid of refractive index* 1·33. *What is the focal length of the combination ?*

Sol. We may think as if there are three lenses of focal lengths f_1 (of glass), f_2 (of liquid) and f_3 (of glass) in contact.

Coordinate convention :

$$\frac{1}{f_1} = (1{\cdot}5 - 1)\left(\frac{1}{\infty} - \frac{1}{+20}\right) \quad \text{or} \quad f_1 = -40 \text{ cm.}$$

$$\frac{1}{f_2} = (1{\cdot}33 - 1)\left(\frac{1}{+20} - \frac{1}{-30}\right) \quad \text{or} \quad f_2 = +36{\cdot}36 \text{ cm.}$$

$$\frac{1}{f_3} = (1{\cdot}5 - 1)\left(\frac{1}{-30} - \frac{1}{\infty}\right) \quad \text{or} \quad f_3 = -60 \text{ cm.}$$

From $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$ we have

$$\frac{1}{F} = \frac{1}{-40} + \frac{1}{36{\cdot}36} + \frac{1}{-60} \quad \text{or} \quad F = -70{\cdot}6 \text{ cm.}$$ **Ans.**

New convention :

$$\frac{1}{f_1} = (1{\cdot}5 - 1)\left(\frac{1}{\infty} + \frac{1}{-20}\right) \quad \text{or} \quad f_1 = -40 \text{ cm.}$$

$$\frac{1}{f_2} = (1{\cdot}33 - 1)\left(\frac{1}{+20} + \frac{1}{+30}\right) \quad \text{or} \quad f_2 = 36{\cdot}36 \text{ cm.}$$

$$\frac{1}{f_3} = (1{\cdot}5 - 1)\left(\frac{1}{-30} + \frac{1}{\infty}\right) \quad \text{or} \quad f_3 = -60 \text{ cm.}$$

From $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$ we have

$$\frac{1}{F} = \frac{1}{-40} + \frac{1}{36{\cdot}36} + \frac{1}{-60} \quad \text{or} \quad F = -70{\cdot}6 \text{ cm.}$$ **Ans.**

Ex. 4. *Two convex lenses each of focal length* 20 cm *are placed* 15 cm *apart. An image of a flagstaff* 20 m *distant and* 1 m *high is formed by the combination. Find the position and size of the image.*

Sol. *Coordinate convention* : $\frac{1}{v'}-\frac{1}{-20}=\frac{1}{+\cdot 2}$ or $v'=+\cdot 202$ m.

The plus sign shows that the image P' is formed on the other side of the lens. But before the rays can converge to P', they are intercepted by the second lens. So P' serves as the *virtual object* for the second lens. The object distance is +ve because P' lies on the positive side of the second lens.

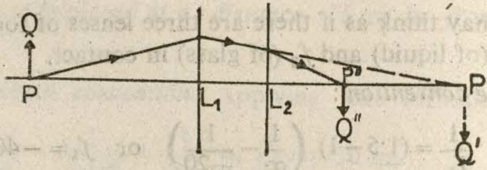

Fig 3-3

$$\therefore \qquad \frac{1}{v}-\frac{1}{+\cdot 052}=\frac{1}{+\cdot 2} \text{ or } v=+\cdot 0413 \text{ m.}$$

m_1 (magnification produced by the first lens) $=\frac{P'Q'}{PQ}-\frac{\cdot 202}{-20}$;

m_2 (,, ,, second ,,) $=\frac{P''Q''}{PQ}=\frac{\cdot 0413}{\cdot 052}$;

m (total magnification) $=\frac{P''Q''}{PQ}=\frac{P''Q''}{P'Q'}\times\frac{P'Q'}{PQ}$

$$=\frac{\cdot 0413}{\cdot 052}\times\frac{\cdot 022}{-20}=-\cdot 008$$

$$\therefore \qquad P''Q''=-\cdot 008\times 1 \text{ m}=-\cdot 008 \text{ m or } -\cdot 8 \text{ cm.}$$

Thus the image is at a distance ·0413 m on the other side of the second lens and it is ·8 cm high. **Ans.**

New convention : $\frac{1}{v'}+\frac{1}{20}=\frac{1}{+\cdot 2}$ or $v'=+\cdot 202$ m.

The plus sign shows that the image is real and it is on the other side (i.e. R-side) of the lens. But before the rays can converge to P', they are intercepted by the second lens. So P' serves as the *virtual object* for the second lens.

$$\therefore \qquad \frac{1}{v}+\frac{1}{-\cdot 052}=\frac{1}{+\cdot 2} \text{ or } v=+\cdot 0413.$$

$$m_1 \text{ (magnification produced by the first lens)}=\frac{P'Q'}{PQ}=\frac{\cdot 202}{20}$$

$$m_2 \text{ (,, ,, second lens)}=\frac{P''Q''}{P'Q'},=\frac{0413}{-\cdot 052};$$

$$m \text{ (total magnification produced)}=\frac{P''Q''}{PQ}=\frac{P''Q''}{P'Q'}\times\frac{P'Q'}{PQ}$$

$$=\frac{\cdot 0413}{-\cdot 052}\times\frac{\cdot 202}{20}=-\cdot 008.$$

$$\therefore \qquad P''Q''=-\cdot 008\times 1 \text{ m}=-\cdot 008 \text{ m or } -\cdot 8 \text{ cm}.$$

Thus the final image is real and it is on the other side (i.e. R-side) of the second lens at a distance ·0413 m and it is ·8 cm high. **Ans.**

Ex. 5. *A concave lens is placed at a distance of* 25 cm *in front of a concave mirror of focal length* 20 cm *and it is found that a pin placed* 70 cm *in front of the lens coincides with its own image. Find the focal length of the lens.*

Sol. Obviously there will be bending of rays three times before final image is formed. Here in this special case when final image and object coincide, second bending is zero. The ray AB must be normal on the mirror i.e. AB being produced must pass through centre of curvature C of the mirror. Clearly from the figure P and C are respectively object and image for lens.

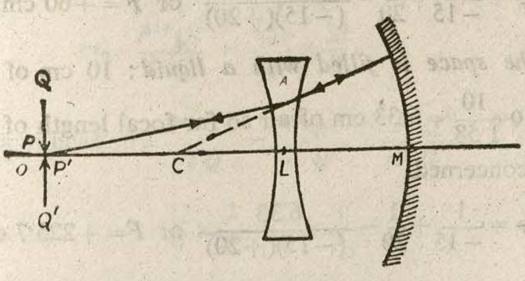

Fig. 3-4

$$LC=MC-ML=40-25=15 \text{ cm}; \; PL=70 \text{ cm}.$$

Coordinate convention : $\frac{1}{-15}-\frac{1}{-70}=\frac{1}{f}$, or $f=-19{\cdot}1$ cm. **Ans.**

New convention : $\frac{1}{-15}+\frac{1}{70}=\frac{1}{f}$, or $f=-19{\cdot}1$ cm. **Ans.**

Ex. 6. *A thin concave lens of focal length* 15 cm *and a thin convex lens of focal length* 20 cm *are placed co-axially at* 10 cm *apart. Calculate the focal length of the combination. If the space between them is filled with a liquid of index of refraction* 1·58, *calculate the new focal length of the combination.*

Sol. *Coordinate convention* : We have, in general,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}.$$

$$\therefore \quad \frac{1}{F}=\frac{1}{-15}+\frac{1}{20}-\frac{10}{(-15)(+20)} \text{ or } F=+60 \text{ cm.} \quad \textbf{Ans.}$$

When the space is filled with a liquid: 10 cm of the liquid is equivalent to $\frac{10}{1{\cdot}58}=6{\cdot}33$ cm of air so far focal length of the equivalent lens is concerned.

$$\therefore \quad \frac{1}{F}=\frac{1}{-15}+\frac{1}{20}-\frac{6\;33}{(-15)(+20)} \text{ or } F=+225{\cdot}7 \text{ cm.} \quad \textbf{Ans.}$$

New convention : We have, in general, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$.

$$\therefore \quad \frac{1}{F}=\frac{1}{-15}+\frac{1}{20}-\frac{10}{(-15)(+20)} \text{ or } F=+60 \text{ cm.} \quad \textbf{Ans.}$$

When the space is filled with a liquid: 10 cm of the liquid is equivalent to $\frac{10}{1{\cdot}58}=6{\cdot}33$ cm of air so far focal length of the equivalent lens is concerned.

$$\therefore \quad \frac{1}{F}=\frac{1}{-15}+\frac{1}{20}-\frac{6\;33}{(-15)(+20)} \text{ or } F=+225{\cdot}7 \text{ cm.} \quad \textbf{Ans.}$$

Ex. 7. *A convex lens of focal length* 20 cm *is placed in front of a convex mirror of focal length* 15 cm *at a distance* 5 cm. *An object of length* 2 cm *is placed perpendicular to the common axis at a distance* 10 cm *from the lens. Find the final position and size of the image.*

Sol. *Coordinate convention* : Applying lens formula to the lens,

$$\frac{1}{v}-\frac{1}{-10}=\frac{1}{+20} \text{ or } v'=-20 \text{ cm.}$$

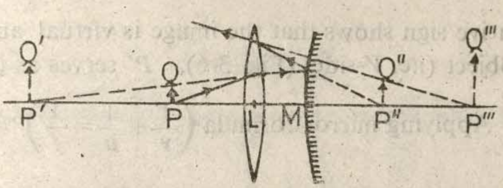

Fig. 3·6

The negative sign shows that the image P' is on the side of the object. P' serves as the object for the mirror. Applying mirror formula $\left(\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\right)$, $\frac{1}{v''}+\frac{1}{-25}=\frac{1}{+15}$ or $v''=+\frac{75}{8}$ cm.

The plus sign shows that the image P'' is on the other side of the mirror. It serves as the object for the lens (second time) with *reversal of the direction of rays and so with reversal of the frame of reference as well* (*−ve rightward* and *+ve leftward*).

$$LP''=\frac{75}{8}+5=\frac{115}{8},$$

$$\frac{1}{v'''}-\frac{1}{-\frac{115}{8}}=\frac{1}{+20} \text{ or } v'''=-\frac{460}{9}=-51{\cdot}1 \text{ cm.}$$

Magnification : $m_1=\frac{P'Q'}{PQ}=\frac{20}{10}=2;\ m_2=\frac{P''Q''}{P'Q'}=\frac{\frac{75}{8}}{25}=\frac{3}{8};$

$$m_3=\frac{P'''Q'''}{P''Q''}=\frac{\frac{460}{9}}{\frac{115}{8}}=\frac{32}{9}.$$

$$\text{Total magnification}=\frac{P'''Q'''}{PQ}=\frac{P'''Q'''}{P''Q''}\times\frac{P''Q''}{P'Q'}\times\frac{P'Q'}{PQ}$$

$$=\frac{32}{9}\times\frac{3}{8}\times\frac{2}{1}=\frac{8}{3}.$$

$$\therefore \quad \text{Size of image}=\frac{8}{3}\times 2 \text{ cm}=5{\cdot}33 \text{ cm.}$$

Thus the image is at a distance 51·1 cm from the lens on the other side and it is 5·33 cm high. **Ans.**

New convention : Applying lens formula to the lens.

$$\frac{1}{v'}+\frac{1}{10}=\frac{1}{+20} \text{ or } v'=-20 \text{ cm.}$$

The negative sign shows that the image is virtual and is on the side of the object (i.e. V-side) (Fig. 3-6). P' serves as the object for the mirror. Applying mirror formula $\left(\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\right)$,

$$\frac{1}{v''}+\frac{1}{25}\quad\frac{1}{-15} \text{ or } v''=-\frac{75}{8} \text{ cm.}$$

The negative sign shows that the image P'' is virtual and is on the other side (V-side) of the mirror. P'' serves as the object for the lens (second time) with reversal of the direction of rays. Now right hand side is the R-side and left hand side of the lens is V-side.

$$LP''=\frac{75}{8}+5=\frac{115}{8} \text{ cm.}$$

$$\frac{1}{v'''}+\frac{1}{\frac{115}{8}}=\frac{1}{20} \text{ or } v'''=-\frac{460}{9}=-51{\cdot}1 \text{ cm.}$$

Magnification : $m_1=\frac{P'Q'}{PQ}=\frac{20}{10}=2;\quad m_2=\frac{P''Q''}{P'Q'}=\frac{\frac{75}{8}}{25}=\frac{3}{8};$

$$m_3=\frac{P'''Q'''}{P''Q''}=\frac{\frac{460}{9}}{\frac{115}{8}}=\frac{32}{9}.$$

$$\text{Total magnification}=\frac{P'''Q'''}{PQ}=\frac{P'''Q'''}{P''Q''}\times\frac{P''Q''}{P'Q'}\times\frac{P'Q'}{PQ}$$

$$=\frac{32}{9}\times\frac{3}{8}\times\frac{2}{1}=\frac{8}{3}.$$

Size of the image $=\frac{8}{3}\times 2 \text{ cm}=5{\cdot}33$ cm.

Thus the image is at a distance 51·1 cm from the lens on the other side and it is 5·33 cm high. **Ans.**

Ex. 8. *A convex lens of focal length* 20 cm *is placed in front of convex mirror of focal length* 15 cm *at a distance* 5 cm. *An object of length* 2 cm *is placed perpendicular to the common axis at a distance* 25 cm *from the lens. Find the position of the image and its size.*

Sol. *Coordinate convention* : Applying lens formula to the lens,

$$\frac{1}{v'}-\frac{1}{-25}=\frac{1}{+20} \text{ or } v'=+100 \text{ cm.}$$

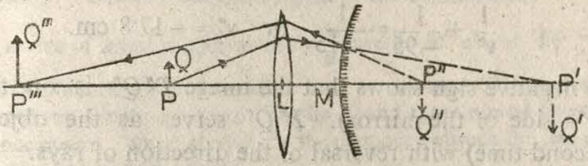

Fig. 3-7

The plus sign shows that the image P' is on the other side of the lens. But before the rays can converge to P' (Fig. 3-7), they are intercepted by the mirror. So P' serves as the *virtual object* for the mirror. $MP'=100-5=95$ cm.

Applying mirror formula $\left(\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\right)$,

$$\frac{1}{v''}+\frac{1}{+95}=\frac{1}{+15} \quad \text{or } v''=+17\cdot8 \text{ cm.}$$

The plus sign shows that the image $P''Q''$ is on the side $P'Q'$. $P''Q''$ serves as the object for the lens (second time) ***with reversal of the direction of the rays and so with reversal of the frame of reference as well (−ve rightward*** and ***+ve leftward).***

$$LP''=17\cdot8+5=22\cdot8 \text{ cm.}$$

$$\frac{1}{v'''}-\frac{1}{-22\cdot8}=\frac{1}{+20} \quad \text{or} \quad v'''=+162\cdot9 \text{ cm.}$$

Magnification

$$=\frac{P'''Q'''}{PQ}=\frac{P'''Q'''}{P''Q''}\times\frac{P''Q''}{P'Q'}\times\frac{P'Q'}{PQ}=\frac{162\cdot9}{22\cdot8}\times\frac{17\cdot8}{95}\times\frac{100}{25}=5\cdot35.$$

Size of the image $=5\cdot35\times2$ cm $=10\cdot7$ cm.

Thus the image is real and at a distance 162·9 cm from the lens in front of it and it is 10·7 cm high. **Ans.**

New convention : Applying lens formula to the lens,

$$\frac{1}{v'}+\frac{1}{25}=\frac{1}{20} \quad \text{or} \quad v'=+100 \text{ cm.}$$

The plus sign shows that the image is real and it is on the other side (R-side) of the lens. But before the rays can converge to P'

(Fig. 3.7), they are intercepted by the mirror. So P' serves as *the virtual object* for the mirror. $MP' = 100 - 5 = 95$ cm.

Applying mirror formula $\left(\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\right)$,

$$\frac{1}{v''}+\frac{1}{-95}=\frac{1}{-15} \quad \text{or} \quad v''=-17{\cdot}8 \text{ cm.}$$

The negative sign shows that the image $P''Q''$ is on the side of $P'Q'$ (V – side of the mirror). $P''Q''$ serves as the object for the lens (second time) with reversal of the direction of rays.

$$LP''=17{\cdot}8+5=22{\cdot}8 \text{ cm.}$$

$$\frac{1}{v'''}+\frac{1}{22{\cdot}8}=\frac{1}{+20} \quad \text{or} \quad v'''=+162{\cdot}9 \text{ cm.}$$

$$\text{Magnification}=\frac{P'''Q'''}{PQ}=\frac{P'''Q'''}{P''Q''}\times\frac{P''Q''}{P'Q'}\times\frac{P'Q'}{PQ}$$

$$=\frac{162{\cdot}9}{22{\cdot}8}\times\frac{17{\cdot}8}{95}\times\frac{100}{25}=5{\cdot}35.$$

Size of the image $=5{\cdot}35\times2$ cm $=10{\cdot}7$ cm.

Thus the image is real and at a distance 162·9 cm from the lens in front of it and it is 10·7 cm high. **Ans.**

EXERCISES

(A)

1. Two lenses are placed in contact and the combination is found to have +2D. The power of one of the lenses is +3D. What is the nature and power of the second lens ?

(**Ans.** power = –1D; the minus sign shows that it is concave)

2. A convex lens of focal length 30 cm is placed 20 cm away from a concave lens of focal length 10 cm. An object is placed 60 cm distant from the convex lens (which is nearer to it) and on the common axis of the system. Determine the position, magnification and size of the image.

(**Ans.** image is virtual at a distance of 13·3 cm from the concave lens; magnification $=\frac{1}{3}$)

3. Two convex lenses each of ·2 m focal length are placed ·05 m apart. An image of a flagstaff 200 m distant and ·10 m high is formed by the combination. Find the position and size of the image.

(**Ans.** ·0858 m from the 2nd lens; the image is real, inverted and $\cdot57\times10^{-2}$ m high)

4. A telephoto lens consists of a convergent lens of focal length ·12 m facing the object and a divergent lens of focal length ·05 m placed ·08 m behind the former. Find the position where the plate to be placed to photograph a distant object. (**Ans.** ·2 m to the right of the 2nd lens)

5. Two equiconvex thin lenses, having radii ·04 m and refractive index 1·5 are placed in air ·02 m apart and the space between them is filled with water of refractive index $\frac{4}{3}$. Calculate the focal length of the combination.

[*Hint* : x metre of water is equivalent to $\frac{x}{\mu}$ metre of air so far the focal length of the combination is concerned.) (**Ans.** ·0246 m, convergent)

6. Two convex lenses of focal lengths f_1 and f_2 are placed co-axially at a certain distance apart. What should be the distance between the lenses so that the combination may be telescopic ?

[*Hint* : Focal length of the combination is to be infinite.] (**Ans.** separation$=f_1+f_2$)

7. A convex lens of focal length 10 cm is placed in front of a plane mirror at a distance 5 cm. An object is placed 5 cm in front of the convex lens. Find the position and nature of the image. Draw path of rays showing formation of intermediate images. (**Ans.** image is real, 20 cm in front of the convex lens)

8. A convex lens of focal length 10 cm is placed in front of a concave mirror of focal length 10 cm at a distance 5 cm. A pin is placed 15 cm in front of the convex lens. Find the position and nature of the image formed. (**Ans.** 1·763 cm in front of the lens; real)

9. A convex lens of focal length 20 cm is placed 12 cm in front of a convex mirror. It is found that when a pin is placed at a distance 30 cm in front of the lens it coincides with its own inverted image formed by the lens and mirror; find the focal length of the mirror.

[*Hint* : Ex. 5] (**Ans.** 24 cm)

10. A thin convergent lens of focal length $f=25$ cm projects the image of an object on a screen at a distance $D=5$ m from the lens. Then the screen was drawn closer by distance $D=18$ cm. By what distance should the object be shifted for its image to become sharp again ? (**Ans.** 0·5 mm)

11. A convex lens of focal length 25 cm is placed in front of a convex mirror at a distance of 15 cm from it. It is found that when a pin is placed 45 cm in front of the lens, it coincides with its own inverted image formed by the lens and the mirror. Find the focal length of the mirror. (**Ans.** 20·63 cm)

12. A convex lens of focal length 10 cm is placed in front of a convex mirror at a distance 5 cm from it. An object is placed at a distance 15 cm in front of the convex lens. Find the position of the image formed by the convex lens and the mirror. Trace the path of rays. (radius of curvature of the mirror$=15$ cm.) (**Ans.** real; 27·5 cm in front of the convex lens)

13. A convex lens of focal length 15 cm is placed in front of a plane mirror at a distance 8 cm. An object is placed at a distance 8 cm in front of the lens. Calculate position of the image formed by the lens and mirror. Trace the path of rays. (**Ans.** real; 27·4 cm in front of the convex lens)

(B)

14. A concave mirror of focal length 30 cm is at a distance of 16 cm from a concave lens of focal length 10 cm. Find the position of the image of a distant object seen through the lens after reflection from the mirror.

[*Hint* : First consider reflection at the mirror and then refraction through lens.] (Ans. 35 cm from the lens, virtual)

15. Two convex lenses of focal lengths ·10 m and ·15 m are separated by a distance ·02 m. An object is placed at a distance ·12 m upon a common axis in front of the first lens. Calculate the position of the final image and of two principal planes. (Ans. ·119 m beyond ·15 m focal length lens; $\alpha = {\cdot}9 \times 10^{-2}$ m and $\beta = 1{\cdot}3 \times 10^{-2}$ m)

16. Two plano-convex lenses of focal lengths ·06 m each are fitted co-axially in a tube at a distance ·04 m from each other. Find the principal planes of the combination and find the position of the image of a distant object formed by the combination.

(Ans. first principal plane ·03 m behind first lens and second principal plane ·03 m in front of second lens, image of distant object is formed at a distance ·015 m behind second lens)

17. Rays of light from a luminous object are brought to a focus at a point A. A convex lens of focal length 12 cm is then placed 12 cm from A, so as to intercept the rays before they meet at A. If they now meet at B, find the distance AB. Where else could the lens be placed so that after passing through it the rays might appear to diverge from B ?

(Ans. $AB = 6$ cm; 6 cm from A on the other side)

18. A concave mirror and a convex mirror are placed face to face at a distance 20 cm apart. An object is placed between the mirrors at a distance 4 cm from the convex mirror. Where is the image formed after reflection of light first at the convex mirror and then at the concave mirror (focal length of either mirror $= 15$ cm). Trace the path of rays showing the formation of the final image. (Ans. 42·6 cm in front of the concave mirror)

19. A source of light is located at 60 cm from a convergent lens of focal length 30 cm. At what distance from the lens should a plane mirror be placed so that the rays reflected from the mirror are parallel after passing through the lens for the second time ? (Ans. 45 cm)

20. A parallel beam of rays is incident on a convergent lens with a focal length of 40 cm. Where should a divergent lens with focal length of 15 cm be placed for the beam of rays to remain parallel after passing through the two lenses ? (Ans. 25 cm from the convergent lens and the focii of both lenses must coincide.)

21. A concave mirror of diameter 6 cm and radius of curvature 5 cm is completely covered by black paper except a thin strip at the edge. At what distance from the pole of the mirror a parallel beam of rays is concentrated ?

(Ans. 1·86 cm)

22. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1·5 is placed at a distance 1 cm from the mirror. Find the final position of the image formed. (IIT 1980)

(Ans. $7\frac{2}{3}$ cm in front of the mirror)

□□□

CHAPTER 4

DISPERSION

1. Dispersion and Dispersive power : When a ray of composite light is passed through a prism, it is split into its component colours by the particular action of the prism that *it always turns a ray towards its base by an angle called angle of deviation* depending on the colour of light. This phenomenon of breaking up of composite light into its component colours is known as Dispersion. Dispersion between any two colours is measured by the separation between these two colours.

The angular separation between the two colours, say, red and violet is called angular dispersion and the linear separation between them is called linear dispersion. Ordinarily, by dispersion we will mean angular dispersion.

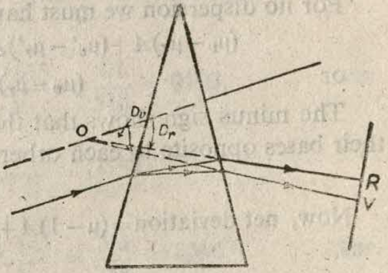

Fig. 4-1

$\therefore$ Dispersion between red and violet $= D_v - D_r$. It is $(D_v - D_r)$ because violet deviates more than red.

It is a fact that dispersion between any two colours is proportional to their mean deviation.

Therefore, $D_v - D_r \propto D$ where $D = \dfrac{D_v + D_r}{2}$

or $D_v - D_r = \omega D$ where ω is a constant of the material of the prism and is called its Dispersive power.

$$\therefore \quad \omega = \left(\frac{D_v - D_r}{D} \right) \qquad \cdots (1).$$

Definition : Put $D = 1$ then $\omega = D_v - D_r$. Thus dispersive power may be defined as the dispersion per unit mean deviation.

2. Dispersive Power in terms of index of refraction :

In a thin prism, $D = (\mu - 1)A$.

$$\therefore \qquad D_v = (\mu_v - 1)A, \quad D_r = (\mu_r - 1)A.$$

$$\therefore \qquad D_v - D_r = (\mu_v - \mu_r)A$$

and $$\omega=\frac{D_v-D_r}{D}=\frac{(\mu_v-\mu_r)A}{(\mu-1)A}=\left(\frac{\mu_v-\mu_r}{\mu-1}\right) \quad .. \quad (2).$$

3. Deviation without dispersion and Dispersion without deviation:

Two thin prisms of different materials may be combined to give deviation without dispersion or dispersion without deviation.

Suppose that two thin prisms of angles A and A' are placed in contact and they are of materials of dispersive powers ω and ω' respectively. Then net deviation produced by them

$$=D+D'=(\mu-1)A+(\mu'-1)A',$$

net dispersion produced by them $=(D_v-D_r)+(D_v'-D_r')$

$$=(\mu_v-\mu_r)A+(\mu_v'-\mu_r')A'.$$

For no dispersion we must have

$$(\mu_v-\mu_r)A+(\mu_v'-\mu_r')A'=0$$

or $$(\mu_v-\mu_r)A=-(\mu_v'-\mu_r')A'.$$

The minus sign shows that the two prisms must be placed with their bases opposite to each other.

Now, net deviation $=(\mu-1)A+(\mu'-1)\left(-\frac{\mu_v-\mu_r}{\mu_v'-\mu_r'}\right)A$

$$=(\mu-1)A\left\{1-\frac{\mu'-1}{\mu-1}\cdot\frac{\mu_v-\mu_r}{\mu_v'-\mu_r'}\right\}$$

$$=(\mu-1)A\left(1-\frac{\omega}{\omega'}\right).$$

Since materials are different $\omega\neq\omega'$ and, therefore, deviation is essentially there.

For no deviation we must have

$$(\mu-1)A+(\mu'-1)A'=0 \quad \text{or} \quad (\mu-1)=-(\mu-1)A'.$$

Again the minus sign shows that the prisms must be placed with their bases opposite to each other.

Now net dispersion $=(\mu_v-\mu_r)A+(\mu_v'-\mu_r')\left(-\frac{\mu-1}{\mu'-1}\right)A$

$$=(\mu-1)A\left(\frac{\mu_v-\mu_r}{\mu-1}-\frac{\mu_v'-\mu_r'}{\mu'-1}\right)$$

$$=(\mu-1)A(\omega-\omega').$$

Here also since $\omega\neq\omega'$, dispersion is essentially there.

EXAMPLES

Ex. 1. *The refractive indices of crown glass for blue and red light are 1·523 and 1·513 respectively and for dense flint glass they are 1·773 and 1·743 respectively. Calculate the dispersive power of the materials. What must be the angle of a flint glass prism which may be combined with a crown glass prism of angle 5° so that the combination produces dispersion without deviation ? What dispersion does the combination produce ?*

Sol. Dispersive Power $=\dfrac{\mu_v-\mu_r}{\mu-1}$.

For crown glass, $\mu=\dfrac{\mu_v+\mu_r}{2}=\dfrac{1{\cdot}523+1{\cdot}513}{2}=1{\cdot}518.$

$\therefore$ ω of crown glass $=\dfrac{1{\cdot}523-1{\cdot}513}{1{\cdot}518-1}=\dfrac{{\cdot}01}{{\cdot}518}={\cdot}0193.$ **Ans.**

For flint glass, $\mu=\dfrac{1{\cdot}773+1{\cdot}743}{2}=1{\cdot}758.$

ω of flint glass $=\dfrac{1{\cdot}773-1{\cdot}743}{1{\cdot}758-1}=\dfrac{{\cdot}03}{{\cdot}758}={\cdot}0396.$ **Ans.**

Condition for no deviation

$$(\mu-1)A=-(\mu'-1)A'$$

$\therefore$ $$(1{\cdot}518-1)5^\circ=-(1{\cdot}758-1)A'$$

or $$A'=\frac{{\cdot}518\times 5^\circ}{{\cdot}758}=-3^\circ 42'.$$ **Ans.**

The minus sign shows that A and A' should be opposite.

$\therefore$ $$A'=3^\circ 42'.$$

Net dispersion produced $=(\mu_v-\mu_r)A-(\mu_v'-\mu_r')A'$

$=(1{\cdot}523-1{\cdot}513)5-(1{\cdot}773-1{\cdot}743)3^\circ 42'$

$=-3'.$

Thus net dispersion is 3′ opposite to that produced by crown glass prism. **Ans.**

Ex. 2. *Calculate dispersion and deviation (mean) produced by a flint glass prism placed in the position of minimum deviation with respect to the mean colour. Angle of the prism = 60°. Also calculate dispersive power of the material of the prism.*

culate dispersion produced and angle of flint glass prism required for producing no deviation. (Ans. 12′ 26″; 5°16′)

6. A prism of 60° angle is made of glass of which the refractive index is 1·50 for red ray and 1·56 for violet ray. Find the angular separation of these rays when a narrow pencil is incident at minimum deviation.

$$\left[Hint:\ \mu = \frac{\sin(A+\delta_m)/2}{\sin A/2}. \right]$$

(Ans. 5°22′)

7. A direct vision spectroscope is made of two crown glass prisms each of angle 5° and two flint glass prisms. The dispersive powers of crown and flint glasses are ·03 and ·06 respectively and mean indices of refraction are 1·500 and 1·600 respectively. Calculate the dispersion produced by the compound prism. What is the advantage of using two of each ?

(Ans. 9′ towards the base of flint glass)

□□□

CHAPTER 5

CHROMATIC ABERRATION OF LENSES

1. Chromatic Aberration : A lens may be supposed to be made up of large number of prisms of gradually increasing angle from the axis to the periphery in case of convex lens but in case of concave lens bases of the prisms are turned away from the axis of the lens. So if a ray coming from a distant point object is passed through a lens it will be split up into its component colours by the prism action of the lens. This results in the formation of a large number of coloured images overlapping one another. Thus an ill-defined coloured image is formed. This colour defect in the image formation by a lens is called chromatic aberration and is measured by the separation of the images of extreme colours (red and violet).

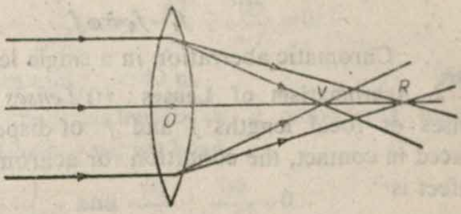

Fig. 5-1

$$\text{Chromatic aberration} = VR = OR - OV = f_r - f_v \quad \cdots \quad (1).$$

For a lens we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Hence,
$$\frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\therefore \quad \frac{1}{f_v} - \frac{1}{f_r} = (\mu_v - \mu_r)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

or
$$\frac{f_r - f_v}{f_r f_v} = (\mu_v - \mu_r) \cdot \frac{1}{f(\mu - 1)} \quad \left[\because \ \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{f(\mu - 1)}\right]$$

$$= \frac{\mu_v - \mu_r}{(\mu - 1) f}.$$

But $\frac{\mu_v-\mu_r}{\mu-1}=\omega$, dispersive power of the material of lens.

$$\therefore \qquad f_r-f_v=\frac{\omega f_r f_v}{f}.$$

$f_r f_v \cong f^2$ because f_r and f_v are very close to each other.

$$\therefore \qquad f_r-f_v=\omega f.$$

$\therefore$ Chromatic aberration in a single lens $=\omega f$ .. (2).

2. Achromatism of Lenses : (i) *Lenses in contact* : When two lenses of focal lengths f and f' of dispersive powers ω and ω' are placed in contact, the condition for achromatism, i.e., for no colour defect is

$$\frac{\omega}{f}+\frac{\omega'}{f'}=0 \qquad \text{.. (3).}$$

(ii) *Lenses separated by a distance* : When the same lenses are separated by a distance x, the condition in general is

$$\frac{\omega}{f}+\frac{\omega'}{f'}-\frac{x}{ff'}(\omega+\omega')=0. \text{ (\textit{in co-ordinate and new convention})}$$

When the lenses are of the same material, the condition is

$$f+f'-2x=0. \text{ (\textit{in co-ordinate and new convention})}$$

EXAMPLES

Ex. 1. *The focal lengths of a thin convex lens are* 1 m *and* ·968 m *for red and blue rays respectively. Calculate the chromatic aberration and dispersive power of the material of the lens.*

Sol. Chromatic aberration $=f_r-f_b=1\text{ m}-{\cdot}968\text{ m}={\cdot}032\text{ m}$. **Ans.**

We have, $\omega=\frac{f_r-f_b}{f}=\frac{f_r-f_b}{\sqrt{f_r f_b}}=\frac{{\cdot}032}{\sqrt{1\times{\cdot}968}}={\cdot}0325.$ **Ans.**

Ex. 2. *An achromatic converging combination of focal length* 60 m *is formed with convex lens of crown glass and a concave lens of flint glass placed in contact with each other. Calculate their focal lengths, if dispersive power of crown glass is* ·03 *and that of flint glass is* ·05.

Sol. We have for a combination of lenses in contact,

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f'},$$

and condition for achromatism,

$$\frac{\omega}{f}+\frac{\omega'}{f'}=0.$$

Coordinate convention : Since the focal length of a converging lens is +ve and that of a divergent lens −ve, we have

$$\frac{1}{+\cdot 60}=\frac{1}{f}+\frac{1}{-f'}\text{ and }\frac{\cdot 03}{f}+\frac{\cdot 05}{-f'}=0.$$

Solving, we get $f=\cdot 24$ m and $f'=\cdot 40$ m. **Ans.**

New convention : Since the focal length of a converging lens is +ve and that of divergent lens −ve, we have

$$\frac{1}{+\cdot 60}=\frac{1}{f}+\frac{1}{-f'}\text{ and }\frac{\cdot 03}{f}+\frac{\cdot 05}{-f'}=0.$$

Solving, we get $f=\cdot 24$ m and $f'=\cdot 40$ m. **Ans.**

Ex. 3. *It is required to form a converging achromatic system of focal length* 30 m *consisting of two lenses made of the same material. If one of the lenses be convex of focal length* ·20 m, *find* (*a*) *the focal length of the other lens, and also* (*b*) *the distance between the lenses.*

Sol. *Coordinate convention* : Since the focal length of a converging lens is +ve and that of diverging lens is − ve, we have

$$\frac{1}{\cdot 3}=\frac{1}{\cdot 2}+\frac{1}{f'}-\frac{x}{\cdot 2f'} \quad \ldots \text{(i)}$$

and $\cdot 2+f'-2x=0$ or $2x=\cdot 2+f'$ .. (ii).

Solving (i) and (ii) for f' and x we have, $f'=+\cdot 6$ m and $x=\cdot 4$ m.

Thus the other lens is convergent and is of focal length ·6 m at a distance ·4 m from the first lens. **Ans.**

New convention : Since the focal length of a converging lens is +ve and that of diverging lens −ve, we have

$$\frac{1}{\cdot 3}=\frac{1}{\cdot 2}+\frac{1}{f'}-\frac{x}{\cdot 2f'} \quad \ldots \text{(i)}$$

and $\cdot 2+f'-2x=0$ or $2x=\cdot 2+f'$ .. (ii).

Solving (i) and (ii) for f' and x, we have $f'=+\cdot 6$ m and $x=\cdot 4$ m.

Thus the other lens is convergent and is of focal length ·6 m at a distance ·4 m from the first lens. **Ans.**

Ex. 4. *A camera lens* ($f=\cdot 20$ m) *is made of two components, one of crown glass* (*dispersive power* ·02) *and the other of flint glass* (*dispersive power* ·04). *Find the focal length of the component lenses.*

Sol. A camera lens is essentially convergent.

Co-ordinate convention : We have $\frac{1}{+\cdot 2}=\frac{1}{f}+\frac{1}{f'}$

and $$\frac{\cdot 02}{f}+\frac{\cdot 04}{f'}=0.$$

(*Note here that we do not put plus or minus sign before f and f' as their nature is not given.*)

Solving for f and f', we have $f=+\cdot 1$ m and $f'=-\cdot 2$ m.

Thus the crown glass component is to be a convergent one of focal length ·1 m and the flint glass component is to be divergent of focal length ·2 m. **Ans.**

New convention : We have $\frac{1}{+\cdot 2}=\frac{1}{f}+\frac{1}{f'}$ and $\frac{\cdot 02}{f}+\frac{\cdot 04}{f'}=0.$

Solving we have $f=+\cdot 1$ m and $f'=-\cdot 2$ m. **Ans.**

EXERCISES

(A)

1. The focal lengths of a thin convergent lens are ·5 m and ·484 m for red and blue rays respectively. Find the chromatic aberration and dispersive power of the lens. (**Ans.** ·016 m; ·0325)

2. The focal lengths of a thin convergent lens are ·20 m and ·21 m for violet and red lights respectively. An object is placed at a distance of ·15 m. Calculate the chromatic aberration in the image. (**Ans.** ·075 m)

3. An achromatic telescope objective of 1·5 m focal length consists of two thin lenses in contact of dispersive powers ·050 and ·075 respectively. Find the focal length of each lens.

(**Ans.** convex lens of dispersive power ·050 is to be of focal length ·5 m and concave lens of dispersive power ·075 is to be of focal length ·75 m)

4. Show that in order to obtain a converging achromatic combination the dispersive power of the material of the concave lens must be greater than that of the material of the convex lens.

5. Two convex lenses of same material and powers +10 D and +12·5 D are available. How will you arrange them to obtain an achromatic combination and what will be the power of the combination ?

(**Ans.** arrange them co-axially at ·09 m apart; +11·25 D)

6. Calculate the focal lengths of a convex lens of crown glass of dispersive power ·012 and concave lens of dispersive power ·020 so that they form when placed in contact an achromatic converging combination of focal length ·3 m.

(**Ans.** convex=·12 m, concave=·2 m)

(B)

7. An achromatic objective of focal length 1·5 m is to be constructed of crown glass convex lens ($\mu=1{\cdot}51$; $\omega={\cdot}21$) and of flint glass concave lens ($\mu=1{\cdot}65$; $\omega={\cdot}45$). One face of the component lenses is in complete contact with each other, and the other face of the concave lens is plane. Calculate the curvature of the surfaces.

(Ans. radius of curvature of common surface=1·11 m; radius of curvature of the other surface of convex lens=·644 m)

8. An achromatic object is to be made with a thin crown glass bi-convex lens and a thin flint glass equi-concave lens. Calculate the focal length of the achromat.

For crown glass : $\mu_r=1{\cdot}480$, $\mu_b=1{\cdot}500$.

For flint glass : $\mu_r=1{\cdot}610$, $\mu_b=1{\cdot}670$.

Radius of curvature of concave lens=·3 m. (Ans. ·18 m, convergent)

9. Show that the power of the achromatic combination of two convex lenses of the same material is equal to the mean power of the lenses.

10. Show that in the achromatic combination of two thin lenses of same material and nature the first principal plane lies at the first principal focus of the first lens and the second principal plane at the second principal focus of the second lens.

□□□

CHAPTER 6

OPTICAL INSTRUMENTS : DEFECT OF VISION : VELOCITY OF LIGHT

1. Microscope : (i) *Simple microscope* : A single convex lens can serve as a microscope when it is called a simple microscope. The magnifying power of a simple microscope is defined and given by

$$M, \text{ Magnifying power} = \frac{\text{Angle subtended by the image at the eye}}{\text{Angle subtended by the object at the eye from least distance of distinct vision}}$$

or $M = \left(1 + \frac{D}{f}\right)$ where D = least distance of distinct vision.

(ii) *Compound microscope* : A compound microscope consists of an achromatic doublet placed in front of the object to be seen and is called the objective and an eye-piece placed behind the objective. The magnifying power of a compound microscope is given by

$$M, \text{ Magnifying power (or Magnification)} = \left(\frac{f_o}{u_o - f_o}\right)\left(1 + \frac{D}{f_e}\right)$$

where f_o = focal length of the objective, f_e = focal length of the eye-piece and u_o = distance of the object from the objective.

Tube length of a microscope $= (v_o + u_e)$.

2. Telescopes : A telescope consists of an achromatic doublet of large focal length and large aperture and an eye-piece placed behind it.

When the object is at infinity the magnifying power of a telescope set for distinct vision is given by

$$M = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) \text{ and that for normal vision, } M = \frac{f_o}{f_e}.$$

When object is not at infinity but at finite but large distance,

$$M, \text{ Magnifying power (Magnification)} = \frac{v_o}{u_e}.$$

Tube length of a telescope $= v_o + u_e$ in general

$= f_o + f_e$ in normal adjustment.

3. Ramsden's eye-piece and Huygen's eye-piece : An eye-piece an ocular consists of two lenses instead of one because in a single lens image suffers from various defects such as chromatic aberration, spherical aberration, astigmatism etc. When two lenses are used all these defects are reduced. The other reason for using two lenses instead of one is that in a single lens the rays passing through the outer portions of the lens do not fall on the eye. Consequently the intensity of the image falls and the field of view is limited. This is why one lens of smaller aperture so as to cover the entire pupil of the eye is placed close to the eye called eye-lens and another of large aperture, called the field lens is placed between the objective and the eye-lens.

Ramsden's eye-piece consists of *two plano-convex lenses of the same material and same focal length* and fitted in a brass tube at a distance *two-third of either focal length from each other*. The curved surfaces face each other.

Huygen's eye-piece consists of two *plano-convex lenses* of focal lengths in the *ratio* 1 : 3 *separated* from each other by a distance equal to their *mean focal* length. The curved surfaces are turned towards the objective. The lens of greater focal length is the field lens and the smaller one is the eye-lens.

How to tackle problems on eye-pieces: (i) As eye-pieces are combination of two lenses separated by a certain distance, the problems on them are to be solved by treating them as combination of lenses. Hence try to recall all the formulae and principle of '*combination lenses separated by a distance*'.

In co-ordinate and new convention :

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2} \quad \text{or} \quad F=\frac{f_1 f_2}{\triangle} \text{ where } \triangle=f_1+f_2-x.$$

(ii) The first principal plane is at a distance α from the first lens towards the second lens and the second principal plane is at a distance β from the second lens towards the first lens. These distances are given by (in all conventions)

$$\alpha=\frac{x}{\triangle}\cdot f_1 \quad \text{and} \quad \beta=\frac{x}{\triangle} f_2.$$

(iii) $\frac{1}{V}-\frac{1}{U}=\frac{1}{F}$ where U=object distance from the principal plane, V=image distance from the second principal plane.

(iv) Magnifying power for *distinct vision* $=\left(1+\frac{D}{F}\right)$.

(v) Magnifying power for *normal vision* $=\frac{D}{F}$ where D is the least distance of distinct vision.

(vi) *Condition for minimum spherical aberration* is :
Distance between lenses = difference of focal lengths of lenses (convex).

(vii) *Condition for minimum chromatic aberration* :
Distance between lenses = mean of the focal lengths of lenses (convex).

4. Defects of vision : (i) *Short sight* : Suppose a man can see only up to distance D.

Coordinate convention : $\frac{1}{v}-\frac{1}{-D}=\frac{1}{f_e}$ .. (i).

When rectified for infinity by using an auxiliary lens of focal length f, we have $\frac{1}{v}-\frac{1}{\infty}=\frac{1}{F}=\frac{1}{f_e}+\frac{1}{f}$ .. (ii).

Subtracting, we have $f=-D$.

The minus sign shows that the lens required is concave.

New convention : $\frac{1}{v}+\frac{1}{D}=\frac{1}{f_e}$ .. (i).

When rectified for infinity by using an auxiliary lens of focal length f, we have $\frac{1}{v}+\frac{1}{\infty}=\frac{1}{F}=\frac{1}{f_e}+\frac{1}{f}$ .. (ii).

Subtracting, we have $f=-D$.

The negative sign shows that the lens required is concave.

(ii) *Long sight* : Suppose a man cannot see up to distance D.

Coordinate convention : $\frac{1}{v}-\frac{1}{-D}=\frac{1}{f_e}$ .. (i).

Suppose we have to rectify it for normal least distance of distinct vision, namely, 25 cm. Then

$$\frac{1}{v}-\frac{1}{-25}=\frac{1}{f_e}+\frac{1}{f} \quad .. \text{(ii)}.$$

Subtracting, we have $f=\dfrac{25D}{D-25}$.

Since $D>25$ cm, f is positive. This means the lens required is convex.

New convention : $\dfrac{1}{v}+\dfrac{1}{D}=\dfrac{1}{f_e}$ .. (i).

Suppose f is the focal length of the rectifying glass. Then

$$\frac{1}{v}+\frac{1}{25}=\frac{1}{f_e}+\frac{1}{f} \quad .. \text{(ii)}.$$

Subtracting, we have $f=\dfrac{25D}{D-25}$.

Since $D>25$ cm, f is positive. This means the lens required is convex.

5. Velocity of light : (i) *Fizeau's method* : $c=\dfrac{4\,mn_s\,D}{2s-1}$ where m is the number of teeth, n_s = number of revolutions per second for sth eclipse and D = distance between wheel and the distant concave mirror. (See author's Introductory Physics Part II.)

(ii) *Foucault's method*: $\triangle\theta=\dfrac{(b+R)x}{2Ra}$ where $\triangle\theta$ is the angle of rotation of the mirror, b = distance between the lens and the rotating mirror, a = distance between the lens and the source, R = radius of curvature of the concave mirror and x = shift of the image. (See author's Introductory Physics Part II.)

$c=\dfrac{4\pi nR}{\triangle\theta}=\dfrac{8\pi nR^2a}{(b+R)x}$ where n = number of revolutions per second.

(iii) *Michelson's method* : $c=mnD$ where m = number of mirrors, n = number of revolutions per second and D = distance traversed by light from the face on which it falls initially to the diametrically opposite face after successive reflections from the intermediate mirrors.

According to the wave theory of light,

$$\mu=\frac{\text{velocity of light in a vacuum}}{\text{velocity in the medium}}.$$

EXAMPLES

Microscopes and Telescopes

Ex. 1. *The focal length of the object glass of a microscope is* ·02 m, *that of the eye-piece is* ·04 *and distance between them is* ·20 m. *What is the distance of the object from the object glass when the image seen by the eye is* ·25 m *from the eye-piece ? Calculate also the magnifying power of the microscope.*

Sol. *Coordinate convention* : *Eye-piece* : Here image for the eye-piece is on the same side as the object, so v_e is negative.

$$\therefore \quad \frac{1}{-\cdot 25}-\frac{1}{-u_e}=\frac{1}{+\cdot 04} \text{ or } u_e=\cdot 0345 \text{ m},$$

Objective : v_o (absolute) $=\cdot 20-\cdot 0345$ m $=\cdot 1655$ m.

$$\therefore \quad \frac{1}{+\cdot 1655}-\frac{1}{u_o}=\frac{1}{+\cdot 02} \text{ or } u_o=-\cdot 023 \text{ m}.$$

$$\therefore \quad M=\frac{f_o}{u_o-f_o}\left(1+\frac{D}{f_e}\right)=\left(\frac{\cdot 02}{\cdot 023-\cdot 02}\right)\left(1+\frac{\cdot 25}{\cdot 04}\right)=48\cdot 3. \quad \textbf{Ans.}$$

New convention : *Eye-piece* : Here image is virtual and so v_e is $-$ve.

$$\therefore \quad \frac{1}{-\cdot 25}+\frac{1}{u_e}=\frac{1}{+\cdot 04} \text{ or } u_e=\cdot 0345 \text{ m}.$$

Objective : Here v_o (absolute) $=\cdot 20-\cdot 0345=\cdot 1655$ m. The image formed by the objective is real and so v_o is $+$ve.

$$\therefore \quad \frac{1}{\cdot 1655}+\frac{1}{u_o}=\frac{1}{+\cdot 02} \text{ or } u_o=+\cdot 023 \text{ m}.$$

$$\therefore \quad M=\frac{f_o}{u_o-f_o}\left(1+\frac{D}{f_e}\right)=\frac{\cdot 02}{\cdot 023-\cdot 02}\times\left(1+\frac{\cdot 25}{\cdot 04}\right)=48\cdot 3. \quad \textbf{Ans.}$$

Ex. 2. *In a simple astronomical telescope the focal length of object glass is* ·75 m, *that of the eye-piece is* ·05 m. *Calculate the magnifying power when the final image of a distant object is seen* (*i*) *a long way off,* (*ii*) *at a distance of* ·25 m. *Find the distance between the two lenses in each case.*

Sol. (i) *When the final image is long way off, the telescope is set for normal vision* when magnifying power is given by

$$M, \text{ magnifying power}=\frac{f_o}{f_e}=\frac{\cdot 75}{\cdot 05}=15. \quad \textbf{Ans.}$$

The distance between the lenses $= \cdot75 + \cdot05 = \cdot80$ m. **Ans.**

(ii) *Eye-piece* : *Coordinate convention* : $\dfrac{1}{-\cdot25} - \dfrac{1}{-u_e} = \dfrac{1}{\cdot05}$

or $u_e = \cdot0417$ m.

New convention : $\dfrac{1}{-\cdot25} + \dfrac{1}{u_e} = \dfrac{1}{\cdot05}$ or $u_e = \cdot0417$ m.

Magnifying power $= \dfrac{f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right) = \dfrac{\cdot75}{\cdot05}\left(1 + \dfrac{\cdot05}{\cdot25}\right) = 18.$ **Ans.**

Distance between lenses $= u_e + f_o = \cdot0417 + \cdot75 = \cdot7917$ m. **Ans.**

Ex. 3. *An object can be seen by a naked eye from a distance D. What is the angular magnification if the same object is viewed through a magnifying glass of focal length f held at a distance d from the eye and so arranged that the image is at a distance L from the eye ? Consider the cases* : (i) $L = \infty$, (ii) $L = D$.

Sol. Let α be the smallest angle subtended by an object at the eye when it is just visible. Then

$$\alpha = \frac{PQ}{D} \text{ where } PQ = \text{size of the object.}$$

When $L = \infty$, the image distance $= f$.

$\therefore$ $\beta =$ angle subtended by the image $= \dfrac{PQ}{f}$.

$\therefore$ Magnification (angular) $= \dfrac{\beta}{\alpha} = \dfrac{D}{f}$. **Ans.**

In the second case, $\dfrac{1}{-(D-d)} - \dfrac{1}{-u} = \dfrac{1}{f}$ or $\dfrac{1}{u} = \dfrac{1}{D-\alpha} + \dfrac{1}{f}$.

$$\therefore \text{ Magnification} = \frac{D-d}{u} = (D-d) \times \left(\frac{1}{D-d} + \frac{1}{f}\right)$$

$$= 1 + \frac{D-d}{f}.$$ **Ans.**

Ex. 4. *A compound microscope has an objective lens of* 1·5 cm *focal length and an eye-piece lens of* 5 cm *focal length. An object is focussed in it from a distance* 1·8 cm. *Calculate the magnification produced by the microscope and the length of the tube.*

Sol. *Co-ordinate convention* : For the objective we have

$$\frac{1}{v_o}-\frac{1}{-1{\cdot}8}=\frac{1}{+1{\cdot}5} \text{ or } v_o=+9 \text{ cm.}$$

The plus sign shows that it is on the other side of the objective.

For the eye-piece we have $\frac{1}{-25}-\frac{1}{u_e}=\frac{1}{+5}$ or $u_e=-4{\cdot}167$ cm.

$\therefore$ Magnification produced $=m_o\times m_e=\frac{9}{1{\cdot}8}\times\frac{25}{4{\cdot}167}=30$. **Ans.**

Tube length $=v_o+u_e=9+4{\cdot}167=13{\cdot}167=13{\cdot}2$ cm. **Ans.**

New convention : For the objective we have $\frac{1}{v_o}+\frac{1}{1{\cdot}8}=\frac{1}{+1{\cdot}5}$

or $$v_o=+9 \text{ cm.}$$

The plus sign shows that the image is real and it lies on the other side of the objective i.e., *R*-side of it.

For the eye-piece we have $\frac{1}{-25}+\frac{1}{u_e}=\frac{1}{+5}$ or $u_e=4{\cdot}167$ cm.

Magnification produced $=m_o\times m_e=\frac{9}{1{\cdot}8}\times\frac{25}{4{\cdot}167}=30$. **Ans.**

Tube length $=v_o+u_e=9+4{\cdot}167=13{\cdot}2$ cm. **Ans.**

Ex. 5. *An astronomical telescope consisting of two convex lenses* 50 cm *and* 5 cm *is focussed on the moon. What is the distance between the two lenses in this position ? If the telescope is turned towards an object* 10 m *away, how much the eye-piece has to be moved to focus the object without altering the accommodation of the eye ? Calculate the magnification (angular) produced by the telescope in the two adjustments.*

Sol. When the telescope is focussed on a distant object and the final image is also at infinity, the distance between lenses $=f_o+f_e$.

$\therefore$ Distance between lenses $=50+5=55$ cm. **Ans.**

When it is turned on a nearer object, the image formed by the objective will not be at its focus, rather a little away from it. The eye-piece has to be shifted exactly by the same distance through which the image is shifted because final image is at the same distance.

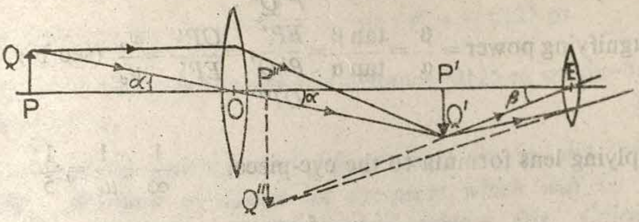

Fig. 6-1

Co-ordinate convention : Applying lens formula to the object glass,

$$\frac{1}{v_o}-\frac{1}{-10}=\frac{1}{+\cdot 5} \quad \text{or} \quad v_o=+\frac{50}{95}\ \text{m}=+52{\cdot}63 \text{ cm.}$$

The plus sign shows that it is on the other side.

∴ The distance by which the eye-piece is to be shifted

$$=52{\cdot}63-50=2{\cdot}63 \text{ cm.}$$ **Ans.**

When the object is not at infinity but at finite but large distance (large in comparison to the tube length),

$$\text{Magnifying power}=\frac{\beta}{\alpha}=\frac{\tan\beta}{\tan\alpha}=\frac{\dfrac{P'Q'}{EP'}}{\dfrac{P'Q'}{OP'}}=\frac{OP'}{EP'}=\frac{v_o}{u_e} \text{ (Fig. 6-1).}$$

Applying lens formula to the eye-piece, $\frac{1}{\infty}-\frac{1}{-u_e}=\frac{1}{+5}$

or $$u_e=5 \text{ cm.}$$

$$\therefore \quad \text{Magnification}=\frac{52{\cdot}63}{5}=10{\cdot}52.$$ **Ans.**

$$\text{Previous magnification}=\frac{f_o}{f_e}=\frac{50}{5}=10 \;(\because\; v_o=f_o \text{ and } u_e=f_e).$$ **Ans.**

New convention : Applying lens formula to the object glass,

$$\frac{1}{v_o}+\frac{1}{10}=\frac{1}{\cdot 5} \quad \text{or} \quad v_o=\frac{50}{95}\ \text{m}=52{\cdot}63 \text{ cm.}$$

The plus sign shows the image is real and it is on the other side.

∴ The distance by which eye-piece is to be shifted

$$=52{\cdot}63-50=2{\cdot}63 \text{ cm.}$$ **Ans.**

When the object is not at infinity but at finite but large (large in comparison to the tube length),

$$\text{Magnifying power} = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\dfrac{P'Q'}{EP'}}{\dfrac{P'Q'}{OP'}} = \frac{OP'}{EP'} = \frac{v_o}{u_e} \text{ (see Fig. 6-1).}$$

Applying lens formula to the eye-piece, $\frac{1}{\infty} + \frac{1}{u_e} = \frac{1}{5}$

or $u_e = 5$ cm.

Magnification $= \frac{52 \cdot 63}{5} = 10 \cdot 52.$ **Ans.**

Previous magnification $= \frac{f_o}{f_e} = \frac{50}{5} = 10.$ **Ans.**

Eye-pieces

Ex. 6. *Light from the sun is falling directly upon the Ramsden's eye-piece consisting of lenses in the usual manner each of focal length* ·05 m. *Find the position of the image formed and trace the path of rays through the eye-piece.*

Sol. *Co-ordinate convention* : $\frac{1}{v'} - \frac{1}{\infty} = \frac{1}{+\cdot 05}$ or $v' = +\cdot 05$ m.

Separation between lenses $= \frac{2}{3} \times \cdot 05$ m.

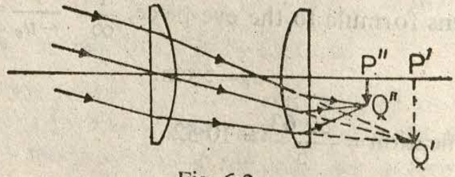

Fig. 6-2

P'Q' serves as the '*virtual object*' for the eye-lens.

$$\therefore \quad \frac{1}{v''} - \frac{1}{(\cdot 05 - \frac{2}{3} \times \cdot 05)} = \frac{1}{+\cdot 05} \quad \text{or} \quad v'' = +\cdot 0125 \text{ m.}$$

Thus the final image is formed at a distance ·0125 m on the other side of the eye-lens. **Ans.**

New convention : $\frac{1}{v'} + \frac{1}{\infty} = \frac{1}{\cdot 05}$ or $v' = \cdot 05$ m.

Separation between lenses $= \frac{2}{3} \times \cdot 05$ m.

P'Q' serves as the '*virtual object*' for the eye-lens.

$$\therefore \quad \frac{1}{v''}+\frac{1}{-(\cdot 05-\frac{2}{3}\times\cdot 05)}=\frac{1}{\cdot 05} \quad \text{or} \quad v''=+\cdot 0125 \text{ m.}$$

Thus the final image is formed at a distance ·0125 m on the other side of the eye-lens. **Ans.**

Ex. 7. *Two plano-convex lenses of focal length ·06 m and ·04 m are available. We have to design an eye-piece which will be free from chromatic aberration. Explain how to achieve this. Calculate the focal length and magnifying power for normal vision. Will it be a positive or negative eye-piece ?*

Sol. Condition for minimum chromatic aberration is that the distance between lenses = mean of the focal lengths of the component lenses

$$=\frac{\cdot 06+\cdot 04}{2}=\cdot 05 \text{ m.}$$

So we will achieve the desired eye-piece keeping the lenses co-axially ·05 m apart. The first lens (·06 m focal length) will be field lens and the other will be eye-lens.

Co-ordinate Convention : Here

$$\Delta=f_1+f_2-x=\cdot 06+\cdot 04-\cdot 05=\cdot 05 \text{ m.}$$

$$F=\frac{f_1 f_2}{\Delta}=\frac{\cdot 06\times\cdot 04}{\cdot 05}=\cdot 048 \text{ m.}$$

$$\text{Magnifying power for normal vision}=\frac{D}{F}=\frac{\cdot 25}{\cdot 048}=5\cdot 2. \qquad \textbf{Ans.}$$

The distance of the first principal plane from the field lens towards the eye-lens is given by

$$\alpha=\frac{x}{\Delta}f_1=\frac{\cdot 05}{\cdot 05}\times\cdot 06=\cdot 06 \text{ m.}$$

The first principal focus is at a distance ·048 m from the first principal plane towards the object. With reference to the field lens it is at a distance (·06 – ·048) = ·012 m on the other side. Hence it is a negative eye-piece. **Ans.**

New convention : Here $\Delta=f_1+f_2-x=\cdot 06+\cdot 04-\cdot 05=\cdot 05$ m

$$F=\frac{f_1 f_2}{\Delta}=\frac{\cdot 06\times\cdot 04}{\cdot 05}=\cdot 048 \text{ m.}$$

$$\text{Magnifying power for normal vision}=\frac{D}{F}=\frac{\cdot 25}{\cdot 048}=5\cdot 2. \qquad \textbf{Ans.}$$

The distance of the first principal plane from the first lens (field lens) towards the eye-lens is given by

$$\alpha = \frac{x}{\triangle} f_1 = \frac{\cdot05}{\cdot05} \times \cdot06 = \cdot06 \text{ m.}$$

The first principal focus is at a distance ·048 m from the first principal plane towards the object. With reference to the field lens it is at distance (·06 – ·048) = ·012 m on the other side. Hence it is a negative eye-piece. **Ans.**

Defects of Vision : Velocity of Light : Rectilinear propagation of light

Ex. 8. *A short-sighted person cannot see distinctly objects situated beyond* 2 m *from him. What should be the power of the lens which he should use for seeing distant objects clearly ?*

Sol. *Co-ordinate convention* : Let f_e be the accommodated focal length of the eye-lens for 2 m distance. Then

$$\frac{1}{v} - \frac{1}{-2} = \frac{1}{f_e} \quad \text{.. (i).}$$

Let f be the focal length of the correcting lens for seeing distant objects. Then

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f_e} + \frac{1}{f} \quad \text{.. (ii).}$$

Subtracting, $f = -2$ m.

∴ Power of the lens required $= \frac{1}{f} = \frac{1}{-2} = -\cdot5$ dioptre. **Ans.**

New convention : Let f_e be the accommodated focal length of the eye-lens for 2 m distance. Then $\frac{1}{v} + \frac{1}{2} = \frac{1}{f_e}$ .. (i).

Let f be the focal length of the correcting lens for seeing distant objects. Then

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{f_e} + \frac{1}{f} \quad \text{.. (ii).}$$

Subtracting, $f = -2$ m.

∴ Power of the lens required $= \frac{1}{f} = \frac{1}{-2} = -\cdot5$ dioptre. **Ans.**

Ex. 9. *A long-sighted person can see distinctly objects beyond* 1 m. *Calculate the power of the lens that should be prescribed for him for normal vision.*

Sol. *Co-ordinate convention* : Let f_e be the accommodated focal length of the eye-lens for distance 1 m. Then

$$\frac{1}{v}-\frac{1}{-1}=\frac{1}{f_e} \quad \cdots \text{(i).}$$

Let f be the focal length of the correcting lens. Then

$$\frac{1}{v}-\frac{1}{-\cdot25}=\frac{1}{f_e}+\frac{1}{f} \quad \cdots \text{(ii).}$$

Subtracting, $\frac{1}{f}=\frac{1}{\cdot25}-\frac{1}{1}$ or $f=+\frac{1}{3}$ m.

$\therefore$ Power of the lens required $=\frac{1}{f}=+3$ D. **Ans.**

New convention : Let f_e be the accommodated focal length of the eye-lens for the distance 1 m. Then $\frac{1}{v}+\frac{1}{1}=\frac{1}{f_e}$ $\cdots$ (i).

Let f be the focal length of the correcting lens. Then

$$\frac{1}{v}+\frac{1}{\cdot25}=\frac{1}{f_e}+\frac{1}{f} \quad \cdots \text{(ii).}$$

Subtracting, $\frac{1}{f}=\frac{1}{\cdot25}-\frac{1}{1}$ or $f=+\frac{1}{3}$ m.

$\therefore$ Power of the lens required $=\frac{1}{f}=+3$D. **Ans.**

Ex. 10. *In a Fizeau's experiment there were* 500 *teeth at the edge of the toothed wheel. The distance between the plane mirror and the wheel was* 10 km. *Calculate the rpm of the wheel when the source disappeared for the first time. Velocity of light in vacuum* $=3\times10^8$ m/s.

Sol. We have $c=\frac{4mn_sD}{2s-1}$. For the first eclipse of the source we have $s=1$.

$\therefore$ $n_1=\frac{c}{4mD}=\frac{3\times10^8}{4\times500\times10^4}=15$ rps $=900$ rpm. **Ans.**

Ex. 11. *A man stands on a vertical long tower of height* $h=20$ m. *Calculate the distance up to which he will be able to see the surface of the earth. Which property of light do you use in the calculation ? R* (*radius of the earth*) $=6\cdot4\times10^6$ km.

Sol. Since light travels in a straight line, the man can see from the foot of the tower up to the point of the surface of the earth where the tangent through him (considered as a point) touches the surface of the earth.

Let θ be the angle subtended by this distance at the centre of the earth. Then the required distance $= R.\ \theta$.

From the geometry of the figure, $\tan\theta = \dfrac{\sqrt{(R+h)^2 - R^2}}{R} = \dfrac{\sqrt{2Rh}}{R}$

$(\because\ R \gg h)$.

$\therefore$ The required distance $= R.\theta = \sqrt{2Rh} = \sqrt{2 \times 6{\cdot}4 \times 10^6 \times 20}$

$= 16$ km. **Ans.**

EXERCISES

Microscopes and Telescopes

1. Two convex lenses of focal lengths ·01 m and ·06 m respectively are arranged to form a microscope. A small object is placed ·012 m from object glass. If the image seen appears to be ·25 m from the eye-piece, what is the distance between the object glass and the eye-piece ? **(Ans. ·1084 m)**

2. Show that the magnifying power of a simple microscope is $\left(1+\dfrac{D-a}{f}\right)$ when D is the least distance of distinct vision, a the distance between the eye and the lens and f the focal length of the lens. What conclusion do you draw about the best position of the eye ?

3. The objective and the eye-piece of a microscope have focal lengths of ·02 m and ·04 m respectively and are placed ·15 m apart. If the final image is situated ·25 m from the eye-piece, how far must the object be placed in front of the objective ? **(Ans. ·0242 m)**

4. A compound microscope consists of two convex lenses of focal lengths ·02 m and ·15 m. An object is placed at a distance of ·025 m from the object glass and the final image is formed at the least distance of distinct vision. Find the magnification produced and the distance between the lenses.

(Ans. 10·67; ·1937 m)

5. The focal lengths of the objective and the eye-piece of an astronomical telescope are ·25 m and ·02 m respectively. The telescope is adjusted to view an object at a distance of 1·5 m from the objective, the final image being ·25 m from the eye of the observer. Calculate the length of the telescope and the magnification produced by it. **(Ans. $31{\cdot}85 \times 10^{-2}$ m; 16·2)**

6. An astronomical telescope in normal adjustment has tube length 93 cm and gives magnification (angular) of 30. If the eye-piece is to be drawn out by 3 cm to focus a near object, with the final image at infinity, find how far away is the object and magnification (angular) in this case.

[*Hint* : See Ex. 4.] **(Ans. 27·9 m; 31)**

7. A telescope is formed of two convex lenses of focal length 10 cm and 1 cm respectively. If the telescope is focussed on a scale 1 m away from the objective and the final image is formed 25 cm away from the eye, calculate the magnification produced. (Similar I. I. T. 1980)

(Ans. 11 56)

8. A simple astronomical telescope is made from two convex lenses of focal length 1 m and ·05 m respectively. Find the magnifying power when the instrument is used to view a distant object if the final image is formed (a) at a great distance, (b) at a distance of ·25 m from the eye-piece. [Ans. (a) 20, (b) 24]

9. A telescope consisting of two convex lenses of focal lengths ·20 m and ·01 m is focussed on a distant object for normal vision. What is the distance between the glasses ? If the same telescope is now focussed on an object distant 2 m away from the object glass without altering the accommodation of the eye by how much the eye-piece has to be shifted ? (**Ans.** ·21 m; by ·022 m away)

10. The focal lengths of the objective and the eye-piece of an astronomical telescope are ·25 m and ·025 m respectively. The telescope is focussed on an object 5 m from the objective, the final image being formed ·25 m from the eye of the observer. Calculate the length of the telescope and its magnifying power.

$$\left[\textit{Hint}: \text{Mag. power}=\frac{v_o}{u_e}\right]$$

(Ans. ·2859 m; 11·6 m)

11. A Gallilean telescope of 10-fold magnification has the length of 45 cm when adjusted to infinity. Find (a) the focal lengths of the telescope's objective and eye-piece. (b) By what distance the eye-piece should be displaced to adjust the telescope to the distance of 50 m ? [**Ans.** (a) 50 cm, 5 cm; (b) ·5 cm]

12. The objective of a telescope focussed for infinity is taken out and replaced by a diaphragm of diameter D. A screen placed on the other side of the eye-piece shows a sharp image of diameter d. What was the magnifying power of the telescope ? (**Ans.** D/d)

13. A telescope with magnification $M=15$ was submerged into water which filled up the inside of the telescope. To make the system work as telescope again within the former dimensions, the objective was replaced. What has the magnification of the telescope become equal to ? μ of the material of the eye-piece $=1·5$ and μ of water $=\frac{4}{3}$. (**Ans.** 3)

Eye-pieces

14. A Huygen's eye-piece is made by arranging two plano-convex lenses of focal lengths ·03 m and ·09 m in the usual manner. Light directly falls on the field lens of the eye-piece. Calculate the position of the final image formed by the eye-piece and trace the path of rays showing the formation of images.

(**Ans.** 1·5 cm behind the eye-lens)

15. An eye-piece is constructed by using two thin lenses of focal lengths ·03 m and ·04 m respectively and are spaced so as to reduce spherical aberration to minimum. Find (a) separation between lenses required, (b) equivalent focal length, and (c) magnifying power when the vision is distinct.

[Ans. (a) ·01 m; (b) ·02 m; (c) 13·5]

16. An eye-piece consists of a concave lens of focal length $2f$ and a convex lens of focal length $3f$ separated by a distance $\frac{f}{2}$. They are of the same material. Examine whether the eye-piece formed is free from spherical and chromatic aberration. Is it suitable for use of cross-wire ?

(*Hint* : An eye-piece is suitable for cross-wire when it is a positive one. See Ex. 6.) (Ans. free from chromatic aberration but not spherical; yes)

17. You are given two plano-convex lenses of focal lengths ·03 m and ·05 m and asked to design an eye-piece free from chromatic aberration. Explain how you will achieve it. Calculate the equivalent focal length of the eye-piece formed and its magnifying power if it is used by a person for whom least distance of distinct vision is ·25 m Will your eye-piece be free from spherical aberration ?

[Ans. (i) by keeping the lenses ·04 m apart, (ii) equivalent focal length ·0375 m, (iii) magnifying power 7·67, (iv) no]

18. Show that in eye-piece both spherical aberration and chromatic aberration can be simultaneously reduced to a minimum only when their focal lengths are in the ratio 1 : 3 .

19. Two thin converging lenses of focal length ·05 m are separated by a distance of ·03 m. Find the focal length of the eye-piece and determine whether this will be a positive or negative eye-piece. Will it be free from spherical and chromatic aberration ? (Ans. ·036 m; positive; no]

20. Two convex lenses of focal lengths $2f$ and f are arranged to form an eye-piece and are spaced by f. The first lens is to be used as field lens and the second one as eye-lens. Find (a) equivalent focal length. (b) See whether it is positive or negative eye-piece. (c) Whether it is free from spherical aberration and chromatic aberration ? (d) What will be disadvantage if f is used as field lens ?

[Ans. (a) $F=f$, (b) positive, (c) free from spherical aberration but not chromatic, (d) eye-lens will fall at the focus of field lens and so dust particles or anything else on field lens will be infinitely magnified and whole field of view will be spoiled.]

Defects of Vision

21. A student with defective eye-sight can see clearly nothing that is farther from his eyes than 50 cm. What is the kind and power of lens that will enable him to see easily and clearly distant objects ?

(Ans. concave lens; power = −2D)

22. A person's range of distinct vision is from 6 cm to 60 cm from the eye. What spectacles would be required to see distant objects clearly and what would be his least distance of distinct vision using the spectacles ?

(Ans. concave lens of focal length 60 cm; 6·67 cm

23. A person can see objects as near as 40 cm and as far as 3 m. What spectacles will be required (a) for reading purposes, (b) for seeing distant

objects ? Least distance of distinct vision=25 cm. What is the range of clear vision with each pair of spectacles ?

[Ans. (a) convex lens of power 1·5 D, (b) concave lens of power ·33 D. With reading glass he can clearly see from 25 cm to 54·5 cm and that with distant glass from 46 cm to infinity]

24. A short-sighted man, the accommodation of whose eye is between 12 cm and 60 cm wears spectacles through which he can see distinctly remote objects. Determine the minimum distance at which the man can read a book through his spectacles. (Ans. 15 cm)

25. Two men, a far-sighted and a short-sighted one, see objects through their spectacles with normal eye-sight. When the far-sighted man inadvertently puts on the spectacle of his short-sighted friend, he found that he could see distinctly only infinitely far objects. At what minimum distance can the short-sighted man read small type if he wears the spectacles of the far-sighted man ? (Ans. 12·5 cm)

Velocity of Light

26. The velocity of light in vacuum is 3×10^8 ms^{-1}. Find the same in glass if index of refraction of glass is 1·5. (Ans. 2×10^8 ms^{-1})

27. A star is at a distance 50×10^{12} km from earth. If the star disappears due to collision with some other star so that its light is extinguished, how long we continue seeing the star (even though it does not exist). Velocity of light in vacuum$=3\times10^8$ ms^{-1}. (Ans. 5·3 years)

28. In Fizeau's experiment the distance between the source and the reflector is 8 km and there are 720 teeth in the toothed wheel. At what speed first, second and third eclipse will occur ? (Velocity of light$=3\times10^9$ ms^{-1})

(Ans. 13 revolutions/sec, 39 revolutions/sec and so on)

29. In an experiment for measuring the velocity of light by Focault's method, the fixed and the revolving mirror were 3 km apart, the latter revolving at 500 revolutions per second. The angular deviation of the return ray was 7°12′. Calculate velocity of light. (Ans. 3×10^8 ms^{-1})

30 The distance between the toothed wheel with 720 teeth and the mirror in the Fizeau experiment is 7 km. Two consecutive speeds of rotation of the wheel for which light disappeared were 283 rps and 313 rps. Find the velocity of light. (Ans. 3×10^8 m/s)

Reetilinear Propagation of Light

31. Two places *A* and *B* on the same longitude are *h* km apart along the surface of the earth, *A* is at the equator. The shadow of a vertical pole is zero at *A* at some hour of observation and that at *B* is just half the length of an identical pole at the same hour of observation. Show that these observations enable us to find the radius of the earth.

$$\left(\text{Ans. } R=\frac{h}{\tan^{-1}\frac{1}{2}}\right)$$

32. In a new-moon day, a man on earth sees the annular eclipse of the sun. How high would he rise above the earth's surface so that he can just see the total eclipse of the sun ? Diameter of the sun$=1{\cdot}4\times10^9$ m, diameter of the moon $=1{\cdot}7\times10^6$ m. Distances of the sun and the moon from the position of the man on earth are respectively $1{\cdot}5\times10^{11}$ m and $3{\cdot}8\times10^8$ m. (Ans. $2{\cdot}0\times10^5$ km)

□□□

CHAPTER 7

WAVE THEORY OF LIGHT : INTERFERENCE OF LIGHT

1. Wave Theory of Light : According to this theory light travels in the form of transverse waves. The colour of light is attributed to the *'frequency of the wave'* which remains unchanged when light passes from one medium into another. The refractive index of a medium is related to the velocity of light in that medium by the relation

$$\mu = \frac{\text{velocity of light in vacuum } (c)}{\text{velocity of light in the medium } (c')}.$$

The intensity of a light wave is proportional to **square of its amplitude.**

2. Interference : Two beams of light from a coherent source may give rise to total darkness or maximum brightness if they meet under certain conditions. This phenomenon is known as Interference.

Two wave trains give rise to complete darkness if their path difference $= (2s+1)\frac{\lambda}{2}$ where $s=0, 1, 2, 3 \ldots$ and maximum brightness if path difference $= s\lambda$.

3. Theory of fringes : Suppose S_1 and S_2 are two coherent sources, d = distance between them; D = distance between screen and sources. O is a point on the screen equidistant from S_1 and S_2. Obviously there will be brightness at this point. This is called central bright fringe. P is any point on the screen at a distance x from O. From geometry of the figure,

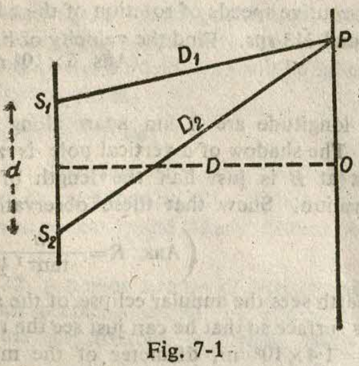

Fig. 7-1

$$D_1^2 = D^2 + \left(x - \frac{d}{2}\right)^2;$$

$$D_2^2 = D^2 + \left(x + \frac{d}{2}\right)^2.$$

$$\therefore \quad D_2^2 - D_1^2 = 2xd, \quad \text{or} \quad D_2 - D_1 = \frac{xd}{D}, \; (\because \quad D_1 + D_2 \simeq 2D).$$

There is darkness at P if $\frac{xd}{D}=(2s+1)\frac{\lambda}{2}$

or $$x=\frac{(2s+1)\lambda D}{2d},$$

and brightness if $$\frac{xd}{D}=s\lambda$$

or $$x=\frac{s\lambda D}{d}.$$

Spacing between consecutive similar pattern, i.e., distance from one darkness to the next or one brigntness to the next is called fringe-width.

$$\therefore \quad \text{Fringe-width } (\beta)=\frac{\lambda D}{d} \qquad \ldots (1).$$

4. **Biprism** : Biprism is a device of producing two coherent virtual sources S_1 and S_2 at a very short distance apart. If A is the angle of prism in radian, μ = index of refraction and l = distance of slit (source) from biprism then δ (deviation produced by each half of the biprism) $=(\mu-1)A$

and d (distance between virtual sources) $=2(\mu-1)Al$

or $$d=2(\mu-1)Al \qquad \ldots (2).$$

5. **Effect of placing a glass plate in the path of the interfering beams** : If a glass plate of thickness t is introduced (index of refraction μ) in the path of a ray, say, S_1P, then the path of wave is increased by $\mu t-t=(\mu-1)t$ because t metre of glass is equivalent to μt metre of air.

$\therefore$ Now, path difference $=D_2-[D_1+(\mu-1)t]=D_2-D_1-(\mu-1)t$

$$=\frac{xd}{D}-(\mu-1)t.$$

Let O' be now the point equidistant from S_1 and S_2.

$\therefore$ Path difference for this point is zero.

$$\therefore \quad \frac{xd}{D}-(\mu-1)t \ 0$$

or $$x \ \frac{(\mu-1)tD}{d}.$$

Shift in central bright fringe and so shift of all other fringes

$$=\frac{(\mu-1)tD}{d}=\frac{(\mu-1)t\beta}{\lambda}.$$

Let this shift be x_0. Then $x_0=\frac{(\mu-1)t\beta}{\lambda}$. .. (3)

Hence knowing x_0 we can calculate 'μ'.

EXAMPLES

Ex. 1. *Calculate the time which light will take to pass normally through a glass plate of thickness* 1 *mm.* (μ *of glass is* 1·5) *and velocity of light in air* 3×10^5 *kilometre per second.*

Sol. Velocity of light in glass $=\frac{10^{-3}}{t}$ ms^{-1} where t is the time in which light travels 10^{-3} m (i.e. 1 mm) of glass.

(Velocity of light in air) $=3\times10^8$ ms^{-1}.

Now μ (of glass) $=\frac{c}{c'}$.

$$\therefore \qquad 1{\cdot}5=\frac{3\times10^8}{\frac{10^{-3}}{t}}=3t\times10^{11}$$

$$\text{or} \qquad t=\frac{1{\cdot}5}{3\times10^{11}}={\cdot}5\times10^{-11}=5\times10^{-12}\text{ s.} \quad \textbf{Ans.}$$

Ex. 2. *The wavelength of red light in air is* 7890$A°$ *in air. What is the wavelength in glass* ($\mu=1{\cdot}5$) *?*

Sol. We have $c'=\nu\lambda$ in glass, $c_0=\nu\lambda_0$ in air.

$$\therefore \qquad \frac{c_0}{c}=\frac{\lambda_0}{\lambda}. \qquad \text{But} \qquad \mu=\frac{c_0}{c}.$$

$$\therefore \qquad \mu=\frac{\lambda_0}{\lambda}$$

$$\text{or} \qquad \lambda=\frac{\lambda_0}{\mu}=\frac{7890}{1{\cdot}5}=5260\text{Å}. \qquad \textbf{Ans.}$$

Ex. 3. *Two interfering beams have their intensities in the ratio* 9 : 4. *Calculate the ratio of maximum and minimum intensities produced by their interference.*

Sol. Since the intensity is proportional to amplitudes, the amplitudes of the intefering beams are in the ratio 3 : 2. Hence the amplitudes are 3 k and 2 k where k is the common ratio. The maximum amplitude of the resultant wave is $3k+2k=5k$ and the minimum amplitude of the resultant wave is $3k-2k=k$.

$$\therefore \quad I_{max} : I_{min} = (5k)^2 : (k)^2 = 25 : 1.$$ **Ans.**

Ex. 4. *Two sources* $\cdot 03 \times 10^{-2}$ m *apart emitting waves of wavelength* λ *in the same phase produce interference fringes on a screen at a distance of* $\cdot 30$ m *from the plane of two sources. The fringe-width is found to be* $\cdot 59 \times 10^{-3}$ m. *Calculate the wavelength.*

Sol. Fringe-width is given by $\beta = \frac{\lambda . D}{d}$.

$$\therefore \quad \cdot 59 \times 10^{-3} = \frac{\lambda \times \cdot 30}{\cdot 03 \times 10^{-2}}$$

or $\lambda = 5{\cdot}9 \times 10^{-7}\ \text{m} = 5900 \times 10^{-10}\ \text{m} = 5900\text{A}^\circ$. **Ans.**

$(\because \ 10^{-10}\ \text{m} = 1\text{A}^\circ)$

Ex. 5. *A Fresnel's biprism of angle* 2° *is illuminated by light of wavelength* $5890A^\circ$ *by a source* $\cdot 10$ m *from it. Calculate the width of the fringes formed on a screen* $\cdot 9$ m *from the biprism.* (*The index of refraction of glass is* 1·5.)

Sol. The deviation δ produced by each half of biprism $= (\mu - 1)A$ where A = angle in radian.

$$d = 2(\mu - 1)Ax$$

$$= 2(1{\cdot}5 - 1)\left(\frac{\pi}{180} 2^\circ\right) \times \cdot 1$$

$$d = \frac{\pi}{900}\ \text{m}.$$

$$D = \cdot 9 + \cdot 1 = 1\ \text{m}.$$

$$\beta = \frac{\lambda D}{d}.$$

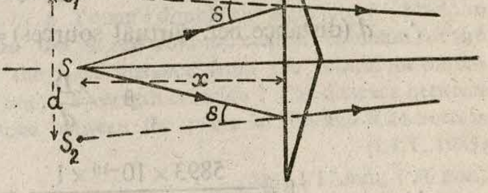

Fig. 7-2

$$\therefore \quad \beta = \frac{(5890 \times 10^{-10}) \times 1}{\pi/900} = 1{\cdot}687 \times 10^{-4}\ \text{m}.$$ **Ans.**

Ex. 6. *Fringes are produced with monochromatic light of wavelength* $5450A^\circ$. *A thin plate of glass of refractive index* 1·5 *is then placed normally in the path of one of the interfering beams and* *the central bright band of the fringe system is found to move into position previously occupied by the third band from the centre. Calculate the thickness of the glass plate.*

Sol. See the article 4 above and show

$$x_0 = \frac{(\mu - 1)t\beta}{\lambda}.$$

Here it is given that $x_0 = 3\beta.$

$$\therefore \quad 3\beta = \frac{(\mu-1)t\beta}{\lambda}$$

or $$t = \frac{3\lambda}{\mu - 1} = \frac{3 \times 5450 \times 10^{-10}}{1{\cdot}5 - 1} = \mathbf{3{\cdot}27 \times 10^{-6}\ m.}$$ **Ans.**

Ex. 7. *Fringes are produced by a Fresnel's biprism in the focal plane of an eye-piece which is* 1 m *from the centre slit. A lens inserted between biprism and eye-piece gives two images of the slit in two positions. In one case the images of the slit are* $4{\cdot}05 \times 10^{-3}$ m *and in the other position* $2{\cdot}9 \times 10^{-3}$ m *apart. If sodium light* ($\lambda = 5893 \times 10^{-10}$ m) *is used, find the distance between interference fringes.*

Sol. It is a fact that when the distance between screen and object is greater than four times the focal length of a convex lens, there are two positions of the lens where it forms real image of the object on the screen. If d_1 and d_2 are the sizes of the images in the two positions then

$$d \text{ (size of the object)} = \sqrt{d_1 d_2}.$$

$$\therefore \quad d \text{ (distance bet. virtual sources)} = \sqrt{4{\cdot}05 \times 10^{-3} \times 2{\cdot}90 \times 10^{-3}}.$$

$$\beta = \frac{\lambda D}{d}.$$

$$\therefore \quad \beta = \frac{5893 \times 10^{-10} \times 1}{\sqrt{4{\cdot}05 \times 2{\cdot}90 \times 10^{-3}}} = 1{\cdot}72 \times 10^{-4}\ \text{m}.$$ **Ans.**

EXERCISES

1. Light travels a certain distance in water in 3 μs. What is the time in which light would travel the same distance in air ? μ of water = $\frac{4}{3}$. (Ans. 2·25 μs)

2. What is the wavelength of yellow light (λ in air = 6000A°) in water ? μ of water = $\frac{4}{3}$. (Ans. 4500A°)

3. Two straight and narrow slits 3×10^{-4} m apart are illuminated by a monochromatic source of wavelength $5{\cdot}9\times10^{-7}$ m. Fringes are obtained at a distance of ·3 m from the slits; find the width of the fringes. (Ans. $5{\cdot}9\times10^{-4}$ m)

4. Two fine slits are placed very close to each other and they are illuminated by strong beam of light of wavelength 5900 A°. Fringes are obtained at a distance of ·3 m from the slits. The fringe width is found to be $5{\cdot}9\times10^{-5}$ m. Calculate the distance between the slits. (Ans 3×10^{-3} m)

5. In Young's experiment the distance between the slits is 1·5 mm and the distance of the screen from the slits is 2 m. The slits are illuminated by a source provided with a red filter ($\lambda=687$ n m). Find the distance between the interference fringes. (Ans. 0·9 mm)

6. In Fresnel's biprism experiment fringes of width $\cdot185\times10^{-3}$ m are observed at a distance of 1 m from the slit. Image of the coherent sources is then produced at the same distance from the slit by placing a convex lens at ·3 m from the slit. The images are found to be separated by $\cdot7\times10^{-2}$ m. Calculate the wavelength of the light used. (Ans. 5550A°)

7. Biprism fringes are produced with the help of sodium yellow light ($\lambda=$ 5893A°). The angle of the biprism is 179·5° and its refractive index 1·5. If the distance between slit and the biprism is ·4 m and the distance between the biprism and the screen is ·6 m, find the distance between successive bright fringes. (Ans. $3{\cdot}376\times10^{-4}$ m)

8. When a thin monochromatic source of light was placed at a distance of ·5 m from a Fresnel biprism of refractive index 1·5, the distance between consecutive bands formed on the screen placed at a distance of 1 m from the biprism was found to be $\cdot12\times10^{-3}$ m. If the wavelength of light was 5893×10^{-10} m, find the magnitude of the obtuse angle of the biprism. (Ans. 178° 19′)

9. A glass plate $1{\cdot}2\times10^{-6}$ m thick is placed in the path of one of the interfering beams in a biprism arrangement using monochromatic light of wavelength 6000A°. If the central band shifts a distance equal to the width of the bands, find refractive index of glass. (Ans. 1·5)

10. A beam of light consisting of two wavelengths 6500Å and 5200Å is used to obtain interference fringes in a Young's double slit experiment. Find the distance of the third fringe on the screen from the central maximum for the wavelength 6000A°. What is the least distance from the central maximum where the bright fringes due to both wavelength coincide ? The distance between slits is 2 mm and the distance between the plane of slits and the screen is 120 cm. (I.I.T. 1985)

(Ans. 1·17 mm; 1·56 mm)

□□□

CHAPTER 8

DIFFRACTION

1. Diffraction : When an obstacle is put in the path of light it bends round the corners and spreads out to a certain extent into the geometricl shadow. This phenomenon is known as Diffraction.

There are two types of diffraction—(i) Fresnel's class of diffraction : In this class of diffraction the source or the screen on which the pattern is observed or both are at finite distance. (ii) Fraunhofer's class : Here the above two are at infinite distances.

2. Diffraction at straight edge : Suppose *S* is a line source and *AB* is an obstacle parallel to it. Join *S* with *A* and produce it to cut the screen at *O*. Then *O* defines the boundary of the geometrical shadow. The wave front incident on *AB* will be cylindrical in shape. Suppose *P* is any point on the screen at a distance *x* from *O*. Join *P* with *S* and let it cut the wave front at *E*. Then *E* is the pole of the wave front with respect to the point *P*. The intensity at *P* depends on the number of half period elements in *EA*. If *EA* contains odd number of half period elements, there is maximum intensity at *P*. On the other hand if there are even number of half period elements in *EA* there is minimum intensity at *P*. For package of odd number of elements in *EA*, it is required that path difference $(AP-EP)$ be odd multiple of $\lambda/2$ and for even number of half period elements, the same path difference must be any integer of λ. Suppose $SA=a$, $AO=b$.

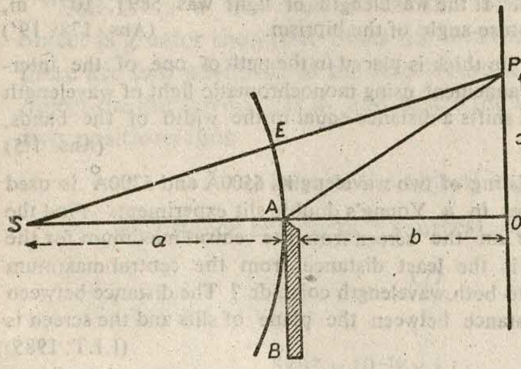

Fig. 8-1

Now, $AP=(b^2+x^2)^{\frac{1}{2}}=b\left(1+\frac{x^2}{b^2}\right)^{\frac{1}{2}}=b\left(1+\frac{1}{2}\frac{x^2}{b^2}\right)$ expanding and neglecting higher powers of x^2/b^2 as $x<<b$.

or
$$AP=b+\frac{x^2}{2b}.$$

Similarly,
$$SP=(a+b)+\frac{x^2}{2(a+b)}.$$

Now,
$$EP=SP-SE=(a+b)+\frac{x^2}{2(a+b)}-a=b+\frac{x^2}{2(a+b)}.$$

$\therefore$ Path difference
$$(AP-EP)=\left(b+\frac{x^2}{2b}\right)-\left\{b+\frac{x^2}{2(a+b)}\right\}$$
$$=b+\frac{x^2}{2b}-b-\frac{x^2}{2(a+b)}$$
$$=\frac{x^2a}{2b(a+b)}.$$

There is maxima at P if

$$\frac{x^2a}{2b(a+b)}=(2n-1)\frac{\lambda}{2}, \text{ where } n=1, 2, 3, ..$$

Writing x_n for distance of nth maxima, we have

$$\frac{x_n{}^2a}{2b(a+b)}=(2n-1)\frac{\lambda}{2}$$

or
$$x_n=\sqrt{\frac{(2n-1)b(a+b)\lambda}{a}}, \text{ where } n=1, 2, 3, 4, ..$$

There is minima at P if

$$\frac{x^2a}{2b(a+b)}=n\lambda.$$

Writing x_n for nth minima,

$$x_n=\sqrt{\frac{2nb(a+b)\lambda}{a}}.$$

3. Diffraction through a narrow slit : Suppose S is a line source and AB is a narrow slit. Join S with A and B and produce to cut the screen at A' and B'. Then A' and B' define the beginning of geometrical shadow.

Suppose, $e=AB$, width of the slit and P is any point in the geometrical shadow at a distance x from the centre of geometrical anti-shadow.

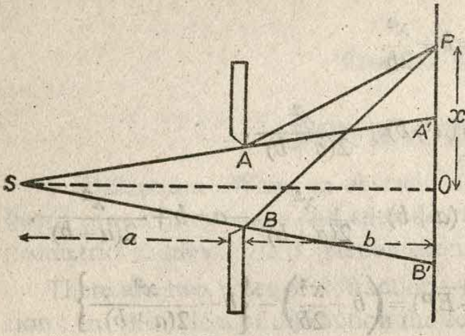

Fig. 8-2

Then, $AP^2 = b^2 + \left(x - \frac{e}{2}\right)^2$

and $BP^2 = b^2 + \left(x + \frac{e}{2}\right)^2$

$\therefore \quad BP^2 - AP^2 = 2xe$

or $BP - AP = \frac{2xe}{BP + AP}$

$= \frac{2xe}{2b} = \frac{xe}{b}$.

$(\because\ BP + AP \simeq 2b)$

For maximum at P, path difference $= (2n-1)\frac{\lambda}{2}$.

$$\therefore \qquad \frac{xe}{b} = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, 4, \ldots.$$

oa
$$x = \frac{(2n-1)b\lambda}{2e}.$$

Writing x_n for nth bright fringe $x_n = \frac{(2n-1)b\lambda}{2e}$.

For minimum at P, path difference $= n\lambda$. $\therefore \quad \frac{xe}{b} = n\lambda$.

Writing x_n for nth minima, $x_n = \frac{nb\lambda}{e}$.

4. Diffraction at Circular hole : Let us consider intensity only at a point P on the axis of the hole. The intensity at P depends on the number of half period zones contained in the hole. Intensity is minimum if it contains even number of half period zones, that is, path difference $(AP - OP)$, or $(SA + AP - SP)$ is any integer of λ. Suppose r = radius of hole; $OP = b$, distance of screen and $SO = a$, distance of source from hole.

$\therefore$ There is minimum intensity at P if

$$(\sqrt{a^2 + r^2} + \sqrt{b^2 + r^2}) - (a + b) = n\lambda$$

or
$$a\left(1 + \frac{r^2}{2a^2}\right) + b\left(1 + \frac{r^2}{2b^2}\right) - (a + b) = n\lambda,$$

expanding and neglecting higher powers of $\frac{r}{a}$ as $r \ll a$

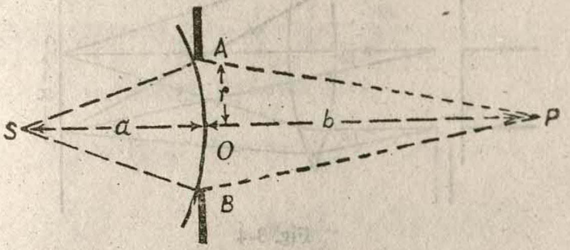

Fig. 8-3

or $$a+\frac{r}{2a}+b+\frac{r^2}{2b}-a-b=n\lambda$$

or $\frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)=n\lambda$, or $\frac{1}{a}+\frac{1}{b}=\frac{2n\lambda}{r^2}$, where $n=1, 2, 3\ldots.$

Similarly there is maxima at P if

$$\frac{1}{a}+\frac{1}{b}=\frac{(2n-1)\lambda}{r^2}, \text{ where } n=1, 2, 3\ldots.$$

5. Fraunhofer diffraction through Narrow slit (single slit) : When a parallel beam of monochromatic light is passed through a narrow slit, the rays are diffracted. On collecting the diffracted rays by a lens they are focussed at different points at the focal plane of the lens.

If e = width of the slit,

θ = angle made by the diffracted rays with the normal to the slit, then the minimum intensity will occur in the directions given by

$$e \sin\theta = n\lambda, \text{ where } n=1, 2, 3,\ldots.$$

6. Fraunhofer diffraction at circular aperture : Exactly in the same way we have minima in the directions given by $d \sin\theta = n\lambda$,

where d = diameter of aperture and $n=1, 2, 3,\ldots.$

and for maxima $$d \sin\theta=(2n+1)\frac{\lambda}{2}.$$

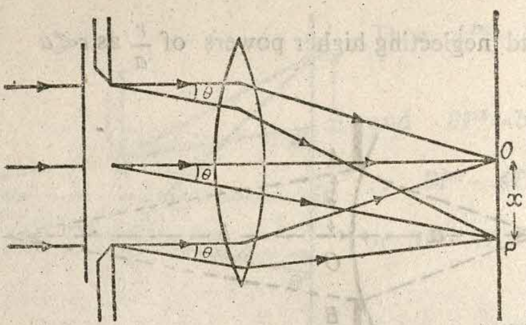

Fig. 8-4

EXAMPLES

Ex. 1. *Light from a narrow slit falls on a st. edge at a distance* ·5 m *from the slit. If the diffracted pattern is observed on a screen* ·75 m *away from the edge, calculate the distance between the first and fourth dark band.* (λ = 6000 A°)

Sol. We have, $x_n=\sqrt{\frac{2n\lambda b(a+b)}{a}}$.

When $n=1$, $x_1=\sqrt{\frac{2\lambda b(a+b)}{a}}$,

when $n=4$, $x_4=\sqrt{\frac{2\times 4\lambda b(a+b)}{a}}$.

$\therefore \quad x_4-x_1=\sqrt{\frac{\lambda b(a+b)}{a}}\,(\sqrt{8}-\sqrt{2})$.

Here, $\lambda=6000\ \text{A}^\circ=6000\times10^{-10}$ m.

$$\therefore \quad x_4-x_1=\sqrt{\frac{6000\times10^{-10}\times\cdot75(\cdot75+\cdot5)}{\cdot5}}\,(2\cdot828-1\cdot414)$$

$$=\sqrt{\frac{6\times\cdot75\times1\cdot25\times10^{-7}}{\cdot5}}\,(1\cdot414)$$

$$=\sqrt{6\times15\times125\times1\cdot414^2\times10^{-10}}$$

$$=2\cdot25\times10^{-6}\ \text{m}.$$ **Ans.**

Ex. 2. *A strong parallel beam of monochromatic light is incident on a thin plate having a small circular hole of diameter* 10^{-3} m. *If the screen be moved through a distance of* ·125 m *from the first* *position when the centre is black to the second similar position, what is the wavelength of light used ?*

Sol. We have for minima $\frac{1}{a}+\frac{1}{b}=\frac{2n\lambda}{r^2}$.

Here $a=$ distance of source $=\infty$.

$$\therefore \quad \frac{1}{b}=\frac{2n\lambda}{r^2}, \quad \text{or} \quad b=\frac{r^2}{2n\lambda}.$$

Put $n=1$, $$b=\frac{r^2}{2\lambda}.$$

Put $n=2$, $$b'=\frac{r^2}{4\lambda}.$$

$$\therefore \quad b-b'=\frac{r^2}{2\lambda}-\frac{r^2}{4\lambda}=\frac{r^2}{4\lambda}.$$

$$\therefore \quad \cdot 125=\frac{(10^{-3}/2)^2}{4\lambda}, \text{ or } \lambda=\frac{10^{-6}}{16\times \cdot 125}$$

or $$\lambda=5\times 10^{-7}\text{ m}=5000\times 10^{-10}\text{ m}=5000\text{ A}^\circ.$$ **Ans.**

$(\because \;\; 10^{-10}\text{ m}=1\text{ A}^\circ)$

Ex. 3. *A source of monochromatic light of wavelength* ·6 μm *is placed at a distance* ·3 m *from a narrow slit, the distance between consecutive maxima is found to be* ·15 mm *on a screen* ·5 m *from the slit. Calculate width of the slit.*

Sol. This is a case of Fresnel diffraction at a narrow slit. The distance of nth bright fringe from the centre of antishadow is given by

$$x_n=\frac{(2n-1)b\lambda}{2e}.$$

$$\therefore \quad x_{n-1}=\frac{(2n-3)b\lambda}{2e}.$$

$$\therefore \quad x_n-x_{n-1}=\frac{b\lambda}{e}.$$ This is independent of n.

Hence fringes are of constant width. Let this constant be β.

Then, β (fringe width) $=\frac{b\lambda}{e}$,

$$\therefore \quad e=\frac{b\lambda}{\beta}=\frac{\cdot 5\times \cdot 6\times 10^{-6}}{\cdot 15}=2\times 10^{-6}\text{ m}.$$ **Ans.**

Ex. 4. *A narrow slit of width* ·025 mm *is illuminated by a parallel beam of light. The diffraction pattern is observed through a telescope. It is found that to reach the first minimum the telescope has to be rotated through* 1°24′ *from the direction of direct ray. Calculate the wavelength of light used.*

Sol. This is a case of Fraunhofer diffraction at a narrow slit.

We have, $e \sin\theta = n\lambda$, where $n = 1, 2, 3 \ldots$ for first, second, third.. minima.

$\therefore \quad \cdot 025 \times 10^{-3} \sin 1^\circ 24' = 1 \times \lambda$

or $\quad \lambda = \cdot 025 \times 10^{-3} \times \cdot 0244 = 6100 \times 10^{-10}\ \text{m} = 6100\ \text{A}^\circ.$ **Ans.**

$(\because \ 1\ \text{A}^\circ = 10^{-10}\ \text{m}).$

Ex. 5. *A strong parallel beam of light* ($\lambda = 5890$ A°) *is incident on a thin plate having a small circular aperture. A screen placed very close to the plate in the beginning is moved slowly away from it. When it is at a distance of* ·3 m *the centre appears to be dark for the first time. Calculate radius of the hole.*

Sol. This is a case of Fresnel diffraction at a circular aperture.

We have for minima $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{2n\lambda}{r}$.

Here, $\quad a$ = distance of the source from the hole $= \infty$, $b = \cdot 3$ m,
$\lambda = 5890\ \text{A}^\circ = 5893 \times 10^{-10}\ \text{m},\ n = 1.$

$$\therefore \quad \frac{1}{\cdot 3} = \frac{2 \times 1 \times 5890 \times 10^{-10}}{r^2}$$

or $\quad r = \sqrt{\cdot 6 \times 5890 \times 10^{-10}} = 5 \cdot 94 \times 10^{-3}\ \text{m}.$ **Ans.**

EXERCISES

1. A narrow slit illuminated by light of wavelength ·64 μ m is placed at a distance of 3 m from a straight edge. If the distance between straigth edge and the screen is 6 m, calculate the distance between the first and fourth dark band.
(Ans. $4 \cdot 8 \times 10^{-3}$ m**)**

2. Light of wavelength 6000 A° falls normally on a narrow circular aperture of radius 9×10^{-4} m. At what distance along the axis will the first maximnm be observed ? **(Ans.** 1·35 m**)**

3. A circular aperture of diameter $1 \cdot 2 \times 10^{-3}$ m is illuminated by plane waves of monochromatic light and the emergent light is allowed to fall on a screen. As the screen is moved gradually towards the aperture it is found that when the screen is at ·3 m a dark spot appears at the centre of the patch of light. Find the wavelength of light. **(Ans.** 6000 A°**)**

4. Light from a narrow source falls on a razor edge at a distance 20 cm from the source. Calculate the separation between first and fourth maxima on a screen held at 30 cm away from the edge. (Wavelength of light=6000 A°)

(Ans. $11{\cdot}04\times10^{-4}$ m)

5. A screen is placed at a distance of 1 m from a narrow slit. Light of wavelength ·5 micron is falling normally on the slit. If the first minima of the diffraction pattern is situated at 5 mm from the central maximum, find the width of the slit.

(*Hint* : It is a case of Fraunhofer diffraction at a narrow slit.)

(Ans. 10^{-4} m)

6. A lens whose focal length is 40 cm forms a Fraunhofer dffraction pattern of a slit ·3 mm wide. The distance of the first dark band from the direct one is ·785 mm. Calculate wavelength of light. (Ans. 5884 A°)

7. Light from a source falls on a straight edge at a distance 50 cm and the distance between first and third maxima is found to be 1·197 mm on a screen at a distance 75 cm from the edge. Calculate wavelength of light falling on the slit.

[*Hint* : $x_n=\sqrt{\dfrac{(2n-1)\lambda b(a+b)}{a}}$ where $n=1,\ 2,\ 3,$..............for first, second, third,.........maxima.]

(Ans. 5002 A°)

8. A circular aperture of radius ·5 mm is illuminated by plane waves of monochromatic light and the emergent light is collected by a lens of focal length 30 cm. The distance between two dark bands is ·18 mm. What is the wavelength of light ?

[*Hint* : This is a case of Fraunhofer diffraction at circular aperture. We have $d\sin\theta=n\lambda$ where $n=1,\ 2,\ 3,$......for first, second, third,...dark bands.

Since θ is small, $\sin\theta_n=\theta_n=\dfrac{x_n}{f}$.

$\therefore\ d.\dfrac{x_n}{f}=n\lambda$ or $x_n=\dfrac{n\lambda f}{d}$.]

(Ans. 6000 A°)

□□□

CHAPTER 9

POLARISATION

1. Polarisation : When light waves acquire the property of onesidedness due to reflection, refraction or transmission through a crystal, it is said to be polarised and this phenomenon of acquiring onesidedness in respect of intensity (and hence in vibration) is called Polarisation.

2. (i) Plane polarised light : When onesidedness is of the kind that vibration takes place only in one direction perpendicular to the direction of wave propagation, light is said to be plane polarised.

(ii) Circularly polarised light : When onesidedness is of the kind that there is circular motion of the ether particles in planes perpendicular to the direction of wave-motion, light is said to be circularly polarised.

(iii) Elliptically polarised light : When onesidedness is of the kind that there is elliptical motion of ether particles in planes perpendicular to the direction of wave-motion, light is said to be elliptically polarised.

3. Brewster's law : When light is incident on the surface of a transparent medium at such an angle i that both reflected and refracted light are completely plane polarised and they are perpendicular to each other we have

$\mu = \tan i.$ This is known as Brewster's law

and that particular angle of incidence is called polarising angle.

4. Quarter wave plate and Half wave plate : A double refracting crystal cut of such thickness that it introduces path difference of $\lambda/4$ or phase difference of $\pi/2$ between ordinary and extraordinary wave is called quarter wave plate. Exactly in the same way a crystal cut introducing a path difference of $\lambda/2$ is called half wave plate.

Let t = thickness of the quarter plate,

v_o = index of refraction for ordinary light,

μ_e = index of refraction for extraordinary light.

Then path difference $=\mu_o t\sim\mu_e t$.

$$\therefore \qquad \frac{\lambda}{4}=(\mu_o\sim\mu_e)t$$

or
$$t=\frac{\lambda}{4(\mu_o\sim\mu_e)}.$$

Similarly, t (thickness of the half wave plate) $=\dfrac{\lambda}{2(\mu_o\sim\mu_e)}$.

EXAMPLES

Ex. 1. *At what angle a beam of light be incident on the surface of water having index of refraction* 1·33 *for the given light so that the reflected light may be plane polarised ?*

Sol. Reflected light is plane polarised when light is incident at polarising angle given by Brewster's law,

$$\tan i=\mu$$

or
$$i=\tan^{-1}\mu=\tan^{-1} 1{\cdot}33=53°4'.$$
Ans.

Ex. 2. *Calculate the phase difference introduced by a quartz plate of thickness* 20 *micron between ordinary and extraordinary light. Given that* $\mu_o=1{\cdot}5443$, $\mu_e=1{\cdot}5533$ *and* $\lambda=6000A°$. *Which light travels faster in quartz ?*

Sol. Path difference introduced $=(\mu_o\sim\mu_e)t$

$$=(1{\cdot}5533-1{\cdot}5443)\times 20\times 10^{-6}$$

$$(\because \ 1 \text{ micron}=10^{-6}\text{ m})$$

$$={\cdot}0090\times 20\times 10^{-6}.$$

Now, Path difference $\lambda\equiv$ Phase difference 2π

" " $1\equiv$ " " $2\pi/\lambda$.

$$\therefore \text{ Phase difference introduced}={\cdot}0090\times 20\times 10^{-6}\times\frac{2\pi}{6000\times 10^{-10}}$$

$$=\frac{4\pi\times 90}{6}\times 10^{-3}={\cdot}06\pi.$$
Ans.

$$\left(\because \text{ velocity}\propto\frac{1}{\mu}\right)$$

As $\mu_e>\mu_o$, $c_e<c_o$.

Thus ordinary light travels faster in quartz. This is why quartz is called a positive crystal.

Ex. 3. *Calculate the thickness of a quarter wave plate for a light of wavelength* $4000 A^\circ$ *for which* $\mu_o = 1{\cdot}5632$ *and* $\mu_e = 1{\cdot}5541$.

Sol. Path difference introduced $= (\mu_o \sim \mu_e)t$.

For quarter wave plate this must be $\frac{\lambda}{4}$.

$$\therefore \qquad \frac{\lambda}{4} = (\mu_o \sim \mu_e)t$$

$$\text{or} \qquad t = \frac{\lambda}{4(\mu_o \sim \mu_e)} = \frac{4000 \times 10^{-10}}{4(1{\cdot}5632 - 1{\cdot}5541)}$$

$$\text{or} \qquad t = \frac{4 \times 10^{-7}}{4 \times {\cdot}0091} = 1{\cdot}099 \times 10^{-8} \text{ m.}$$ **Ans.**

Ex. 4. *Show that when a ray of light is incident on the surface of a transparent medium at polarising angle, reflected ray and refracted ray are at right angles to each other.*

Sol. When a ray is incident at polarising angle we have

$$\mu = \tan i. \qquad \text{Brewster's law.}$$

$$\text{Also,} \qquad \mu = \frac{\sin i}{\sin r}. \qquad \text{Law of refraction.}$$

$$\therefore \qquad \tan i = \frac{\sin i}{\sin r}, \text{ or } \frac{\sin i}{\cos i} = \frac{\sin i}{\sin r}$$

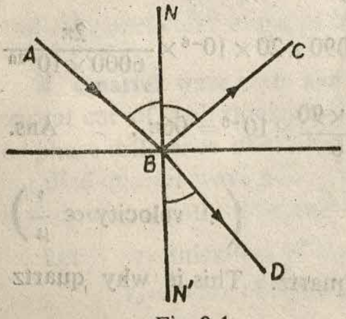

Fig. 9-1

$$\text{or} \qquad \cos i = \sin r = \cos(90^\circ - r)$$

$$\text{or} \qquad i = 90^\circ - r.$$

$$\therefore \qquad i + r = 90^\circ.$$

Now, included angle between reflected ray (BC) and refracted ray (BD) $= \angle CBD$

$$= 180 - (\angle NBC + \angle N'BD)$$

$$= 180 - (i + r)$$

$$= 180 - 90 = 90^\circ.$$ **Proved.**

EXERCISES

1. The index of refraction of glass, for sodium light is 1·63. At what angle a beam of sodium light must fall on the surface of a glass slab so as to be plane polarised by reflection ? (Ans. 58° 28′)

2. For a slab of flint glass, the angle of polarisation is found to be 57° 24′. Calculate the refractive index of flint glass. (Ans. 1·564)

3. A ray of light is incident on the surface of a glass plate of index of refraction 1·55 at the polarising angle. Calculate the angle of refraction. (Ans. 32° 49′)

4. Calculate the thickness of a quartz quarter wave-plate for sodium light $\lambda = 5893$ A° given that the index of refraction of quartz for ordinary and extra-ordinary rays are 1·5442 and 1·5533 respectively. (Ans. $1{\cdot}62 \times 10^{-5}$ m)

5. Calculate the phase difference introduced by a calcite crystal plate of thickness 30 micron between ordinary and extraordinary light of wavelength 6000A°. Given that $\mu_o = 1{\cdot}554$ and $\mu_e = 1{\cdot}547$. (Ans. ·7π radian)

6. You have to design a half wave-plate of quartz crystal for light of wave-length 5800A°. Calculate the thickness of the plate required for the purpose if $\mu_e = 1{\cdot}5508$ and $\mu_o = 1{\cdot}5418$. (Ans. $3{\cdot}2 \times 10^{-5}$ m)

7. A sheet of cellophane is a half wave-plate for light of wavelength 4000A°. If the index of refraction does not change with wavelength, how the sheet would behave with respect to light of wavelength 8000A° ?

(Ans. It will behave as a quarter wave-plate)

□□□

CHAPTER 10

PHOTOMETRY

1. Luminous intensity of a source : The amount of light emitted by a source per second in all directions through unit solid angle is called luminous intensity of the source.

The unit of luminous intensity is *candela* (cd) (formerly candle power).

2. Illuminance at a point of a surface : The amount of light falling in 1 second normally on unit area surrounding a point is called Illuminance (formerly Intensity of illumination) of the surface at that point. The unit of illuminance is *lux* (lx) (formerly metre candle).

3. Law of photometry : If a surface is illuminated by a source of luminous intensity P from a distance r, then illuminance of the surface is given by

$I=\frac{P\cos}{r^2}i$ where i is the angle of incidence of light on the surface.

If light falls normally on the surface, then $i=0$ and

$$I=\frac{P}{r^2}\text{ lux (lx).}$$

4. Principle of photometers : Photometers are devices to constant illuminating powers of sources. The sources of light are placed at certain distance from a surface (a grease spot in Bunsen's photometer, a white screen in Rumford's photometer) and then distance of one of them is adjusted till the two produce *equal* illuminance on the same surface. If P_1 and P_2 are the illuminating powers of the sources and r_1 and r_2 are their distances when they produce equal illuminance, we have $\frac{P_1}{r_1^2}=\frac{P_2}{r_2^2}$.

5. Luminance or Brightness : The amount of light emitted by unit area of an extended source along the normal in unit time is called its luminance. If L is the luminance of the surface of an extended source, its luminous intensity in a particular direction is given by $P=\Sigma L\ \Delta S \cos\theta$ where θ is the angle between normal to ΔS and that particular direction.

$$B \text{ (brightness)}=\pi L.$$

6. **Luminous flux :** If I is the energy current density, i.e., intensity of the light wave, then $\vec{I}.\ \Delta\vec{S}$ is called the luminous flux through ΔS. Thus $\Delta\varphi=\vec{I}.\Delta\vec{S}$ and $\varphi=\Sigma\ \vec{I}.\Delta\vec{S}=\Sigma I.\Delta S \cos\theta$.

Unit, Lumen (lm).

EXAMPLES

Ex. 1. *Two 16 candela lamps are placed on the same side of a screen at distances 2 m and 5 m from it. Calculate the distance at which a single 32 candela lamp must be placed in order to give the same intensity of illumination.*

Sol. The illuminance on the screen due to the two lamps,

$$=\frac{16}{2^2}+\frac{16}{5^2}=4+{\cdot}64=4{\cdot}64 \text{ lux.}$$

Let x be the required distance of 32 candela lamp.

Then $$4{\cdot}64=\frac{32}{x^2} \text{ or } x=\sqrt{\frac{32}{4{\cdot}64}}=2{\cdot}63 \text{ m.}$$ **Ans.**

Ex. 2. *Two lamps balance on the shadow photometer at distances of 0·60 m and 0·42 m from the screen. The stronger is then covered with a glass shade which transmits 80% of the incident light. How far must the other lamp be displaced in order to restore balance ?*

Sol. Let P_1 and P_2 be the luminous intensities of the sources.

Then $$\frac{P_1}{{\cdot}60^2}=\frac{P_2}{{\cdot}42^2} \text{ or } \frac{P_1}{P_2}=\frac{{\cdot}60^2}{{\cdot}42^2}.$$

Obviously $$P_1>P_2.$$

When P_1 is covered by the glass shade, its luminous intensity is reduced to $\frac{80}{100}P_1$ or ${\cdot}8P_1$.

Let x be the distance of P_2 when the balance is restored.

Then $$\frac{{\cdot}8P_1}{{\cdot}60^2}=\frac{P_2}{x^2} \text{ or } \frac{P_1}{P_2}=\frac{{\cdot}60^2}{{\cdot}8x^2}$$

$$\therefore\ \frac{\cdot60^2}{\cdot42^2}=\frac{\cdot60^2}{\cdot8x^2}\quad\text{or}\quad x^2=\frac{\cdot42^2}{\cdot8}\quad\text{or}\quad x=\cdot47\text{ m.}$$

∴ The other lamp has to be displaced by $\cdot47-\cdot42=\cdot05$ m $=5$ cm away from the screen. **Ans.**

Ex. 3. *A lamp of* 120 *candela hangs at a height of* 4 *m exactly above the centre of a circular table of diameter* 6 *m. Calculate the illuminance at the centre and at the edge of the table.*

Sol. In general, $I=\frac{P\cos i}{r^2}$ where i is the angle of incidence.

For the centre of the table $i=0°$.

$$\therefore\quad I=\frac{120\cos 0°}{4^2}=\frac{120}{16}=7\cdot5\text{ lux.}\qquad\textbf{Ans.}$$

For the edge, $\cos i=\frac{4}{\sqrt{4^2+3^2}}=\frac{4}{5}$.

$$\therefore\quad I=\frac{120\times4/5}{(\sqrt{4^2+3^2})^2}=\frac{120\times4}{5^3}=3\cdot84\text{ lux.}\qquad\textbf{Ans.}$$

Ex. 4. *A convex lens of diameter D and focal length f projects a small object at a great distance from the lens on to a screen. Calculate the illuminance of the image in terms of luminance L of the object, diameter D and focal length f.*

Sol. When the object is at a large distance, the image is formed at the focal plane of the lens. The magnification is $m=\frac{v}{u}=\frac{f}{u}=\frac{r}{R}$ where R is the radius of the object (considered as a small disc) and r is the radius of the image. The object will behave like a point object of luminous intensity P along the axis given by

$$P=\Sigma L\triangle S\cos 0°=L\times\pi R^2.$$

By definition this is the amount of luminous flux through unit solid angle. Therefore, luminous flux through the lens is $\varphi=P\times\frac{{}^{2}}{4u^2}$

$$\left(\because\ \text{solid angle}=\frac{\text{area}}{\text{distance}^2}=\frac{\pi D^2}{4u^2}\right).$$

This flux is concentrated on the area of the image.

$$\therefore\ \text{Illuminance of the image}=\frac{\varphi}{\pi r^2}=\frac{1}{4}\pi L\frac{D^2}{f^2}.\qquad\textbf{Ans.}$$

EXERCISES

(A)

1. An electric lamp hangs 3 m above one end of a table 4 m long. Compare intensity of illumination at the two ends of the table. (Ans. 4·6 : 1)

2. Two sources of lights of illuminating powers in the ratio 1 : 2 are placed on a line 2 metre apart. At what point is the space between them equally illuminated ? (Ans. 1·17 m from the stronger source)

3. In the grease spot photometer a lamp having a dirty chimney is found to balance an electric lamp of constant illuminating power at a distance of ·1 m from the spot. When the chimney is cleaned, the electric lamp has to be shifted by 2 cm to obtain a balance. Calculate the percentage of light absorbed by the chimney. (Ans. 36%)

4. A photographic print is found satisfactory when the exposure was for 16 s at a distance of 20 cm from a 25 watt lamp. At what distance must the same paper be held from a 50 watt lamp in order that an exposure of 25 sec will give the same result ? (Ans. 35·4 cm)

5. A lamp of luminous intensity 50 candela is placed at a distance of 1 m from a wall. What is the illuminance of the wall directly in front of the lamp ? If another lamp of 100 cd is placed on the same line at a distance 125 cm, what is the illuminance now ? (Ans. 50 lux; 114 lux)

6. A book placed at a distance of 30 cm from a candle can be read. What is the maximum distance at which lamp of 32 candela can be placed for reading the book ? (Ans. 1·697 m)

7. A screen is illuminated by a lamp which is one metre away. A sheet of glass is placed between the screen and the lamp which has to be moved 10 cm nearer to maintain the same intensity of illumination as before. What fraction of light is absorbed by sheet of glass ? (Ans. 19%)

8. A lamp of luminous intensity 100 cd is held at a distance of 2 m from a vertical wall. A plane mirror is fixed on the opposite wall which is at a distance of 4 m from the lamp. Calculate the illuminance on the wall opposite to the mirror at a point on the horizontal line through the lamp. Also calculate the luminous flux on a small area ·02 m^2 surrounding the point. (Ans. 26 lx, ·52 lm)

(B)

9. An infinite number of lamps, each of luminous intensity 100 cd are placed 1 m apart in a horizontal line perpendicular to a wall, the nearest lamp being 1 m from the wall. Calculate the illuminance on the wall at a point on the wall lying on the line through the lamps.

Given that $\sum_{0}^{\infty} 1/n^2 = \pi^2/6$. (Ans. 164·5 lux)

10. At what height should a lamp be hung above the centre of a round table of radius R to obtain the maximum illumination on its edges ?

$$\left(\text{Ans. } h = \frac{R}{\sqrt{2}}\right)$$

11. A cylindrical hall of diameter D and height h is illuminated by a lamp fixed at the centre of the ceiling. Compare the minimum illumination of the wall and of the floor. $\left(\text{Ans. } \frac{D}{2h}\right)$

12. A point light source illuminates a screen, the maximum illuminance being I_0. How will the illuminance of this point change if a large plane mirror of 100% reflectance is placed behind the source at half the distance between the source and the screen ? $\left(\text{Ans. } \frac{5}{4}I_0\right)$

13. A small lamp having the form of a uniformly luminous sphere of radius $R=6$ cm is suspended at a height $h=3$ m above the floor. The luminance of the lamp is equal to $L=2\times10^4$ cd/m². Find the illuminance of the floor below the lamp. (Ans. 25 lx)

(*Hint* : $P=\Sigma L\Delta S\cos\theta=L\times\pi R^2$)

14. Street lamps of 10 cm diameter and $1{\cdot}8\times10^5$ cd/m² luminance are suspended at a height of 12 m, the distance between them being 40 m. Find the illuminance under each lamp and mid-way between them.

(Ans. 10·38 lx; 2·7 lx)

□□□

MAGNETISM

CHAPTER 1

INTRODUCTORY NOTES

1. Introduction : Usually Coulombs law of force between two magnetic poles is taken as the starting point from which one develops all 'magnetic' terms and equations. This law as basis has been objected mainly due to two reasons, *firstly* that the very concept of so called magnetic, 'pole, is not clear and its realisation in isolated state is impossible; *secondly*, that it is now an established fact that the magnetic effects of magnetised matter are due to its revolving or spinning electrons. Hence it is quite reasonable that we turn to electricity to base the magnetic terms and equations on the general law of force between currents or moving charges.

2. Current element and a moving charged particle : A short conductor carrying current is termed as current element. If $\triangle l$ is the length of conductor and I is the current through it then we say that current element is $I\triangle l$ ampere metre (Am). A moving positive charge q_0 of velocity v is as good as a current element of 'q_0v' ampere metre (Am).

3. Ampere's law of current-current interaction :

(i) *Like parallel currents attract and unlike parallel currents repel.*

(ii) *The force of attraction between two like parallel current elements in a vacuum is proportional to the product of current elements, inversely proportional to the square of the distance between them and proportional to the 'sine' of the angle between either element and the line joining them.* The direction of the force exerted by the first on the second is perpendicular to the second and directed towards the line of extension of the first and that of the force exerted on the first by the second is perpendicular to the first and directed towards the line of extension of the second.

Suppose $I_1\triangle l_1$ and $I_2\triangle l_2$ are two *parallel* current elements at a distance r from each other.

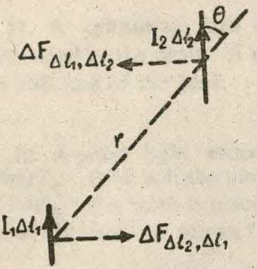
θ
$I_2 \Delta l_2$
$\Delta F_{\Delta l_1, \Delta l_2}$
r
$I_1 \Delta l_1$
$\Delta F_{\Delta l_2, \Delta l_1}$

Fig. 1-1

Then $\triangle F_0 \propto (I_1 \triangle l_1)(I_2 \triangle l_2)$

$$\propto \frac{1}{r^2}$$

$$\propto \sin\theta$$

or $$\triangle F_0 = \frac{\mu_0}{4\pi} \cdot \frac{(I_1 \triangle l_1)(I_2 \triangle l_2) \sin\theta}{r^2} \quad .. \quad (1)$$

where μ_0 is a universal constant called 'universal magnetic constant'. In SI it is set to $4\pi \times 10^{-7}$ and the unit of current, namely, ampere is defined from this formula. It is also called 'permeability of vacuum' or air. $\frac{1}{4\pi}$ is introduced in (1) just to 'rationalise' it.

When the same current elements are submerged in an *extended homogeneous* material medium, the force between them is changed. The ratio of the force between two given current elements in the material medium to the force between the same two elements in the same position of the elements when the material medium is replaced by vacuum or air is called the relative permeability of the medium. It is denoted by μ_r.

Thus, $$\mu_r = \frac{\triangle F}{\triangle F_0}, \quad \text{or} \quad \triangle F = \mu_r \triangle F_0$$

or $$\triangle F = \frac{\mu_0 \mu_r}{4\pi} \cdot \frac{(I_1 \triangle l_1)(I_2 \triangle l_2) \sin\theta}{r^2} \quad .. \quad (2)$$

or $$\triangle F = \frac{\mu}{4\pi} \cdot \frac{(I_1 \triangle l_1)(I_2 \triangle l_2) \sin\theta}{r^2} \quad .. \quad (2a)$$

where $\mu = \mu_0 \mu_r$, or $\mu_r = \frac{\mu}{\mu_0}$. μ_r is thus ratio of the permeability of the medium to the permeability of vacuum or air. This is why it is called the relative permeability and μ is called the absolute permeability of the medium. μ_r has no unit but μ has definite unit. It is henry per metre (Hm^{-1}).

4. Magnetic field and its unit : A magnetic field of strength B is said to exist at a point if a current element placed at that point or a moving positive charge passing through that point experiences sideways force given in magnitude by the formula

$$\triangle F = I \triangle l\, B \sin\beta, \quad \text{or} \quad \triangle F = q_0 v B \sin\beta$$

where β is the angle between $\overrightarrow{\triangle l}$ and $\vec{B}$ or $\vec{v}$ and $\vec{B}$ and in direction by the *direction of translational of a screw perpendicular to the element or the direction of the moving positive charge and the magnetic field and rotating from the former to the latter through the smaller angle.*

In vector notation, $\triangle\vec{F}=I\triangle\vec{l}\times\vec{B}$ or $q_0\vec{v}\times\vec{B}$.

The unit of magnetic field in SI is newton per ampere metre (N/Am) or tesla (T). **Tesla** is in common use.

Hence $$\triangle B=\frac{\triangle F}{I\triangle l\sin\beta}\text{ tesla} \qquad .. \quad (3).$$

5. **Biot-Savart-Laplace's law (BSL law)** : This law derivable from Ampere's law of current-current interaction gives the magnitude and the direction of the magnetic field due to a current element. According to this law the magnetic field due to a current element in vacuum is given by

$$\triangle B_0=\frac{\mu_0}{4\pi}\cdot\frac{I\triangle l\sin\theta}{r^2} \qquad .. \quad (4)$$

in the direction of a screw perpendicular to $\triangle\vec{l}$ and $\vec{r}$ and rotating from $\triangle\vec{l}$ to $\vec{r}$ through the smaller angle.

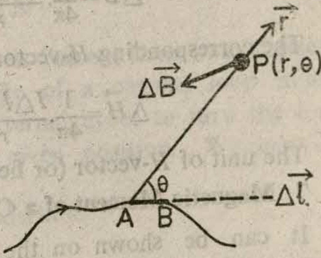

Fig. 1-2

Vectorially, $\triangle\vec{B}_0=\frac{\mu_0}{4\pi}\cdot\frac{I\triangle\vec{l}\times\vec{r}}{r^2}$, or $\frac{\mu_0}{4\pi}\frac{I\triangle\vec{l}\times\hat{r}}{r^2}$,

where $\hat{r}$ is the unit vector along $\vec{r}$.

6. **H-vector of a magnetic field** : The maximum force experienced by a unit current element is defined as the *B*-vector of the magnetic field. We see from Ampere's law that the force of interaction between two current elements depends not only on the lengths and strength of current elements but also on the medium between the two. Hence the force experienced by a test current element or moving test charge will not only depend on the 'prime' current element but also on the medium. The medium affects the force on the test element by reshuffling of its atomic current loops. The phenomenon of reshuffling of atomic current loops due to the presence of the 'prime current' is called magnetic induction. This is why *B*-vector is also called the *magnetic induction field*. A vector multiplied by scalar is also vector of the same direction but of different magnitude and dimension. Thus we may introduce any number of auxiliary of vectors by introducing a scalar with the *B*-vector. The auxiliary vector obtained by

multiplying B-vector by $\frac{1}{\mu_0}$ for vacuum and $\frac{1}{\mu}$ for any other medium is called H-vector or intensity vector of the field. *The characteristic property of this vector is that it depends solely on the prime element* and *not* on the medium. The second characteristic property of this vector is that it is this vector which originates or terminates from *Magnetic Poles.*

Thus, by definition, $\vec{B}=\mu_0 \vec{H}$.

We have already seen that B-vector at a point due to a prime element $(I\triangle l)$ is given by

$$\triangle\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{I\triangle\vec{l}\times\vec{r}}{r^3} \quad \text{or} \quad \frac{\mu_0}{4\pi}\cdot\frac{I\triangle\vec{l}\times\hat{r}}{r^2}.$$

The corresponding H-vector is given by

$$\triangle\vec{H}=\frac{1}{4\pi}\cdot\frac{I\triangle\vec{l}\times\vec{r}}{r^3} \quad \text{or} \quad \frac{1}{4\pi}\cdot\frac{I\triangle\vec{l}\times\hat{r}}{r^2}. \qquad \text{..} \quad (5)$$

The unit of H-vector (or field) is *ampere per metre* (Am^{-1}).

7. Magnetic Moment of a Current Loop and its unit :

It can be shown on the basis of Ampere's law that when a current loop is kept in a magnetic field it experiences a torque given by

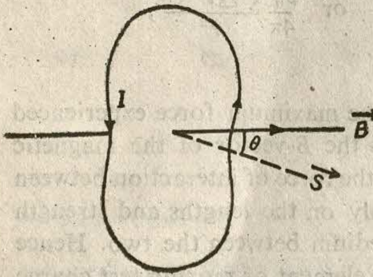

Fig. 1-3

$$\tau=ISB\sin\theta$$

where I=current through loop, S=area of loop, B=magnetic field in tesla and θ=angle between normal to loop and B. (See author's *Introductory Physics*, Part II, Ch II)

The quantity 'IS' which is obviously the torque experienced by the loop when its normal is at right angles to a unit uniform magnetic field is defined as the magnetic moment of the loop in Sommerfeld system of units. An alternative system due to Kenelly uses the relation

$$\tau=\frac{1}{\mu_0}(\mu_0 IS)B\sin\theta$$

and takes $(\mu_0 IS)$ as the magnetic moment of a loop. We, however, follow Sommerfeld system of units.

Thus magnetic moment of a current loop $(m) = IS$ ampere metre^2 .. (6).

We know that an area may be considered as a vector along the normal in the direction of traverse of the contour around the area by a screw. Hence magnetic moment of a loop is also a vector of magnitude 'IS' along the normal in the direction of traverse of the contour around the loop by a screw. Vectorially, therefore,

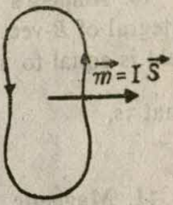

Fig. 1-4

$$\vec{m} \text{ (magnetic moment of a loop)} = I\vec{S}$$

and $$\vec{\tau} = I\vec{S} \times \vec{B} = \vec{m} \times \vec{B} \qquad (7).$$

8. Magnetic Potential Energy of a Current Loop in a Magnetic Field : The magnetic potential energy of a current loop in any position is the work done by an external agent to turn the loop from its zero-energy position to the given position. We take by convention $\theta = \frac{\pi}{2}$ as the zero-energy position.

$$\therefore \quad U = \int_{\pi/2}^{\theta} \tau d\theta = \int_{\pi/2}^{\theta} mB \sin\theta \, d\theta = -mB \cos\theta \text{ joule (J)} \quad .. \quad (8).$$

In vector notation this relation can be written as

$$U = -\vec{m}.\vec{B}.$$

Equation (8) provides us an alternative unit of magnetic moment. Here, $m = \frac{U}{B\cos\theta}$. U is in joule (J) and B is in tesla. Hence alternative unit of magnetic moment is joule per tesla (JT^{-1}).

9. Gauss's Flux Theorem : This theorem states that the magnetic induction flux across any closed surface drawn in a magnetic field is zero. That is, $\int_{\text{CS}} \vec{B}.d\vec{s} = 0$ where 'CS' stands for 'closed surface'. This theorem is of universal applicability. According to this theorem 'source' and 'sink' of B-vector occur in equal amount in any close space so that on the whole it contains neither 'source' nor 'sink' of B-vector.

10. Ampere's Circuital Law : This theorem states that the line integral of B-vector along any closed path (CP) drawn in a magnetic field is equal to μ_0 times the total current enclosed by that path.

That is, $$\int_{CP} \vec{B}.\ d\vec{l} = \mu_0 i.$$

11. Magnetic Charge or Pole : According to the above two fundamental laws of magnetism, the transverse ends (perpendicular to the direction of magnetisation of a magnetised body) are either source or sink of H-vector because the flux of H-vector across a closed surface enclosing the free ends is not zero. Just as electric positive and negative charges are 'source' and 'sink' of electric field vector, the H-vector must be due to similar magnetic charges situated at the free transverse ends of the magnetised body. These are called magnetic charges or poles. For details see author's *Introductory Physics*, Part II, Magnetism, Ch II. If M is the magnetisation, that is, algebraic sum of magnetic moments of electronic current loops per unit volume and S is the area of cross-section, then the pole strength of the magnetised body is

$$q_m = MS \text{ ampere metre (Am)} \quad \text{.. (9).}$$

EXAMPLES

Ex. 1. *A current element of strength* 2×10^{-4} *Am is at the corner A of a cube ABCDEFGH of side length* 10 *cm, the element lying along AB. Calculate the B-field and H-field of the element at the diagonally opposite corner G of the cube.*

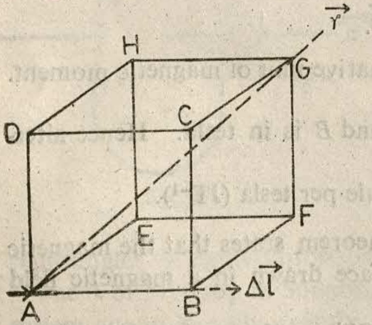

Fig. 1-5

Sol. Here, $r = \sqrt{10^2 + 10^2 + 10^2}$ $= 10\sqrt{3}$ cm $= 10\sqrt{3} \times 10^{-2}$ m.

The inclination of the diagonal with the edge (taken as x-axis) is given by, $\tan\theta = \dfrac{\sqrt{z^2 + y^2}}{x} = \sqrt{2}$

($\because$ here $x = y = z$).

$\therefore$ $\sin\theta = \sqrt{2/3} = \sqrt{\cdot 6667} = \cdot 8162$ (taken from log table)

$$\triangle B=\frac{\mu_0}{4\pi}\cdot\frac{I\triangle l\sin\theta}{r^2}=10^{-7}\frac{(2\times10^{-4})\times{\cdot}8162}{(10\sqrt{3}\times10^{-2})^2}$$

$$=\frac{2\times{\cdot}8162}{3}\times10^{-9}=5{\cdot}44\times10^{-10}\text{ tesla.}\quad\textbf{Ans.}$$

The direction is 'out of the plane' of the triangle ABG.

$$\triangle H=\frac{\triangle B}{\mu_0}=\frac{5{\cdot}44\times10^{-10}}{4\pi\times10^{-7}}=4{\cdot}33\times10^{-4}\text{ Am}^{-1}.\qquad\textbf{Ans.}$$

Ex. 2. *Calculate the mass of a charged particle which will have its horizontal velocity at right angles to the magnetic meridian unchanged. Charge of the particle*$=4{\cdot}9\times10^{-9}$ C. *Earth's horizontal intensity*$=36\times10^{-6}$ T *and velocity of particle*$=4\times10^4$ ms^{-1}.

Sol. When the magnetic force experienced by the particle will balance its weight, it will suffer no deflection and its velocity will remain unchanged.

$$F_m=q_0vB\sin\theta=\omega g;$$

$$\therefore\quad \omega=\frac{q_0vB\sin\theta}{g}=\frac{4{\cdot}9\times10^{-9}\times4\times10^4\times36\times10^{-6}.\sin 90^\circ}{9{\cdot}8}$$

or $$\omega=7{\cdot}2\times10^{-10}\text{ kg.}$$

Ex. 3. *In a hydrogen atom an electron carrying* $1{\cdot}6\times10^{-19}$ *coulomb* (C) *charge makes* 6×10^{15} *revolutions per second. What is the magnetic moment associated with this orbital motion of the electron ? (radius of the orbit*$={\cdot}5$A°).

Sol. I (current in the orbit)$=6\times10^{15}\times1{\cdot}6\times10^{-19}=9{\cdot}6\times10^{-4}$ ampere (A).

Therefore, magnetic moment $(m)=IS=I\pi r^2$

$$=9{\cdot}6\times10^{-4}\times3{\cdot}142({\cdot}5\times10^{-10})^2$$

$$(\because\quad 1\text{A}^\circ=10^{-10}\text{ m})$$

$$=7{\cdot}54\times10^{-24}\text{ Am}^2\text{ or JT}^{-1}.\ \textbf{Ans.}$$

Ex. 4. *The coil of a galvanometer has* 500 *turns and each turn is of average area* 3×10^{-4} m^2. *Calculate the magnetic moment of the coil when a current of* ½ *ampere is passed through it. If a torque of* 1·5 Nm *is required to place this coil carrying same current to set it parallel to a magnetic field, calculate the strength of the magnetic field.*

Sol. Magnetic moment of a current loop = area × current Am²

$$= 3 \times 10^{-4} \times \tfrac{1}{2} \times 500 \text{ Am}^2$$

$$= \cdot 075 \text{ Am}^2.$$

We have, $\tau = ISB \sin\theta$

where τ = torque, I = current in ampere, S = area and B = magnetic field and θ = angle between B and normal to S.

Here, $\theta = 90°$

$\therefore$ $1 \cdot 5 = \tfrac{1}{2}(3 \times 10^{-4} \times 500) B \sin 90°$

or $B = \dfrac{1 \cdot 5 \times 2}{3 \times 5 \times 10^{-2}} = 20$ tesla (T). **Ans.**

Ex. 5. *A current loop of magnetic moment* ·02 Am² (*or* JT^{-1}) *is suspended freely in a magnetic field. In which position will it rest in the magnetic field ? From this position of rest if it is turned through* 60°, *what is the work done ?* (*$B = \cdot 2$ tesla*)

Sol. The torque on the loop is, $\tau = ISB \sin\theta$. Obviously the loop will rest in that position where $\tau = 0$. This needs $\sin\theta = 0$ or $\theta = 0°$.

In this position, magnetic potential energy $= -mB \cos 0° = -mB$.

When it has been rotated through 60°, magnetic potential energy $= -mB \cos 60°$.

$\therefore$ Change in potential energy $= (-mB \cos 60°) - (-mB)$

$$= mB(1 - \cos 60°) = mB.\tfrac{1}{2}.$$

$\therefore$ Work done in turning = change in potential energy $= \cdot 02 \times \cdot 2 . \tfrac{1}{2}$

$= \cdot 002$ joule. **Ans.**

Ex. 6. *A ball is suspended by a thread* 1 m *long and is then charged with* $+\cdot 1$ *microcoulomb. It is then whirled very rapidly in a vertical plane at right angles to the magnetic meridian at the rate* 10 *revolutions per second. What is the magnetic moment associated with this motion of ball ? If to an observer to the north of it, the ball appears to rotate in clockwise direction, what is the specific direction of this magnetic moment ?*

Sol. The current associated with the motion is

$$I = \frac{q}{t} = \frac{\cdot 1 \times 10^{-6}}{\frac{1}{10}} = 10^{-6} \text{ ampere.}$$

$\therefore$ $m = \text{area} \times \text{current} = \pi 1^2 \times 10^{-6} = 3 \cdot 14 \times 10^{-6} \text{ Am}^2$.

It is at right angles to the plane of rotation of ball and specifically from north to south. **Ans.**

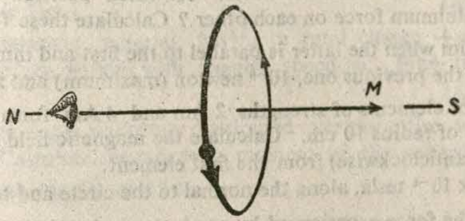

Fig. 1-6.

Ex. 7. *A rod of transverse section* 3×10^{-6} m^2 *is magnetised to saturation lengthwise by a strong magnetic field. The magnetic moment of each atom of the material of the rod is* $10\ \mu_B$ (μ_B *is the Bohr magneton*). *Calculate the pole strength of the rod if its length is* 10 cm. *There are* 5×10^{21} *atoms per unit volume of the rod.* ($\mu_B=9{\cdot}2\times10^{-24}$ Am^2)

Sol. M (magnetisation of the rod) $=5\times10^{21}\times10\mu_B=5\times10^{22}\mu_B$

$$=5\times10^{22}\times9{\cdot}2\times10^{-24}\ \text{Am}^{-1}$$

$$={\cdot}46\ \text{Am}^{-1}.$$

$\therefore$ Pole strength of the rod $=MS={\cdot}46\times3\times10^{-6}$

$$=1{\cdot}38\times10^{-6}\ \text{Am}.$$ **Ans.**

EXERCISES

(A)

1. A current element of 2 milliampere metre is at the corner A of a unit cube $ABCDEFGH$, the element being along AB. Calculate the field at the remaining seven corners of the cube (Fig. 1.5).

(Ans. at the corner B, field is zero; at $C={\cdot}7\times10^{-10}$ T along $\overrightarrow{GC}$; at $D=2\times10^{-10}$ T along $\overrightarrow{HD}$; at $E=2\times10^{-10}$ T along $\overrightarrow{EH}$; at $F={\cdot}7\times10^{-10}$ along $\overrightarrow{FG}$; at $G={\cdot}5443\times10^{-10}$ T perpendicular 'out of the plane' of the triangle ABG; at $H=2\times10^{-10}$ T out of the plane of the triangle ABH)

2. Two current elements 2×10^{-4} Am and 5×10^{-4} Am are placed at the vertices A and B of an equilateral triangle along AB in the same sense. Calculate the magnetic field at C. (Each side of the triangle measures 10 cm)

(Ans. $6{\cdot}06\times10^{-9}$ tesla)

3. Four current elements of length ·01 cm and carrying 10^{-3} ampere lie at the four corners of a square $ABCD$ along AB, BC, CD and DA. Calculate the magnetic field at the centre of the square. Each side of the square $=10$ cm.

(Ans. $5{\cdot}6\times10^{-12}$ T along the normal to the square and out of the plane of it)

4. A current element of ·2 Am is fixed at the centre of a circle of radius 10 cm. Another element of strength ·5 Am is placed along the circumference of the circle and taken once round the circle. At which positions will they exert maximum and minimum force on each other ? Calculate these forces.

[Ans. maximum when the latter is parallel to the first and minimum when it is perpendicular to the previous one, 10^{-6} newton (maximum) and zero (minimum)].

5. Two current elements of strengths ·2 Am and ·4 Am form a cross at the centre of a circle of radius 10 cm. Calculate the magnetic field at a point on the circle 30° away (anticlockwise) from the first element.

(Ans. $2{\cdot}46\times10^{-6}$ tesla, along the normal to the circle and towards its plane)

6. Calculate the force experienced by an electron when it enters into a uniform magnetic field of strength 2 tesla with velocity $\cdot 2\times10^{8}$ ms^{-1} at right angles to the field. What kind of motion will it have in the magnetic field ? (Charge of electron$=1{\cdot}6\times10^{-19}$ coulomb.)

(Ans. $6{\cdot}4\times10^{-12}$ newton, circular motion because magnetic force is always perpendicular to velocity)

7. An electron of velocity 2×10^{6} ms^{-1} is moving in a magnetic field of strength 1 tesla making an angle 45° with it. Find the magnitude and direction of the magnetic force experienced by it. Charge of electron$=1{\cdot}6\times10^{-19}$ C.

[*Hint* : $\vec{F}=q_0\,\vec{v}\times\vec{B}$.) (Ans. $2{\cdot}26\times10^{-13}$ N)

8. A ball having a mass of 2×10^{-3} kg carries a charge of 2×10^{-8} C. This ball is given an initial horizontal velocity 5×10^{4} ms^{-1} in the earth's gravitational field. What is the magnetic field required to keep the ball moving without deflection ?

[*Hint* : F_m (magnetic force)$=q_0vB\sin\theta=\omega g$] (Ans. 19·6 tesla)

9. The plane of a rectangular coil of wire of dimensions ·05 m × ·10 m and carrying a current of 10 A, is parallel to a magnetic field of ·1 tesla. What is the torque on the coil ? What will be the torque when the normal to the coil makes an angle of 30° with the direction of the field ? (Ans. 5×10^{-3} Nm, $2{\cdot}5\times10^{-3}$ Nm)

10. Considering $\theta=\frac{\pi}{2}$ as the zero energy position of a current loop of magnetic moment m placed in a magnetic field of strength B, find the minimum potential energy. In what position ? Is this an equilibrium position ? What will be the work done in turning it through α from this position ? Examine whether you get the same position for equilibrium and same value for work done if $\theta=0$ is taken as zero-energy position.

[Ans. $-mB$; when $\theta=0$ i.e. coil plane is perpendicular to B and its axis is along B; yes, because $\tau=0$ in this position; $mB(1-\cos\alpha)$]

(B)

11. A thin disc of dielectric material, having a total charge $+q$ distributed uniformly over its surface rotates with angular speed ω about an axis perpendicular to the disc and passing through its centre. Find the magnetic moment of the disc. The radius of the disc is r.

[*Hint* : Consider a ring of radius x and thickness dx. Calculate charge on it and then the current constituted by its rotation. Find its magnetic moment and integrate for the whole disc.] **(Ans. $1/4\, q\omega r^2$)**

12. A ring of dielectric material, having a total charge $+q$ distributed uniformly over its surface, rotates with angular speed ω. Find the magnetic moment of the ring. **(Ans. $1/2\, q\omega r^2$)**

13. Electric charge $+q$ is uniformly distributed over the entire volume of a sphere of radius r. Calculate the magnetic moment of the sphere if it spins about its diameter with angular speed ω. **(Ans. $\frac{1}{5}\, q\omega r^2$)**

* * *

CHAPTER 2

MAGNETIC INDUCTION

1. Magnetic Induction : Wherever we may fix up a closed conductor there is always a material medium surrounding it and that medium is air. If we like we may, however, partly or completely remove air by some other material medium. Now whatever may be the surrounding medium we may depict it as an assemblage of large number of electronic current loops of magnetic moment = '$i \times s$' Am^2 arranged in a random way due to thermal agitation of atoms. Here i is the current constituted by the orbital motion of the electron and s is the area of the orbit. When we pass current through the conductor all these tiny loops are reshuffled and aligned (not necessarily in the direction of the field due to current) with their moments in the same direction. *This alignment of electronic current loops is known as magnetic induction.*

2. Intensity of magnetisation or simply magnetisation : Due to magnetic induction the magnetised matter will have magnetic moment everywhere. This magnetic moment per unit volume is called magnetisation of matter. This is denoted by M.

$\therefore\ \frac{m}{M} = V$ where m = magnetic moment of the body of matter,
V = volume of the body of matter.

For simplicity sake let us consider matter in the form of a ring surrounded by a solenoidal conductor, fig. 2.1 (a). When we pass current through it, matter in the ring is magnetised. Let us consider a transverse section of the ring. The small circles in the figure 2.1 (b) represents the electronic circuits. Except at the periphery of the section, every portion of each circuit is adjacent to another circuit in which at the points of contact the current is in opposite direction. Hence at interior points the currents mutually cancel each other. The outer portions of the outermost circuits, however, are uncompensated and therefore the whole net work of electronic circuits is equivalent to a current in the periphery of the section as hown in fig. 2-1 (c).

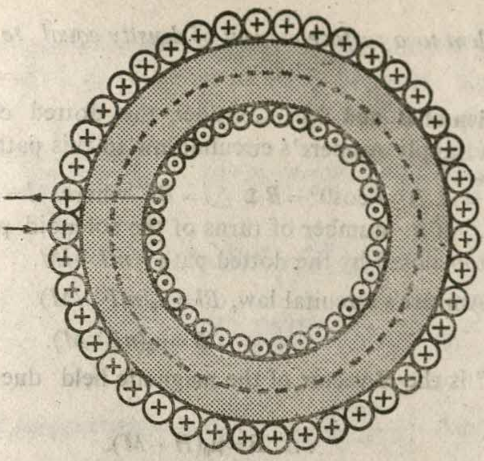

Fig. 2-1 (a).

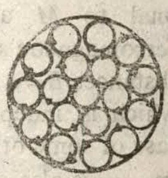

Fig. 2-1 (b).

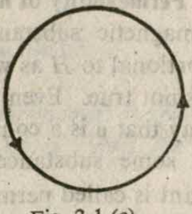

Fig. 2-1 (c).

This current is called 'surface current'. From this we conclude that the *existence of magnetisation within a medium implies the existence of surface current.*

Let us consider a short length $\triangle l$ of the ring.

Suppose $\triangle m$ = magnetic moment of this element of the ring; S = area of cross-section.

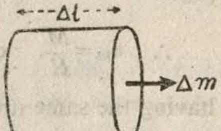

Fig. 2-2

Then $M = \dfrac{\triangle m}{S \triangle l}$, or $\triangle m = MS \triangle l$.

Let us replace this by a cylindrical shell of the same length and sectional area and imagine a current I circulating round it. Then this current will have magnetic moment IS.

$\therefore \quad MS\triangle l = IS$, or $M \triangle l = I$

or $\dfrac{I}{\triangle l} = M$, or $J = M$, where $J = \dfrac{I}{\triangle l}$.

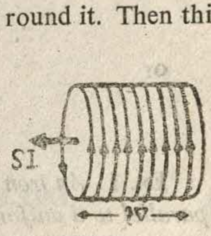

Fig. 2-3

This 'J' is called '*surface current density*'.

Thus '*a magnetised matter of magnetisation*

M is equivalent to a surface current of density equal to magnetisation itself'.

3. **Relation of B and M** : Consider the dotted closed path in figure (a) and apply Ampere's circuital law to this path. Now,

$\Sigma\, \vec{B}.\triangle\, \vec{l} = \Sigma\, B\triangle l \cos 0° = B\, \Sigma\, \triangle l = Bl$ where l = length of the dotted path. If n = number of turns of the solenoid per unit length then current enclosed by the dotted path $= nlI + lM$.

$\therefore$ By Ampere's circuital law, $Bl = \mu_0(nlI + lM)$

or $$B = \mu_0(nI + M).$$

Now 'nI' is the H-vector of the magnetic field due to solenoidal current.

$$\therefore \quad B = \mu_0(H + M).$$

4. **Permeability of medium** : In some substances (like diamagnetic and magnetic substances) B is proportional to M and hence proportional to H as well but in some (like ferromagnetic substances) this is not true. Even then we put for all substances $B = \mu\, H = \mu_0\mu_r\, H$ and say that μ is a constant of the medium which is independent of H in some substances and dependent in case of some other. This constant is called permeability of the medium. μ_r is also a constant of the medium called relative permeability of the medium.

5. **Magnetic susceptibility (κ_m)** : The ratio of magnetisation (M) to the H-vector of magnetic field is called magnetic susceptibility (κ_m).

$\therefore\ \kappa_m = \dfrac{M}{H}$. κ_m has no unit as it is the ratio of two quantities having the same unit namely Am^{-1}.

6. **Relation μ_r and κ_m** : We have, $B = \mu_0(H + M)$.

Dividing by H we have, $\dfrac{B}{H} = \mu_0\left(1 + \dfrac{M}{H}\right)$.

$$\therefore \quad \mu_0\mu_r = \mu_0(1 + \kappa_m), \quad \text{or} \quad \mu_r = \kappa_m + 1$$

or $$\kappa_m = \mu_r - 1.$$

EXAMPLES

Ex. 1. *An iron bar* ·1 m *long on being placed with its length parallel to a uniform magnetic field of* 10 Am^{-1} *is magnetised and the strength of the field due to the bar at a point on its axis is at a distance*

$\cdot30$ m *from its middle point is found to be* $\cdot25\times10^{-6}$ *tesla. If the cross-section of the bar is* $\cdot3\times10^{-4}$ m^2, *calculate magnetic susceptibility of iron.*

Sol. $B=\frac{\mu_0}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}$, where m = magnetic moment of the bar

or $$\cdot25\times10^{-6}=10^{-7}\frac{2m\times\cdot3}{(\cdot3^2-\cdot05^2)^2}.$$

$$\therefore\quad m=\frac{\cdot25\times10^1\times\cdot0875^2}{\cdot6}=\cdot0319\text{ Am}^2.$$

$$M\text{ (magnetisation)}=\frac{m}{V}=\frac{\cdot0319}{\cdot3\times10^{-4}\times\cdot1}\text{ Am}^{-1}$$
$$=1\cdot0633\times10^4\text{ Am}^{-1}.$$

$$\therefore\quad \kappa_m=\frac{M}{H}=\frac{1\cdot0633\times10^4}{10}=1063\cdot3.\qquad\textbf{Ans.}$$

Ex. 2. *The magnetic moment of a steel bar-magnet weighing* 66 gm *is* 2·5 Am^2. *If the density of steel is* 7900 kg/m^3, *find the intensity of magnetisation.*

Sol. $$V=\frac{66\times10^{-3}}{7900}=\frac{66}{75}\times10^{-5}\text{ m}^3.$$

$$M=\frac{m}{V}=\frac{2\cdot5}{\frac{66}{75}\times10^{-5}}=\frac{2\cdot5\times75\times10^5}{66}\text{Am}^{-1}=\mathbf{2\cdot84\times10^5\text{ Am}^{-1}}.\qquad\textbf{Ans.}$$

Ex. 3. *A ring made of annealed iron has winding of insulated copper wire of* 1000 *turns to the metre. When the current in the winding is* ·15 A, *the field inside the ring is* 1·0 T. *Calculate relative permeability and magnetic susceptibility of annealed iron.*

Sol. $$H=nI=1000\times\cdot15=150\text{ Am}^{-1}.$$

$$\mu=\frac{B}{H}=\frac{1}{150}\text{ Hm}^{-1}$$

or $$\mu_r\mu_0=\frac{1}{150}$$

or $$\mu_r=\frac{1}{150\times4\pi\times10^{-7}}=\frac{10^7}{600\pi}=\frac{10^5}{6\pi}=\mathbf{5304}.\qquad\textbf{Ans.}$$

$$\kappa_m=\mu_r-1=\mathbf{5303}.\qquad\textbf{Ans.}$$

EXERCISES

(A)

1. A magnet of magnetic moment 3 Am^2 weighs 75 gm. If the density of the material of the magnet is 7900 kgm^{-3}, find its magnetisation.

(**Ans.** $3{\cdot}16\times10^{5}$ Am^{-1})

2. A ring of iron of permeability $8{\cdot}2\times10^{-3}$ henry per metre has 500 windings on it. The mean circumference of the ring is 50 cm. It carries a current of 100 mA. Calculate, (i) the magnetic field H, (ii) the magnetic field B inside core, (iii) the relative permeability of the core. (iv) By what factor is the magnetic field B due to the electronic loop currents in the core greater than that due to free current ?

[*Hint* : (i) $H=nI$, (ii) $B=\mu H$, (iii) $\mu_r=\frac{\mu}{\mu_0}$, (iv) Required factor $=\frac{\mu_0 M}{\mu_0 H}=\kappa_m$]

[**Ans.** (i) 100 Am^{-1}, (ii) ·82 T, (iii) $6{\cdot}5\times10^{3}$, (iv) $6{\cdot}5\times10^{3}$]

3. The dipole moment of an iron atom is $1{\cdot}8\times10^{-23}$ Am^2. If all the atoms of an iron rod of length 10 cm and diameter 1 cm are aligned along its length, what is the dipole moment of the bar ? (Density of iron is 7870 kgm^{-3}, and its atomic weight is 55·87 and Avogadro's number$=6{\cdot}02\times10^{26}$ kg mol^{-1}) (**Ans.** 12·0 Am^2)

4. A heavy steel beam of length 10 m and area of cross-section ·25 m^2 runs parallel to the magnetic meridian. The horizontal component of H-field of earth is 30 Am^{-1}. A small compass needle is moved along the vertical line passing through the middle point of the beam and is found to turn through 180° at a distance 5 m from it. Calculate the intensity of magnetisation of the bar and also susceptibility of its material. (**Ans.** $5{\cdot}3\times10^{4}$ Am^{-1}; 1767)

5. A ring has a cross-section 2 cm^2, a mean length 30 cm and is wound with 400 turns. Find the current in the winding that is required to set up a magnetic field of ·1 tesla in the ring. (Permeability of the material of the ring$=16\times10^{-4}$ Hm^{-1})

(**Ans.** ·046 A)

6. A solenoid of length 20 cm contains 150 turns. If a rod of mild steel of volume 2 cc. and relative permeability 2000 is placed at the centre of the solenoid and 1 ampere current is passed through the solenoid,, what maximum magnetic moment will be acquired by the rod ?

$\left(\textit{Hint}: H=nI;\ \kappa_m=\frac{M}{H};\ \kappa_m=\mu_r-1.\right)$ (**Ans.** 3 Am^2)

(B)

7. A current flows uniformly along a solid rod of radius R. Show that the B-field inside this rod, at a distance r from the axis, is $B=\frac{\mu_0 Ir}{2\pi R^2}$.

[*Hint* : Apply Ampere's circuital law.]

□□□

CHAPTER 3

MAGNETIC FIELD AND POTENTIAL

1. With the formalism developed in the way indicated in the introductory notes we can now take up the magnetic effect of magnetised matter (magnets) introducing 'poles' of a magnetised body as only a mathematical artifice which are the 'sources' and 'sinks' of magnetic *H*-vector. A 'north pole' is the 'source' of magnetic *H*-lines and a 'south pole' is the 'sink' of *H*-lines. A magnet of magnetisation *M* and area of cross-section *S* will have pole strength (q_m) given by $q_m = MS$ ampere metre (Am). .. (1)

2. **Coulomb's law of force** : The law states that the force between two magnetic poles q_{m_1} and q_{m_2} situated *d* distance apart is given by $F = \frac{\mu_0 \mu_r}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{d^2}$ where μ_r is the relative permeability of the medium between the poles and μ_0 is the permeability of vacuum $4\pi \times 10^{-7}$ henry per metre (Hm^{-1}).

Unit magnetic pole : The force between two magnetic poles in a vacuum is $F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{d^2}$. If $q_{m_1} = q_{m_2} = 1$ and $d = 1$ m, then $F = \frac{\mu_0}{4\pi}$ $= 10^{-7}$N. Thus in *SI unit magnetic pole is defined as the one which situated at* 1 m *distance in a vacuum from an equal pole experiences a force of* 10^{-7} *newton.* Alternatively, it can be defined as that pole strength which placed in a unit magnetic field experiences a force of 1 newton. Unit, *ampere-metre* (Am).

3. **Magnetic moment of a magnet** : If a magnet of pole strength q_m and length $2l$ is placed in a uniform magnetic field of strength *B*, it experiences a torque given by $\tau = 2q_m lB \sin\theta$ when θ is the angle between the magnetic field and the axis ($\overrightarrow{SN}$) of the magnet.

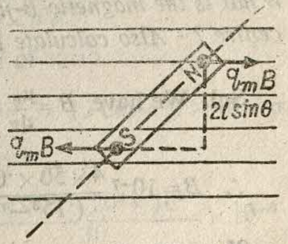

Fig. 3-1

The quantity '$2q_m l$' which is obviously the torque experienced by the magnet when it is perpendicular to a unit uniform magnetic field is defined as the 'magnetic moment' of magnet.

Thus m (magnetic moment of the magnet)

$$=2q_m l \text{ ampere metre}^2 \text{ (Am}^2\text{)} \quad .. \quad (2)$$

$$\tau \text{ (torque on the magnet)} = mB\sin\theta \text{ newton metre (Nm)}. \quad .. \quad (3)$$

4. Potential energy of a magnet in a magnetic field : The magnetic potential energy of a magnet in any position θ is the work that an external agent must do to turn the magnet from its zero energy position ($\theta=0°$) to the given position.

$$\text{Thus} \quad U=\int_0^\theta mB\sin\theta\, d\theta = mB(1-\cos\theta) \text{ joule (J)}.$$

$$U=mB(1-\cos\theta) \text{ J}. \quad .. \quad (4)$$

5. Field due to a bar magnet :

(a) End-on position : The magnetic field at a point situated on the axis at a distance 'd' from the centre of a bar magnet of moment m is given by

$$B=\frac{\mu_0}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2} \text{ tesla (T) and } H=\frac{1}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2} \text{ Am}^{-1}. \quad .. \quad (5)$$

(b) Broad-side-on position : At a point situated on the perpendicular bisector at a distance d from the centre of the magnet is given by

$$B=\frac{\mu_0}{4\pi}\cdot\frac{m}{(d^2+l^2)^{\frac{3}{2}}} \text{ tesla (T) and } H=\frac{1}{4\pi}\cdot\frac{m}{(d^2+l^2)^{\frac{3}{2}}} \text{ Am}^{-1}. \quad .. \quad (6)$$

EXAMPLES

Ex. 1. *A bar magnet is* ·1 m *long and has poles of strength* 50 Am. *What is the magnetic B-field at a point on its axis* ·15 m *from its centre ? Also calculate H-field at that point.*

Sol. We have, $B=\frac{\mu_0}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}=\frac{\mu_0}{4\pi}\cdot\frac{4q_m ld}{(d^2-l^2)^2} \quad (\because \quad m=2q_m l)$

$$\therefore \quad B=10^{-7}\frac{4\times 50\times \cdot 05\times \cdot 15}{(\cdot 15^2-\cdot 05^2)^2} \qquad \left(\because \quad \frac{\mu_0}{4\pi}=10^{-7} \text{ Hm}^{-1}\right)$$

or $$B=3{\cdot}75\times 10^{-4} \text{ T}.$$ **Ans.**

$$H=\frac{B}{\mu_0}=\frac{3{\cdot}75\times10^{-4}}{4\pi\times10^{-7}}=298\ \text{Am}^{-1}.\qquad \textbf{Ans.}$$

Ex. 2. *The intensities of magnetic field at two points on the axis of a bar magnet at distances of* 10 cm *and* 20 cm *respectively from the middle point of the magnet are in the ratio* 18 : 1. *Find the distance between the poles of the magnet.*

Sol. We have for the magnetic field at a point on the axis of a bar magnet,
$$B=\frac{\mu_0}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}.$$

$$\therefore\quad B_1=\frac{\mu_0}{4\pi}\cdot\frac{2m.10\times10^{-2}}{(10^2-l^2)^2 10^{-8}};\ B_2=\frac{\mu_0}{4\pi}\cdot\frac{2m.20\times10^{-2}}{(20^2-l^2)^2\times10^{-8}}$$

$$\therefore\qquad B_1 : B_2=\frac{1}{(10^2-l^2)^2}:\frac{2}{(20^2-l^2)^2}.$$

But $\qquad B_1 : B_2=18 : 1.$

$$\therefore\qquad \frac{18}{1}=\frac{(20^2-l^2)^2}{2(10^2-l^2)^2},\quad\text{or}\quad 36=\left(\frac{20^2-l^2}{10^2-l^2}\right)^2$$

$$\text{or}\quad 6=\frac{400-l^2}{100-l^2},\quad\text{or}\quad 600-6l^2=400-l^2,\quad\text{or}\quad 5l^2=200$$

$$\text{or}\quad l^2=40,\quad\text{or}\quad l=\sqrt{40}=6{\cdot}325.$$

$\therefore$ Length of magnet $=2l=2\times6{\cdot}325=12{\cdot}650$ cm $={\cdot}1265$ m. **Ans.**

Ex. 3. *A neutral point is found on the prolongation of the axis of a bar magnet at a distance of* 10 cm *from the nearer pole. If the length of the bar be* 10 cm *and* $H=28$ Am^{-1}, *find the pole strength of the magnet.*

Sol. Neutral point is that point where the earth's magnetic field is neutralised by the magnetic field of the magnet. Since here H-vector of earth's magnetic field is given we will use expression for H-vector due to a magnet. We have, $H=\frac{1}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}.$

At the neutral point, H of magnet $=H$ of earth.

$$\therefore\quad \frac{1}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}=28.\quad\text{Here } d=10+5=15\text{ cm}=15\times10^{-2}\text{ m},$$

$$l=5\text{ cm}=5\times10^{-2}\text{ m}.$$

$$\therefore\quad m=\frac{4\pi\times28\times(15^2-5^2)^2\times10^{-8}}{2\times15\times10^{-2}}=\frac{4\pi\times28\times20^2\times10^2}{30}\times10^{-6}$$

or $m=\frac{4\pi\times 28\times 20^2\times 10^{-4}}{30}=\cdot 469\text{ Am}^2.$

$\because$ $m=2q_m l,$

$\therefore$ $q_m=\frac{m}{2l}=\frac{\cdot 469}{10\times 10^{-2}}=4\cdot 69\text{ Am.}$ **Ans.**

Ex. 4. *A magnet, whose poles are* 10 cm *apart is placed in the magnetic meridian. The intensity due to magnet balances the earth's field* ($3\cdot 6\times 10^{-5}$ T) *at a point* 9 cm *from either pole. Find the pole strength of the magnet.*

Sol. Since the neutral point is equidistant from the poles, it must essentially be on the perpendicular bisector, *B*-vector of the magnetic field on broad-side-on position is

$B=\frac{\mu_0}{4\pi}\cdot\frac{m}{(d^2+l^2)^{3/2}}=\frac{\mu_0}{4\pi}\cdot\frac{m}{x^3}$ where x is the distance from each pole.

At neutral point, *B*-field of magnet = *B*-field of earth.

$$\frac{\mu_0}{4\pi}\cdot\frac{m}{x^3}=3\cdot 6\times 10^5$$

or $10^{-7}\frac{m}{9^3\times 10^{-6}}=3\cdot 6\times 10^{-5}$, or $m=9^3\times 3\cdot 6\times 10^{-4}$

or $m=\cdot 262\text{ Am}^2.$

$\because$ $m=2q_m l,$

$\therefore$ $q_m=\frac{m}{2l}$

or $q_m=\frac{\cdot 262}{10\times 10^{-2}}=2\cdot 62\text{ Am.}$ **Ans.**

Ex. 5. *The magnetic moment of a magnet is* 1 *joule per tesla. How much work is done in turning it through* 90° *from the magnetic meridian at a place where* $B=1\cdot 6\times 10^{-5}$ T ?

Sol. See theory above and show $U=mB(1-\cos\theta)$ taking $\theta=0°$ as the zero-energy position.

$\therefore$ $U=1\times 1\cdot 6\times 10^{-5}(1-\cos 90°)=1\cdot 6\times 10^{-5}$ joule. **Ans.**

Ex. 6. *Find the stable equilibrium position of a magnet in a magnetic field.*

Sol. A body is said to be in equilibrium when torque and force on it vanish. In a magnetic field torque on a magnet is, Torque $= mB \sin\theta$ where θ is the angle between field B and magnetic moment m (SN-direction). The forces on the poles being equal and opposite, resultant force on the magnet is already zero. $\therefore$ For equilibrium of magnet we have $mB \sin\theta = 0$, or $\theta = 0°$ or π. Thus there are two positions where magnet will be in equilibrium. The position in which potential energy is minimum is the stable equilibrium position. Taking $\theta = 0°$ as the position of zero energy we have, $U = mB\,(1-\cos\theta)$; when $\theta = 0$, $U = 0$ and when $\theta = \pi$, $U = 2mB$.

Hence $\theta = 0°$ is the stable equilibrium position. **Ans.**

Ex. 7. *A magnetic needle of length* 10 cm *is pivoted so as to be free to rotate in a horizontal plane at a place where earth's horizontal field is* $3{\cdot}6 \times 10^{-5}$ *tesla. It is then pulled by a weightless string passing over a frictionless pulley and carrying* 10 mgm *at its other end and the needle rotates through* 60°. *Calculate magnetic moment and pole strength of the needle.*

Sol. There are two torques on the needle.

Magnetic torque $= mB \sin\theta$. Mechanical torque = moment of tension about the centre $= Tl \cos\theta$ where l = half the length of the needle.

For equilibrium we have

$$mB \sin\theta = Tl \cos\theta.$$

Here $T = 10$ mgm wt.

$$= (10 \times 10^{-6}) \times 9{\cdot}8 \text{ N}$$

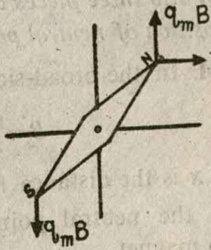

Fig. 3-2

$$\therefore \quad m = \frac{Tl \cos\theta}{B \sin\theta} = \frac{(10 \times 10^{-6}) \times 9{\cdot}8 \times 5 \times 10^{-2} \cos 60°}{3{\cdot}6 \times 10^{-5} \sin 60°}$$

or $$m = {\cdot}0786 \text{ Am}^2 \text{ (or JT}^{-1}\text{)}$$

$$q_m = \frac{m}{2l} = \frac{{\cdot}0786}{10 \times 10^{-2}} = {\cdot}786 \text{ Am.}$$ **Ans.**

Ex. 8. *Two magnets of magnetic moment m and 3m respectively are mounted so as to cross each other at an angle 60°. The combination rests on a cork floating in water. Find the direction which the weaker magnet makes with the meridian.*

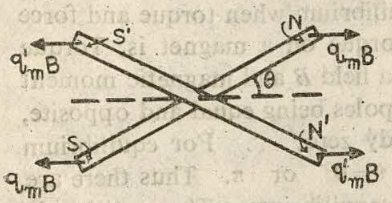

Fig. 3-3

Sol. Two torques act on the combination.

Torque on $m = mB \sin\theta$ where B = magnetic field of the earth and θ is the angle made by it with magnetic meridian.

Torque on $3m = (3m)B \sin(60^\circ - \theta)$.

For equilibrium we have, $mB \sin\theta = 3mB \sin(60^\circ - \theta)$

or $\sin\theta = 3 \sin(60^\circ - \theta)$, or $\sin\theta = 3 \sin 60^\circ \cos\theta - 3 \cos 60^\circ \sin\theta$

or $\sin\theta = \frac{3\sqrt{3}}{2}\cos\theta - 3.\frac{1}{2}\sin\theta$, or $\frac{5}{2}\sin\theta = \frac{3\sqrt{3}}{2}\cos\theta$

or $\tan\theta = \frac{3\sqrt{3}}{2} = 1{\cdot}039$

or $\theta = 46^\circ 6'$. (Angles taken from the log table.) **Ans.**

Ex. 9. *A magnet placed in the north pointing north position balances the earth's field at a point 27 cm from either pole. If it is broken into three pieces and one such piece is similarly placed, find the position of neutral point.*

Sol. In the broad-side-on position,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)^{3/2}} \quad \text{or} \quad \frac{\mu_0}{4\pi} \cdot \frac{m}{x^3}$$

where x is the distance from either pole.

At the neutral point, B-vector of earth's field = B-vector of field due to magnet.

$$\therefore \quad B_0 = \frac{\mu_0}{4\pi} \cdot \frac{m}{x^3} \text{ where } B_0 = B\text{-vector of earth's field.}$$

When a magnet is broken, its pole strength remains the same but not the magnetic moment.

$$m = q_m \times 2l.$$

If m' = magnetic moment of the broken piece then

$$m' = q_m \times \frac{2l}{3} = \frac{m}{3}.$$

$$\therefore \quad B_0 = \frac{\mu_0}{4\pi} \cdot \frac{m/3}{x'^3}; \quad \therefore \frac{\mu_0}{4\pi} \cdot \frac{m}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{m}{3x'^3}$$

$$\text{or} \quad 3x'^3 = x'^3, \text{ or } x' = \frac{x}{\sqrt[3]{3}}$$

or $$x' = \frac{27\times10^{-2}}{3^{1/3}}m = 18{\cdot}73\times10^{-2}\ m.$$ **Ans.**

Ex. 10. *Two short magnets of moments* 2 Am^2 *and* 5 Am^2 *are placed along two lines drawn at right angles to each other on a sheet of paper. Find the field at the point of intersection of their axis when their centres are respectively* ·3 m *and* ·4 m *from the point.*

Sol. B_1, field at the point of intersection

$$=\frac{\mu_0}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}=\frac{\mu_0}{4\pi}\cdot\frac{2m}{d^2}\,(d^2 \gg l^2)=\frac{\mu_0}{4\pi}\cdot\frac{2\times2}{(\cdot3)^3}=10^{-7}\frac{4}{\cdot027}$$

or $$B_1=1{\cdot}48\times10^{-5}.$$

$$B_2=\frac{\mu_0}{4\pi}\cdot\frac{2\times5}{(\cdot4)^3}=10^{-7}\times\frac{10}{\cdot064}=1{\cdot}56\times10^{-5}.$$

The two fields are at right angles to each other.

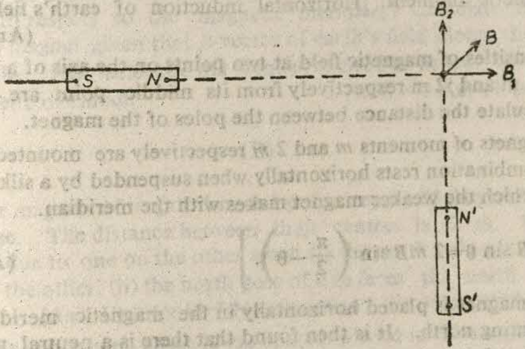

Fig. 3-4

$\therefore$ B (resultant field) $=\sqrt{B_1^2+B_2^2}=\sqrt{1{\cdot}48^2+1{\cdot}56^2}\times10^{-5}$

or $$B=2{\cdot}15\times10^{-5}\text{ tesla.}$$ **Ans.**

Let it make an angle θ with B_1. Then $\tan\theta=\frac{B_2}{B_1}=\frac{1{\cdot}56}{1{\cdot}48}$

or $\theta=45°51'$. (Angle taken from the log table)

EXERCISES

(A)

1. An electron is moving with velocity of 2×10^6 ms^{-1} in the vicinity of a ·1 m long bar magnet of pole strength 50 Am in a direction perpendicular to the axis of the magnet at a distance ·15 m from the nearer pole. What is the force on the electron ? Charge of electron$=1{\cdot}6\times10^{-19}$ C)

[*Hint* : Calculate *B*-vector of the field and apply $F=q_0 v\ B\ \sin\theta$.] (Ans. $4{\cdot}6\times10^{-17}$ N)

2. A magnetic needle of magnetic moment ·9 Am² and pole strength 5 Am is pivoted so that it is free to turn in a horizontal plane where earth's *B*-field is $3{\cdot}6\times10^{-5}$ tesla. It is in equilibrium at an angle 30° from the magnetic meridian when it is pulled by a string attached to its north pole in the easterly direction. What is the tension of the string ? (Ans. $2{\cdot}08\times10^{-4}$ N)

3. A magnet ·1 m long and pole strength ·1 Am is freely suspended in a horizontal uniform magnetic field of intensity $\cdot18\times10^{-4}$ T. Find the torque tending to restore the magnet to its original position of rest when it is deflected by 30° from that position. (Ans. 9×10^{-8} Nm)

4. A bar magnet lies in a magnetic field of intensity $\cdot5\times10^{-4}$ tesla. The torque required to deflect it through 30° is $2{\cdot}5\times10^{-5}$ Nm. Calculate the magnetic moment of the magnet. (Ans. 1 Am²)

5. A magnet ·2 m long balances the earth's horizontal field at a point ·2 m from the nearer pole. The magnet is placed with its north pole pointing south. Find its magnetic moment. [Horizontal induction of earth's field$=3{\cdot}6\times10^{-5}$ tesla (T).] (Ans. 3·84 Am²)

6. The intensities of magnetic field at two points on the axis of a bar magnet at distances ·1 m and ·2 m respectively from its middle point are in the ratio 12·5 : 1. Calculate the distance between the poles of the magnet. (Ans. ·1 m)

7. Two magnets of moments *m* and 2 *m* respectively are mounted to form a cross. The combination rests horizontally when suspended by a silk fibre. Find the direction which the weaker magnet makes with the meridian.

$$\left[\textit{Hint}:\ mB\sin\theta=2\,mB\sin\left(\frac{\pi}{2}-\theta\right).\right]$$ (Ans. 63° 26′)

8. A short magnet is placed horizontally in the magnetic meridian with its north pole pointing north. It is then found that there is a neutral point 20 cm from the middle of the magnet. If the magnet is turned through 180°, where will the neutral point be ?

[*Hint* : When magnet is placed north pointing north neutral point occurs on the broad-side-on position. When it is rotated through 180°, neutral point will shift on to the end-on position.) (Ans. 25·2 cm)

9. The intensities of a magnetic field at two points on the axis of a bar magnet at distances of ·15 m and ·2 m respectively from its middle point are in the ratio 75 : 20. Find the distance between the poles of the magnet. (Ans. ·180 m)

10. A short bar magnet is placed horizontally in the magnetic meridian with its north pole pointing north and the neutral points are found ·15 m from the centre of the magnet. Find the distance between the neutral points if the magnet is reversed. (Ans. ·378 m)

11. A neutral point is found on the prolongation of the axis of a bar magnet at a distance ·1 m from the nearest pole. If the length of the bar be ·1 m and

B-field of earth$=\cdot 36\times 10^{-4}$ tesla, find the pole strength of the magnet.

(Ans. 4·8 Am)

12. A magnet whose poles are 12 cm apart is placed in the magnetic meridian. The field due to this magnet counter balances the earth's horizontal field ($H=30$ Am^{-1}) at a point 10 cm from each pole. Find the pole strength of the magnet. (Ans. 3·142 Am)

13. A uniformly magnetised bar of brittle steel is broken into two pieces, one twice as long as the other, and the pieces are fastened together at right angles to each other. How would the combination thus formed set itself under the action of the earth's magnetic force, if made to float on water ?

(Ans. the combination will set with the longer one making an angle 26° 36′ with the magnetic meridian)

14. Calculate moment of a magnet which when placed at right angles to the horizontal magnetic field of the earth of intensity 30 Am^{-1} experiences a couple 3×10^{-4} newton metre.

[*Hint* : $B_0=\mu_0 H_0=4\pi\times 10^{-7}\times 30$ tesla and $\tau=B_0 m \sin\theta$.] (Ans. 7·958 Am^2)

15. A neutral point is found on the prolongation of the axis of a bar magnet of length 10 cm at a distance 10 cm from the nearer pole. How has the magnet been placed with respect to the magnetic meridian ? Calculate the magnetic moment of the magnet given that B-vector of earth's field along the horizontal $=3\cdot 6\times 10^{-5}$ tesla. Also calculate the torque required to deflect it through 30° from the magnetic meridian. (Ans. ·48 Am^2, $8\cdot 64\times 10^{-6}$ Nm)

(B)

16. Two bar magnets of length ·1 m and pole strength 75 Am each, are placed on the same line. The distance between their centres is ·2 m. What is the resultant force due to one on the other when (i) the north pole of one faces the south pole of the other, (ii) the north pole of one faces the north pole of the other ? [Ans. (i) $3\cdot 4\times 10^{-2}$ N (attraction), (ii) $3\cdot 4\times 10^{-2}$ N (repulsion)]

17. A bar magnet having poles 10 cm apart is placed in the magnetic meridian with its north pole pointing south. The neutral point is at a distance of 20 cm from the nearest pole. Find the intensity of the resultant field at a point on the perpendicular bisector of the axis of the magnet and at a distance of 10 cm from the centre of the magnet. (B-vector of earth's field along the horizontal$=4\times 10^{-5}$ tesla.) (Ans. 2·88 Am^2; $2\cdot 46\times 10^{-4}$ tesla)

18. A long vertical magnet is placed at perpendicular distance of 8 m from the centre of a horizontal magnetic needle of length ·12 m and pole strength ·6 Am. If the moment of the couple acting upon the needle is 72×10^{-7} Nm, find the pole strength of the long magnet.

[*Hint* : Since the magnet is long, neglect the effect of distant pole.]

(Ans. 645 Am)

19. A magnet placed in the magnetic meridian with its north pole pointing north of the earth produces neutral point at a distance ·15 m from either pole. It is then broken into two equal pieces and one such piece is placed in a similar position. Calculate the position of the neutral point. Where will be the neutral point if the other piece is placed over the previous one with like poles together ?

(**Ans.** ·119 m from either pole of the piece; ·15 m from either pole of the combination)

20. Two short magnets of the same type, but of moments *m* and 2*m* are mounted on a frame so as to form a cross. If the combination is suspended at the centre with a vertical fibre, find the direction in which it will set in the earth's magnetic field. Calculate also the magnetic *B*-field at a distance '*d*' from the centre of the cross on the prolongation of one of the arms.

(**Ans.** It will set at an angle 26°36′ with the magnetic meridian; $\frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}\,m}{d^3}$; $\frac{\mu_0}{4\pi}\cdot\frac{m\sqrt{17}}{d^3}$)

21. Assuming $\theta=\frac{\pi}{2}$ as the zero-energy position of a magnet in a magnetic field, show that potential energy of a magnet can be written as $U=-\vec{B}\cdot\vec{m}$. Hence find the stable equilibrium position of a magnet in a magnetic field.

22. Show that the force of attraction between two short magnets of moment *m* and *m′* with their centres *d* distance apart and their axes in the same direction along the same line is $\frac{\mu_0}{4\pi}\cdot\frac{6\,mm'}{d^4}$.

□□□

CHAPTER 4

MAGNETIC DIPOLE

1. Definition of Magnetic Dipole : A short bar magnet is called a magnetic dipole.

2. Magnetic potential at a point due to a magnetic dipole : The work done in bringing a unit north pole by an external agent from infinity up to a point in the field due to the dipole is called magnetic potential due to the dipole at that point. The magnetic potential at a point due to magnetic dipole at a distance r from its centre in a direction making an angle θ with the axis (SN-direction) of the dipole is given by

$$V=\frac{\mu_0}{4\pi}\cdot\frac{m\cos\theta}{r^2} \text{ joule per ampere metre } (\mathrm{JA^{-1}m^{-1}}).$$

3. Magnetic field at a point due to a magnetic dipole : B_r, component of magnetic field at the point (r, θ) along $r=\frac{\mu_0}{4\pi}\cdot\frac{2m\cos\theta}{r^3}$.

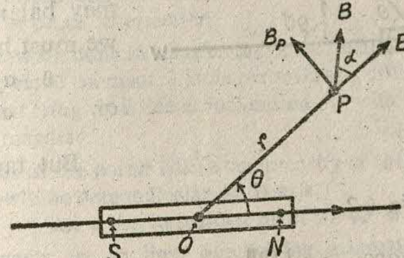

Fig. 4-1

B_v, component of magnetic field perpendicular to r

$$=\frac{\mu_0}{4\pi}\cdot\frac{m\sin\theta}{r^3}.$$

$\therefore$ Magnetic field (resultant) at that point

$=\frac{\mu_0}{4\pi}\cdot\frac{m}{r^3}\sqrt{1+3\cos^2\theta}$ tesla in a direction making angle α with r or $(\theta+\alpha)$ with the axis (SN-direction) is given by

$$\tan\alpha=\frac{\tan\theta}{2}.$$

EXAMPLES

Ex. 1. *Find the value of the potential at a point situated on a line passing through the middle point of a short magnet of moment* ·3 Am² *at an angle* 60° *with its axis at a distance* ·05 m *away from the mid point of the magnet.*

Sol. We have, $V=\frac{\mu_0}{4\pi}.\frac{m\cos\theta}{r^2}\ \text{JA}^{-1}\text{m}^{-1}.$

$$V=10^{-7}.\frac{\cdot 3\cos 60^\circ}{(\cdot 05)^2}=6\times 10^{-6}\ \text{J/Am}. \qquad \textbf{Ans.}$$

Ex. 2. *A short magnet of magnetic moment* 2 Am² *is placed in east-west direction. Find the point where there will occur neutral point.* (*Earth's horizontal induction* $=3{\cdot}6\times 10^{-5}$ *tesla*)

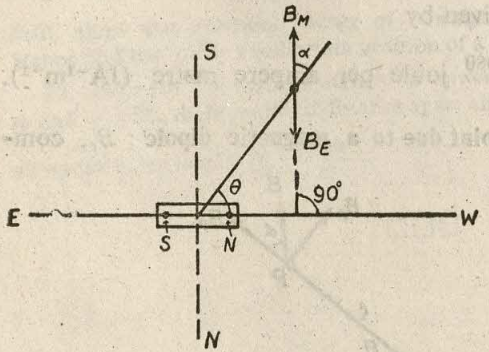

Fig. 4-2

Sol. At the neutral point the earth's field (B_E) neutralises the field (B_M) due to magnet. In order that B_E and B_M may balance each other we must have

$$\theta+\alpha=90^\circ$$

or $$\alpha=90^\circ-\theta.$$

But $\tan\alpha=\frac{\tan\theta}{2}$, (always)

$\therefore \qquad \tan(90^\circ-\theta)=\frac{\tan\theta}{2}$

or $\qquad \tan^2\theta=2$, or $\tan\theta=1{\cdot}414$ or $\theta=54^\circ 42'$.

Also at the neutral point, $B_M=B_E$

or $$\frac{\mu_0}{4\pi}.\frac{m}{r^3}\sqrt{1+3\cos^2\theta}=3{\cdot}6\times 10^{-5}$$

or $$r^3=\frac{10^{-7}\times 2\sqrt{1+3\cos^2 54^\circ 42'}}{3{\cdot}6\times 10^{-5}}$$

or $$r^3=\frac{\sqrt{1+3\times\cdot 578^2}\times 10^{-2}}{1{\cdot}8}=\frac{\sqrt{1+1{\cdot}0024}\times 10^{-2}}{1{\cdot}8}$$

or $$r=\cdot 199\ \text{m}.$$

Thus the neutral point will occur in a direction 54°42′ at a distance of 19·9 cm from the centre of the magnet. **Ans.**

EXERCISES

1. A very short magnet is placed at a point O with its axis horizontal and perpendicular to the magnetic meridian. If P is a neutral point, show that the angle between OP and the axis ($\overrightarrow{SN}$ direction) of the magnet is $\tan^{-1}\sqrt{2}$.

2. Find the magnitude and direction of the magnetic field due to a small bar magnet of moment ·03 joule per tesla at a point situated on a line through the centre of the magnet and at an angle 60° with its axis, the point being at a distance of ·05 m from the centre of the magnet.

(Ans. $\cdot 32\times10^{-4}$ tesla in a direction 100°54′ with the axis)

3. Find the potential due to a magnet at a point on its axis at a distance 09 m form its centre in the end-on position. The length of the magnet is ·02 m and its pole strength 9 Am.

$$\left(\text{Ans. } 2{\cdot}25\times10^{-6}\frac{\text{J}}{\text{Am}}\right)$$

4. Two magnetic dipoles of moments ·108 and ·122 Am² are placed along two lines drawn on the table at right angles to each other; find the intensity at the point of intersection of their axes, their centres being ·3 and ·4 m respectively from the pt. of intersection. (Ans. 10^{-6} tesla)

5. Calculate the work done in transferring a unit magnetic pole from a point on the axis of a dipole of moment ·3 joule per tesla at a distance 10 cm from its centre on to a point lying on a line at inclination 60° with the axis at the same distance from the magnet. (Ans. $\cdot 15\times10^{-5}$ joule)

6. Find the locus of the points where the intensity of magnetic field due to a magnetic dipole will be perpendicular to its axis.

(Ans. a line at inclination of 54°42′ with axis of the dipole)

7. Two bar magnets are in line, one having magnetic length ·08 m and pole strength 5 Am and the other magnetic length ·04 m and pole strength 4 Am, their mid-points being ·16 m apart. Find how much work is done in turning one of these magnets over end for end. (Ans. $7{\cdot}4\times10^{-6}$ joule)

8. The earth may be considered as a big dipole of moment $6{\cdot}4\times10^{12}$ Am placed at its centre with its axis (SN direction) in the SN direction of it. Radius of the earth is 6000 km. Calculate the horizontal and vertical component of the earth's magnetic field at a place of latitude 22°N.

$$\left(\textit{Hint}: B_r=\frac{\mu_0}{4\pi}\cdot\frac{2m\cos\theta}{r^3} \text{ and } B_1=\frac{\mu_0}{4\pi}\cdot\frac{m\sin\theta}{r^3};\quad \text{here } \theta=90^\circ+22^\circ=112^\circ\right)$$

[Ans. vertical component$=2{\cdot}22\times10^{-6}$ tesla (downward)
and horizontal component$=2{\cdot}75\times10^{-6}$ tesla]

9. A point lies on a line making an angle of 30° with the axis (SN-direction) of a magnetic dipole. If the dipole is turned through 90° away from the line, how the magnetic field at the point will be affected ?

(Ans. field will be reduced $\sqrt{\frac{7}{18}}$ times)

□□□

CHAPTER 5

MAGNETIC MEASUREMENTS

1. Tangent law : If a small magnetic needle capable of rotating freely in a horizontal plane is subjected to two perpendicular magnetic fields B_1 and B_2 and if it makes an angle θ with B_2 then

$$B_2 = B_1 \tan\theta.$$

This is called *Tangent law*.

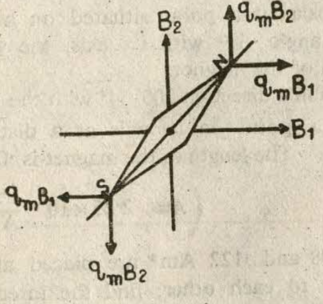

Fig. 5-1

2. Deflection magnetometer : (i) Tan *A*-position : When the magnetometer is placed with its arms in the east-west direction and the experimental magnet is placed along the arms so that the compass needle of the magnetometer lies on the end-on position of the magnet, this arrangement of magnetometer is called Tan *A*-position. Here one field is due to earth's horizontal induction (B_0) and the other (B) is due to the magnet. These two fields are at right angles to each other. So we have $B = B_0 \tan\theta$.

But $$B = \frac{\mu_0}{4\pi} \cdot \frac{2md}{(d^2 - l^2)^2}$$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2md}{(d^2 - l^2)^2} = B_0 \tan\theta$$

or $$\frac{m}{B_0} = \frac{4\pi}{\mu_0} \cdot \frac{(d^2 - l^2)^2}{2d} \tan\theta \text{ joule per square tesla (JT}^{-2}\text{)}.$$

(ii) Tan *B*-position : When the magnetometer is placed with its arms in the north-south direction and the experimental magnet is placed perpendicular to the arms so that the compass needle of the magnetometer lies on the perpendicular bisector of the magnet, this arrangement of magnetometer is called tan *B* position.

Here $$B = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)^{3/2}}$$

$$\therefore \quad \frac{m}{B_0}=\frac{4\pi}{\mu_0}(d^2+l^2)^{3/2}\tan\theta \text{ JT}^{-2}.$$

3. Oscillation magnetometer : The time period of oscillation of a magnet free to oscillate in a magnetic field B is given by

$$T=2\pi\sqrt{\frac{I}{mB}}$$

where I = moment of inertia of the vibrating magnet about the axis of suspension, m = magnetic moment of magnet.

If the vibrating magnet is a bar of length l and breadth b then

$$I=\omega\left(\frac{l^2+b^2}{12}\right)\text{kgm}^2 \quad \text{where } \omega = \text{mass of the magnet.}$$

If it is cylindrical in shape then $I=\omega\left(\frac{l^2}{12}+\frac{r^2}{4}\right)\text{kgm}^2$

where r = radius of the cylinder.

4. Searle's vibration magnetometer : In this magnetometer a magnetic needle which is extremely short is attached to a brass cylinder. As it is very short it measures the magnetic field of a point.

Here also we have $T=2\pi\sqrt{\frac{I}{mB}}$.

If n = number of oscillations per sec. then $n=\frac{1}{T}$.

$$\therefore \quad n^2=\frac{1}{T}=\frac{mB}{4\pi^2 I}$$

$$\therefore \quad n^2=kB$$

where $k=\frac{m}{4\pi^2 I}$ is a constant of the magnetometer

or $n^2=k'H$ where H is H-vector of the magnetic field.

EXAMPLES

Ex. 1. *A small magnetic needle placed* ·2 m *to the east of a magnet of length* ·08 m *and having its axis at right angles to the* *meridian is deflected through* 60°. *Calculate the magnetic moment of the magnet and its pole strength.* ($B_0 = 36\times10^{-6}$ *tesla*)

Sol. We have, $B=\frac{\mu_0}{4\pi}\cdot\frac{2md}{(d^2-l^2)^2}=B_0\tan\theta$

$$m=\frac{4\pi}{\mu_0}\cdot\frac{(d^2-l^2)^2}{2d}B_0\tan\theta.$$

Here $d=\cdot2+\cdot04=\cdot24$ m $\left(\because\ \frac{\mu_0}{4\pi}=10^{-7}\right)$

$l=\cdot04$ m.

$$\therefore\quad m=10^7\ \frac{(\cdot24^2-\cdot04^2)^2}{2\times\cdot24}\cdot36\times10^{-6}\tan60^\circ$$

$$=\frac{(\cdot0576-\cdot0016)^2}{\cdot48}\times360\times1\cdot732=\frac{\cdot003136}{\cdot48}\times360\times1\cdot732$$

$$=4\cdot073\text{ Am}^2$$

Pole strength $=\frac{4\cdot073}{\cdot08}=50\cdot9$ Am. **Ans.**

Ex. 2. *Two bar magnets are bound together side by side and suspended, so as to swing in horizontal plane freely. The combination makes one oscillation in* 12 *seconds when like poles are together and* 16 *seconds when the direction of one of the magnets is reversed. Compare their magnetic moments.*

Sol. We have, $t_1=2\pi\sqrt{\frac{I_1+I_2}{(m_1+m_2)B}}$

where B = earth's horizontal induction

and $t_2=2\pi\sqrt{\frac{I_1+I_2}{(m_1-m_2)B}}$.

$$\therefore\quad \frac{t_1^2}{t_2^2}=\frac{m_1-m_2}{m_1+m_2}$$

$$\therefore\quad \frac{m_1-m_2}{m_1+m_2}=\frac{12^2}{16^2}=\frac{9}{16}$$

or $\frac{m_1}{m_2}=\frac{25}{7}$. **Ans.**

Ex. 3. *A compass needle makes* 30 *oscillations per minute in the earth's field. When a bar magnet is placed near it so as not to alter the direction of the needle, it makes* 40 *oscillations in one minute. How many times will the needle oscillate per minute if the magnet is reversed ?*

Sol. We have, $30^2 = kB$ where B = earth's horizontal induction. Let B' be the field due to the magnet. Since the direction of the needle is not changed and frequency of vibration has increased, B and B' must be in the same direction.

$$\therefore \quad 40^2 = k(B+B')$$

$$\therefore \quad \frac{40^2}{30^2} = \frac{B+B'}{B}, \quad \text{or} \quad \frac{B'}{B} = \frac{7}{9}.$$

Hence $B < B$.

When the magnet is reversed, B' is reversed. So the resultant field is now $B - B'$ because $B > B'$.

$$\therefore \quad n^2 = k(B-B').$$

$$\therefore \quad \frac{n^2}{30^2} = \frac{B-B'}{B} = 1 - \frac{B'}{B} = 1 - \frac{7}{9} = \frac{2}{9}.$$

$$\therefore \quad n = \frac{30\sqrt{2}}{3} = 10 \times 1{\cdot}414 = 14{\cdot}14. \qquad \textbf{Ans.}$$

Ex. 4. *A cylindrical bar magnet weighing* 150 gm *and measuring* 2 cm *in diameter and* 10 cm *in length oscillates in the earth's field and makes* 10 *oscillations in* 40 *seconds. Calculate the magnetic moment of the magnet. Also calculate its magnetisation. (B-vector of earth's field* $= 3{\cdot}0 \times 10^{-5}$ *tesla.)*

Sol. We have for cylindrical body,

$$I = w\left(\frac{l^2}{12} + \frac{r^2}{4}\right) = 150 \times 10^{-3}\left(\frac{10^2}{12} + \frac{1^2}{4}\right) \times 10^{-4} \text{ kgm}^2$$

$$= 150\left(\frac{100}{12} + \frac{1}{4}\right) \times 10^{-7} \text{ kgm}^2$$

$$= 1287{\cdot}5 \times 10^{-7} \text{ kgm}^2.$$

$$t = \frac{40}{10} = 4 \text{ s}.$$

Now $$t=2\pi\sqrt{\frac{I}{mB}} \quad \text{or} \quad mB=\frac{4\pi^2 I}{t^2}$$

$$\therefore \quad m=\frac{4\pi^2\times 1287{\cdot}5\times 10^{-7}}{3\times 10^{-5}\times 4^2}$$

$m=8{\cdot}4\ \text{Am}^2$. **Ans.**

Magnetisation is defined as magnetic moment per unit volume.

Volume of cylinder $=\pi r^2 l=\pi(1\times 10^{-2})^2\ (10\times 10^{-2})$

$=\pi\ \ 10^{-5}\ \text{m}^3$.

$\therefore \quad M=\frac{M}{V}=\frac{8{\cdot}4}{\pi\times 10^{-5}}\ \text{Am}^{-1}=2{\cdot}67\times 10^5\ \text{Am}^{-1}$. **Ans.**

Ex. 5. *A compass needle makes* 10 *oscillations per minute in the earth's field alone. A bar magnet is placed in such a way that its field at the needle is at right angles to the earth's field and deflects the needle through* 30°. *If the needle is made to oscillate in the deflected position, find the number of oscillations that it will make in one minute.*

Sol. We have, $B=B_0\tan\theta$ (where $B_0=B$-vector of earth's magnetic field)

$$=B_0\tan 30^\circ=\frac{B_0}{\sqrt{3}}.$$

$\therefore$ Resultant field $=\sqrt{B^2+B_0^2}=\sqrt{\frac{B_0^2}{3}+B_0^2}=\frac{2B_0}{\sqrt{3}}$.

According to the principle of vibration magnetometer,

$$B_0=k.10^2$$

and $$\frac{2B_0}{\sqrt{3}}=k.n^2.$$

$$\therefore \quad \frac{n^2}{10^2}=\frac{2}{\sqrt{3}},\ \text{or}\ n^2=115{\cdot}4,\ \text{or}\ n=10{\cdot}7.$$ **Ans.**

Ex. 6. *Two magnets of equal magnetic moment and inertia are fastened together so that they cross each other at angle of* 60°. *If the combination is suspended by unspun silk fibre, what will be the time period of small oscillations of it, if time period of each magnet in a similar condition is* 5 s ?

Sol. Magnetic moment is a vector quantity. Hence the two magnets will behave like a single magnet of moment given by

$$m' = \sqrt{m^2+m^2+2mm\cos 60^2}$$
$$= \sqrt{3m^2} = m\sqrt{3}.$$

But moment of inertia is not a vector.

Hence, $I' = I + I = 2I$ (where I = moment of inertia of each magnet)

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{m'B}} = 2\pi\sqrt{\frac{2I}{\sqrt{3}\,mB}} = \sqrt{\frac{2}{\sqrt{3}}} \times 2\pi\sqrt{\frac{I}{mB}}$$

$$= \sqrt{\frac{2}{\sqrt{3}}} \times T \qquad \left(\because \; T = 2\pi\sqrt{\frac{I}{mB}}\right)$$

$$\text{or} \quad T = \sqrt{\frac{2}{1{\cdot}732}} \times 5 = \frac{1{\cdot}414}{1{\cdot}316} \times 5 = 5{\cdot}37 \text{ s.}$$ **Ans.**

(For roots see log table)

Ex. 7. *A small magnetic needle when placed 29 cm to the east of a magnet of length 8 cm and having its axis at right angles to the meridian, it is deflected through 60°. Calculate the moment of the magnet and its pole strength. (H-vector of earth's field along horizontal* = 30 Am^{-1}).

Sol. Since here H-vector is given, we use expression for H-field due to a magnet.

$$H = \frac{1}{4\pi} \cdot \frac{2md}{(d^2-l^2)^2} = H_0 \tan\theta \quad \text{or} \quad m = 4\pi\frac{(d^2-l^2)^2}{2d} \cdot H_0 \tan\theta.$$

In this case, $H_0 = 30$ Am^{-1}, $d = 20+4 = 24$ cm $= 24 \times 10^{-2}$ m;

$l = 4$ cm $= 4 \times 10^{-2}$ m and $\theta = 60°$.

$$\therefore \quad m = 4\pi\frac{(24^2-4^2)^2 \times 10^{-8}}{2 \times 24 \times 10^{-2}} \times 30 \times \tan 60°$$ **Ans.**

or $m = 3{\cdot}53$ Am^2. **Ans.**

$$\text{Pole strength} = \frac{3{\cdot}53}{8 \times 10^{-2}} = 44 \text{ Am}^{-1}.$$

Ex. 8. *An iron rod 20 cm long and radius 2 cm is uniformly magnetised and its time of oscillation is 4 s. It is broken into four parts. Find the time period of each part.*

Sol. $$I \text{ (m.i. of rod)} = w \times \left(\frac{20^2}{12} + \frac{2^2}{4} \right) \times 10^{-4} \text{ kgm}^2$$

$$= w \times 34{\cdot}33 \times 10^{-4} \text{ kgm}^2.$$

When a magnet is broken, the only thing that remains unchanged is its pole strength.

$\therefore$ m' (magnetic moment of each part) $= q_m \times \frac{20}{4} \times 10^{-2}$ where q_m is the pole strength of the rod $= 5q_m \times 10^{-2}$.

$$I' = \frac{w}{4} \times \left(\frac{5^2}{12} + \frac{2^2}{4} \right) \times 10^{-4} \text{ kgm}^2 = \frac{w}{4} \times 3{\cdot}08 \times 10^{-4} \text{ kgm}^2$$

$$\therefore \quad \frac{I}{I'} = \frac{4 \times 34{\cdot}33}{3{\cdot}08}; \quad \frac{m}{m'} = \frac{q_m.20 \times 10^{-2}}{5q_m \times 10^{-2}} = 4.$$

Now, $$T = 2\sqrt{\frac{I}{mB}} \text{ and } T' = 2\pi\sqrt{\frac{I'}{m'B}}.$$

$$\frac{T}{T} = \sqrt{\frac{I'}{I} \times \frac{m}{m'}} = \sqrt{\frac{3{\cdot}08}{4 \times 34{\cdot}33} \times 4} = \sqrt{\frac{3{\cdot}08}{34{\cdot}33}}$$

or $$T' = \sqrt{\frac{3{\cdot}08}{34{\cdot}33}} \times 4 = 1{\cdot}2 \text{ s.}$$ **Ans.**

Ex. 9. *A magnet is placed in the magnetic meridian with north pole pointing pole north and its neutral point is located. A vibration magnetometer is placed at the neutral point and the magnet is then taken away and the magnetometer is set to oscillations. It makes* 10 *oscillations per minute. The magnet is then replaced in its original position, but now north pole pointing south. If the magnetometer is set to oscillations, how many oscillations will it make in one minute ?*

Sol. At a neutral point, field due to earth is equal and opposite to the field due to magnet and the total field is zero. But when the magnet is replaced with north pole taking the place of south pole, the field due to magnet is reversed and so at the same point there will be a resultant field double the field due to earth.

From theory of magnetometer, $10^2 = k\, B_0$

and $$n^2 = k\,(B_0 + B_0) = k\; 2B_0$$

$$\therefore \quad \frac{n^2}{10^2} = \frac{k\; 2B_0}{B_0}, \text{ or } n = 14{\cdot}14.$$ **Ans.**

EXERCISES

(A)

1. Two short bar magnets are placed one above the other with their (a) similar poles, (b) dissimilar poles together. The first combination is placed at a distance of ·2 m from the needle of a deflection magnetometer in tan A position. The deflection is observed to be 60°, the second combination is placed at half of this distance in tan B position of the magnetometer and the deflection is now 30°. Compare the moments of magnet. (Ans. 13 : 11)

2. A bar magnet of length ·1 m, breadth ·02 m and thickness ·005 m is placed at a distance ·3 m from the centre of a deflection magnetometer arranged in tan A position and it produces a deflection of 38°. Calculate magnetisation of the magnet. (Horizontal induction of earth $= 3 \times 10^{-5}$ tesla.)
(Ans. $2{\cdot}99 \times 10^5$ Am^{-1})

3. A short bar magnet is placed on the arm of a deflection magnetometer arranged in the usual manner in tan B position and it produces a deflection of 60°. It is now broken into two equal parts and one part is placed exactly at the same distance. What deflection does it produce ? At what distance should it be placed to produce the same deflection ? If the other piece is placed on the top of it at right angles to it, will there be any change in deflection ?
(Ans. 40°54′ ; ·793 times the previous distance; yes, restoring field will be changed)

4. A compass needle vibrates 10 times per minute in earth's field alone. When the north pole of a long magnet is brought towards the south of the needle and 10 cm away from it, it makes 20 vibrations per minute. Calculate the number of vibrations of the needle when the distance of the pole of the magnet is reduced to 8 cm. (Ans. 23·8 vibrations per minute)

5. A rectangular bar magnet of length ·1 m and breadth ·01 m oscillates once in 10 s in earth's horizontal field of intensity 18×10^{-6} T. If the mass of the magnet is 40×10^{-3} kg, find the magnetic moment of the magnet.
(Ans. ·74 Am^2)

6. Two short bar magnets exactly alike, of magnetic moment 60×10^{-3} Am^2 and 80×10^{-3} Am^2, are fastened together with their axes parallel and horizontal and their centres in a vertical line about which the system is free to oscillate. Compare the times of oscillation in the earth's field when the axes of magnets are in the same direction and in the opposite direction. (Ans. 1 : 2·646)

7. A small magnet vibrating in earth's horizontal field completes 4 oscillations in 16 seconds, and when a small magnet is brought near it oscillates 5 times in 16 seconds. Compare the intensity of the second magnet with earth's horizontal intensity when (i) both the fields are in the same direction (ii) in opposite direction. [Ans. (i) 9 : 16, (ii) 41 : 16]

8. An iron cylinder 1 m in length and ·02 m in diameter in section is uniformly magnetised. Now a short length ·05 m is cut from it and placed on the frame of an oscillation magnetometer. It makes 10 oscillations in 30 seconds. Calculate magnetisation of the rod. (Density of iron $= 7000$ kg m^{-3} and $B = 4 \times 10^{-5}$ tesla.) (Ans. $1{\cdot}792 \times 10^5$ Am^{-1})

9. A thin iron rod 30 cm long is uniformly magnetised and its time of oscillation is 4 seconds. It is broken into 3 equal parts. Find the time of oscillation of each part. (**Ans.** 1·339 s)

10. Two bar magnets, the moment of one of which is double that of the other, but otherwise similar, are tied together so as to form a 'cross'. The 'cross' is now freely suspended in such a way that it may oscillate in horizontal plane. How the 'cross' will rest in earth's field ? When it is set to oscillation, what will be its time period if the time period of the weaker magnet in similar position is 5 sec ? (**Ans.** At 63°30′; 4·73 s)

11. A compass needle makes 10 oscillations per minute in the earth's field alone. A bar magnet is placed in such a way that its field at the needle is at right angles to the earth's field and deflects the needle through 45°. If the needle is made to oscillate in the deflected position find the number of oscillations per minute. (**Ans.** 11·87 per minute)

12. Two identical bar magnets are tied together at their centres at an inclination of 60° with each other. Calculate the time period of oscillation of the combination when the time of oscillation of each magnet is 2 sec. (**Ans.** 2·15 s)

13. A magnet of moment ·5 Am^2 is deflected through an angle of 60° in a magnetic field of strength 4×10^{-5} tesla and then released. Its angular velocity at the instant it passes through its equilibrium position is ·5 radian per second. Calculate its moment of inertia. (**Ans.** 8×10^{-5} kgm^2)

14. A short magnet produces a deflection 45° in a deflection magnetometer when placed in the usual manner. A similar magnet is put on it with like poles together. What deflection do the two magnets jointly produce ? (**Ans.** 63°26′)

15. A bar magnet measuring 3 cm × 2 cm × 1 cm is placed on the frame of an oscillation magnetometer with its 2 cm edge vertical and its time period of oscillation in the earths field is 5 s. What will be the time period when (a) 1 cm edge is vertical, (b) 3 cm edge is vertical ? (**Ans.** 5·7 s; 3·54 s)

16. A short bar magnet when suspended horizontally and perpendicular to the earth's field experiences a torque of 3×10^{-4} Nm. The same magnet produces a deflection of 45° in the needle of a deflection magnetometer set in tan A position when the distance of the magnet from the centre of the magnetometer is ·3 m. Calculate the moment of the magnet. (**Ans.** 6·37 Am^2)

17. A bar magnet weighing 250 gm and measuring 10 cm × 2 cm × 1 cm is suspended by unspun silk fibre passing through its centre of gravity with 1 cm edge vertical. It takes 1 minute 40 s in 20 oscillations. Calculate mB_0 of the magnet. Also calculate the torque required to deflect it in earth's field through 90° and the work done by the torque.

(**Ans.** $3{\cdot}42 \times 10^{-4}$ joule; $3{\cdot}42 \times 10^{-4}$ newton metre; $3{\cdot}42 \times 10^{-4}$ joule)

(B)

18. *P* is a neutral point on the perpendicular bisector of a bar magnet. A small compass needle is placed at *P*. The bar magnet is then removed and the needle is set in oscillations. Finally the bar magnet is carefully replaced in its position, but north pole pointing south, and the needle is again in oscillation. Compare the periodic times of oscillation in the two cases. (**Ans.** 1·414 : 1)

19. A deflection magnetometer is arranged with its arms along east-west direction. A small bar magnet with its north pole pointing east is placed lengthwise along the eastern arm. The needle is found to be deflected through 30°. What would be the deflection of the needle if the magnetometer arms are rotated anti-clockwise through (a) 90°, (b) 45° ?

[**Ans.** (a) no deflection; (b) 16°]

20. A bar magnet of length 10 cm and breadth 2 cm weighs 200 gm. When it is put on the frame of an oscillation magnetometer, it makes 20 oscillations in 1 minute 20 seconds. It produces a deflection of 45° from a distance of 10 cm of its nearer pole from the needle of a deflection magnetometer arranged in tan *A* position. Calculate magnetic moment and *B*-vector of the earth's magnetic field along the horizontal.

$$\left(\textit{Hint}: mB_0=\frac{4\pi^2 I}{t^2} \quad \text{and} \quad \frac{m}{B_0}=\frac{4\pi}{\mu_0}\cdot\frac{(d^2-l^2)^2}{2d}\tan\theta\right)$$

(**Ans.** 2·39 Am^2; $1{\cdot}8\times10^{-4}$ tesla)

21. A deflection magnetometer is set in tan *A* position. A short bar magnet is placed at a certain distance on the arms of the magnetometer with its north pole pointing north. There is no deflection. When the needle of the magnetometer is set to oscillation by an auxiliary magnet, it makes 5 oscillations in one minute. When the magnet is turned through 180° in its own position, it now makes 20 oscillations per minute.. What permanent deflection will be produced when the magnet will be along the arm of the magnetometer at the same position ?

(**Ans.** 60°24′)

* * *

CHAPTER 6

TERRESTRIAL MAGNETISM

1. Elements of earth's magnetic field : There are three elements of earth's magnetic field :

(*a*) *Declination* : Angle between the geographical meridian and the magnetic meridian.

(*b*) *Inclination or Dip* : Angle between the direction of resultant intensity and a horizontal line in the magnetic meridian.

(*c*) *Horizontal intensity* : It is the resolved part of the total intensity along the horizontal in the magnetic meridian.

2. Resultant intensity and its components :

Let $B = B$-vector of earth's total field.

B_o = horizontal component

B_v = vertical component.

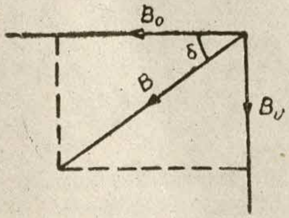

Fig. 6-1

Then it follows from the principle of resolution of vectors,

$$B_o = B\cos\delta$$
$$B_v = B\sin\delta$$

where δ = angle of dip.

$$\therefore \quad \tan\delta = \frac{B_v}{B_o}.$$

Exactly in the same way if $H = H$-vector of earth's magnetic field,

$$\tan\delta = \frac{H_v}{H_o}.$$

3. Apparent dip : When the plane of a dip circle is not in the magnetic meridian, the angle made by the dip needle with the horizontal is called apparent dip. Suppose α = angle made by the plane of the dip circle with the magnetic meridian. Obviously the vertical component is in the plane of the dip circle but the horizontal component is out of the plane of the dip circle by α. Resolving horizontal

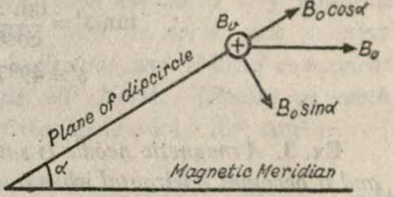

Fig. 6-2

component along and perpendicular to the plane of the dip circle we have $B_0 \cos\alpha$ along the plane and $B_0 \sin\alpha$ perpendicular to it. The normal component is ineffective on the needle because the needle is free to turn only in the plane of the dip circle. Hence in the plane of the dip circle the effective intensities are (i) B_v along the vertical, (ii) $B_0 \cos\alpha$ along the horizontal.

If δ' = angle made by the dip needle with the horizontal in the plane of the dip circle,

then $$\tan\delta' = \frac{B_v}{B_0 \cos\alpha}. \quad \text{But } \tan\delta = \frac{B_v}{B_0}.$$

$$\therefore \quad \tan\delta' = \frac{\tan\delta}{\cos\alpha}.$$

EXAMPLES

Ex. 1. *The angle of dip at a particular place where horizontal intensity is* 30 Am^{-1} *is found to be* 38°. *Calculate the total intensity of the earth's magnetic field there.*

Sol. We have, $$H_0 = H\cos\delta \qquad \text{(just as } B_0 = B\cos\delta\text{)}$$

$$\therefore \qquad H = \frac{H_0}{\cos\delta} = \frac{30}{\cos 38^\circ}\ \text{Am}^{-1}$$

or $$H = \frac{30}{\cdot 788} = 38{\cdot}06\ \text{Am}^{-1}. \qquad \textbf{Ans.}$$

Ex. 2. *The true dip at a certain place is* 30°. *What is the apparent dip when the dip circle is turned* 60° *out of the magnetic meridian ?*

Sol. See the article and deduce

$$\tan\delta' = \frac{\tan\delta}{\cos\alpha}.$$

$$\therefore \quad \tan\delta' = \frac{\tan 30^\circ}{\cos 60^\circ} = \frac{\cdot 5774}{\cdot 5} = 1{\cdot}155$$

$$\delta' = 49^\circ 7'.$$ **Ans.**

(Angle taken from the log table)

Ex. 3. *A magnetic needle is suspended by a thread at its centre and it becomes horizontal when a weight of* 100 mgm *is placed at its front end. If the pole strength of the needle is* 5 Am, *find the vertical intensity of the earth's field.* ($g = 9{\cdot}8$ m/s^2)

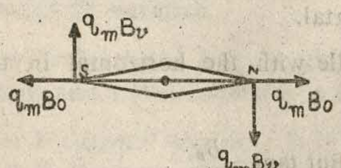

Fig. 6-3

Sol. The forces acting on the needle are shown in the Fig. 6-3. For equilibrium of the needle, the algebraic sum of the moments of all forces must vanish.

$$\therefore \quad wgl - q_mB_vl - q_mB_vl = 0$$

(taking anti-clockwise moment positively)

or $$wg = 2q_mB_v$$

$$\therefore \quad B_v = \frac{wg}{2q_m} = \frac{100 \times 10^{-6} \times 9{\cdot}8}{2 \times 5} \text{ tesla}$$

or $B_v = {\cdot}98 \times 10^{-4}$ tesla. $\therefore \quad H_v = \dfrac{\cdot 98 \times 10^{-4}}{4\pi \times 10^{-7}} = 78{\cdot}0 \text{ Am}^{-1}$. **Ans.**

Ex. 4. *A dip circle is rotated till the needle sets vertical. From this position it is rotated through* α *and the needle shows a deflection of* ψ. *Show that the true dip is* $\tan^{-1}(\tan\psi \sin\alpha)$.

Sol. We have, $\tan\delta' = \dfrac{\tan\delta}{\cos\alpha}$ where δ' = apparent dip, δ = true dip and α = angle made by the dip circle with the magnetic meridian.

When the needle sets vertical, $\delta' = 90^\circ$.

$\therefore \quad \tan 90^\circ = \dfrac{\tan\delta}{\cos\alpha}$ or $\cos\alpha = 0$ or $\alpha = 90^\circ$. Thus when needle sets vertical, the plane of the circle is 90° away from magnetic meridian. If it is rotated through α from this position, the dip circle will remain $(90-\alpha)^\circ$ away from the magnetic meridian.

$$\therefore \quad \tan\psi = \frac{\tan\delta}{\cos(90-\alpha)}, \text{ or } \tan\delta = \tan\psi\sin\alpha$$

or $$\delta = \tan^{-1}(\tan\psi\sin\alpha).$$ **Proved.**

Ex. 5. *Assume the earth's magnetic field due to a very powerful magnetic dipole placed at its centre with its north pole pointing south of the earth. Calculate the horizontal and vertical component of earth's field at a place of latitude* 60° *North.* (*Radius of earth* $=6400$ km *and magnetic moment of the dipole* $=6{\cdot}4\times10^{22}$ Am^2.)

Sol. The angular distance of a place from equator is called latitude of the place. We know that the components of the field due to a short bar magnet (a magnetic dipole) are $\frac{\mu_0}{4\pi}.\frac{2m\cos\theta}{r^3}$ along r and $\frac{\mu_0}{4\pi}.\frac{m\sin\theta}{r^3}$ perpendicular to r.

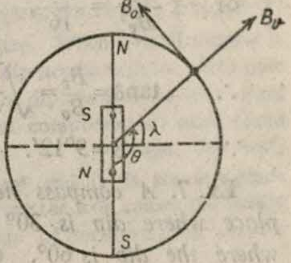

Fig. 6-4

Here the component along r is the vertical component and the horizontal component is the other one.

$$\therefore \qquad B_0=\frac{\mu_0}{4\pi}\cdot\frac{m\sin\theta}{r^3} \text{ and } Br=\frac{\mu_0}{4\pi}.\frac{2m\cos\theta}{r^3}.$$

Here $\qquad \theta=90°+\lambda=90°+60°=150°.$

$$\therefore \; B_o=10^{-7}\frac{6{\cdot}4\times10^{22}.\sin 150°}{(6400\times10^3)^3} \text{ and } B_v=10^{-7}\frac{2\times6{\cdot}4\times10^{22}\cos 150°}{(6400\times10^3)^3}$$

$$(\because \; \mu_o=4\pi\times10^{-7})$$

or $\quad B_o=1{\cdot}22\times10^{-5}$ tesla and $B_v=-4{\cdot}2\times10^{-5}$ tesla.

The minus sign shows that the vertical field will not be away from the centre but it will be towards the centre. **Ans.**

Ex. 6. *A dip circle is rotated till the needle sets vertical. In this position needle makes* 5 *oscillations in* 15 *seconds. When the dip circle is rotated through* 90°, *it now makes the same number of oscillations in* 6 *seconds. Calculate dip of the place.*

Sol. When dip needle is vertical only vertical component is effective on it. Therefore according to the theory of vibration magnetometer,

$$B_v=k\left(\frac{5}{15}\right)^2=\frac{k}{9} \qquad \ldots (1).$$

When it has been brought in the magnetic meridian the needle is now oscillating in the total field.

$$\therefore \quad B=k(\tfrac{5}{6})^2=k(\tfrac{25}{36})$$

$$\therefore \quad \frac{B}{B_v}=\frac{25k}{36}\times\frac{9}{k}=\frac{25}{4}, \quad \text{or} \quad \frac{\sqrt{B_o^2+B_v^2}}{B_v}=\frac{25}{4}$$

$$\text{or} \quad \frac{B_o^2+B_v^2}{B_v^2}=\frac{625}{16}. \quad \text{or} \quad 1+\frac{B_o^2}{B_v^2}=\frac{625}{16}$$

$$\text{or} \quad \frac{B_o^2}{B_v^2}=\frac{609}{16}, \quad \text{or} \quad \frac{B_v}{B_o}=\sqrt{\frac{16}{609}}$$

$$\therefore \quad \tan\delta=\frac{B_v}{B_o}=\sqrt{\frac{16}{609}}=\cdot 162$$

$$\therefore \quad \delta=9°12'.$$

Ans.

Ex. 7. *A compass needle makes* 50 *oscillations per minute at a place where dip is* 30° *and* 40 *oscillations per minute at a place where the dip is* 60°. *Compare the earth's total intensity at the two places.*

Sol. We have from theory of vibration magnetometer,

$$50^2=kB_o \text{ and } 40^2=kB_o'.$$

$$\therefore \quad \frac{25}{16}=\frac{B_o}{B_o'}. \text{ Also } B_o=B\cos 30° \text{ and } B_o'=B'\cos 60°.$$

$$\therefore \quad \frac{B_o}{B_o'}=\frac{B\cos 30°}{B'\cos 60°}. \quad \therefore \quad \frac{25}{16}=\frac{B}{B'}\cdot\frac{\cos 30°}{\cos 60°}$$

$$\text{or} \quad \frac{B}{B'}=\frac{25}{16}\times\frac{\cos 60°}{\cos 30°}=\frac{25}{16}\times\frac{\cdot 5}{\cdot 866}$$

$$\text{or} \quad B:B'=\cdot 902:1.$$

Similarly, H-vector of the fields at the two places,

$$H:H'=\cdot 902:1.$$

Ans.

EXERCISES

1. At a place where horizontal component is 25×10^{-6} T, and the dip 45°, calculate the total induction at that place. (Ans. $35{\cdot}35\times10^{-6}$ T)

2. The vertical component of the earth's field at a place is 40 Am^{-1}. Calculate the value of H_0, if dip of the place is 30°. (Ans. 69 28 Am^{-1})

3. The apparent dip at a place in a plane 30° away from the magnetic meridian is 60°. Calculate the true dip at the place. (Ans. 56° 18′)

4. The apparent dips in two mutually perpendicular planes are found to be δ_1 and δ_2. Show that true dip δ is related with δ_1 and δ_2 by

$$\cot^2\delta=\cot^2\delta_1+\cot^2\delta_2.$$

(*Note* : This is the principle of cot-method of finding true dip of a place.)

5. Considering that the earth's magnetism is due to a very powerful short magnet embedded at its centre placed with its axis (*SN* direction) along *NS* direction of the earth, show that latitude (λ) of any place bears a definite relation with the dip (δ) of the place and that definite relation is $\tan\delta=2\tan\lambda$.

6. The earth is a big dipole of moment $6{\cdot}4\times10^{22}$ Am^2. Calculate the horizontal and vertical components of earth's magnetic field at a place of latitude 30° South. (Radius of the earth = 6000 km)

(*Hint* : See Ex. 5.) (**Ans.** 3×10^{-5} T; $2{\cdot}5\times10^{-5}$T)

7. A dip needle indicates a dip of 60° at a place. When a small magnet is placed on the horizontal line through the centre of dip needle with its north pole pointing north at a distance ·2 m from the needle, the dip changes to 45°. Find the magnetic moment of the magnet if the horizontal component of earth's field is $\cdot2\times10^{-4}$ tesla. (**Ans.** ·585 Am^2)

8. The magnetic moment of a dip needle is ·5 Am^2 and it is set at a place where dip is 60°. A weight of ·05 gm placed 4 cm from the axle causes the needle to turn into horizontal position. Calculate the value of horizontal and vertical components of the earth's field. (**Ans.** $B_v=\cdot392\times10^{-4}$ tesla; $B_0=\cdot225\times10^{-4}$ tesla)

9. The period of oscillation of a dip needle when vibrating in the magnetic meridian is 1·5 s. In a plane at right angles to the magnetic meridian it is 2 s; find the dip of the place. (**Ans.** 34° 15′)

10. In an experiment of finding dip by cot-method it is found that the apparent dips in two mutually perpendicular planes are 30° and 20°. Calculate the true dip of the place. (**Ans.** 17°7′)

11. The needle of a dip circle completes 4 oscillations in one minute while vibrating in a vertical plane at right angles to magnetic meridian. The same needle oscillates 3 times per minute in a horizontal plane at the same place. Find the value of dip at the place. (**Ans.** 60° 36′)

12. A magnetic needle is suspended by a thread at its centre and it shows a dip of 60°. When a weight of 20 mgm is placed at its front end, the dip is reduced to 30°. If the vertical component of the earth's field is 4×10^{-5} tesla, find the pole strength of the needle. (**Ans.** 3·67 Am^{-1})

13. At a place where the *H*-vector of magnetic field along the horizontal is 30 Am^{-1} andthe dip 40°. Calculate earth's total induction (*B*-vector) at that place.

$$\left(\textit{Hint}: H_0=H\cos\delta;\quad \therefore\ H=\frac{H_0}{\cos\delta};\quad B=\mu_0 H.\right)$$

(**Ans.** $4{\cdot}9\times10^{-5}$ tesla)

14. At a place where the angle of dip is 45° and the total intensity of earth's field is 40 Am^{-1}, a magnet vibrating horizontally makes 10 oscillations per minute. Calculate the number of oscillations per minute it would make at another place where the dip is 60° and the total intensity is 50 Am^{-1}. (**Ans.** 9·4 oscillations/min.)

15. A dip circle after levelling is set in any position and dip is found to be α. Then it is rotated through 45° away from the magnetic meridian from this position and dip is found to be β. Show that true dip is given by $\cot^2\delta=2(\cot^2\alpha+\cot^2\beta-\sqrt{2}\cot\alpha\cot\beta)$.

□□□

CHAPTER 1

ELECTRIC FIELD AND POTENTIAL

1. Coulomb's Law of force in electrostatics : (i) Likely charged bodies repel and unlikely charged bodies attract. (ii) The force of attraction or repulsion between two point charges in a vacuum is proportional to the product of charges and inversely proportional to the square of the distance between them. Mathematically Coulomb's law states

$$F_0 \propto \frac{q_1 q_2}{r^2}, \quad \text{or} \quad F_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad \cdots \quad (1)$$

where ϵ_0 is a universal constant called 'universal electric constant'. It is also called 'permittivity' of a vacuum or air. When the same two point charges are placed in an *extended homogeneous* material medium, the force between them is changed. *The ratio of the force between the two given point charges in a vacuum to the force between the same two charges in the same positions in a material medium is called the relative permittivity of the medium.* It is denoted by ϵ_r.

Thus,
$$\epsilon_r = \frac{F_0}{F} \quad \text{or} \quad F = \frac{F_0}{\epsilon_r}$$

or
$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q_1 q_2}{r^2} \qquad \cdots \quad (2)$$

$$= \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \qquad \cdots \quad (2a)$$

where $\epsilon = \epsilon_0 \epsilon_r$ or $\epsilon_r = \frac{\epsilon}{\epsilon_0}$. Thus ϵ_r is the ratio of the permittivity of the medium to the permittivity of a vacuum or air. This is why it is called the relative permittivity and ϵ is called the absolute permittivity of the medium. ϵ_r has no unit but ϵ has definite unit. It is *'farad per metre* (Fm^{-1})'. $\frac{1}{4\pi}$ is introduced in (1) just to 'rationalise' the law. (See author's Introductory Physics, Part II, p 4. Electrostatics chapter.)

Taking the law of force in electrostatics in this form and Ampere's law of force between current elements in the form

$$\triangle F=\frac{\mu_0\mu_r}{4\pi}\cdot\frac{(I_1\triangle l_1)(I_2\triangle l_2)\sin\theta}{r^2}.$$

Maxwell showed in his famous 'theory of electromagnetic waves' that μ and ϵ of a medium are related to each other by the relation $c=\frac{1}{\sqrt{\mu\epsilon}}$ where c is the velocity of electromagnetic wave (and therefore of light as well because light is also an electromagnetic wave) in that medium. For a vacuum, $c_0=\frac{1}{\sqrt{\epsilon_0\mu_0}}$. We have already selected arbitrarily $\mu_0=4\pi\times10^{-7}$ Hm^{-1}. Hence we have no choice for ϵ_0. It has to be adjusted according to this formula.

$$\therefore\quad \epsilon_0=\frac{1}{c_0{}^2\mu_0}=\frac{1}{9\times10^{16}\times4\pi\times10^{-7}}\quad(\because\quad c_0=3\times10^8\ \text{ms}^{-1})$$

$$\text{or}\quad \epsilon_0=\frac{1}{36\pi\times10^9}$$

$$\text{or}\quad \frac{1}{4\pi\epsilon_0}=9\times10^9 \text{ and } \epsilon_0=8{\cdot}85\times10^{-12}\ \text{Fm}^{-1}.$$

2. Electric Field : An electric field of strength E is said to exist at a point if a test charge '$\triangle q$' placed at that point experiences force given by the formula, $\triangle F=\triangle qE$

or vectorially, $\vec{E}=\frac{\triangle\vec{F}}{\triangle q}$ volt per metre (Vm^{-1}).

3. Electric field at a point due to a point charge : The electric field due to a point charge q at a distance r from it, is

$$E=\frac{1}{4\pi\epsilon_0\epsilon_r}\cdot\frac{q}{r^2}\ \text{Vm}^{-1}\text{ along } r$$

or vectorially, $\vec{E}=\frac{1}{4\pi\epsilon_0\epsilon_r}\cdot\frac{q\,\hat{r}}{r^2}$.

4. Electric Potential : The electric potential at a point in an electric field is defined as the amount of work done in bringing unit positive charge by an external agent from infinity up to that point. Unit, volt (V).

$$V_P \text{ (potential at a point)} = \frac{\triangle W_{\infty \to P} \text{ (by an external agent)}}{\triangle q}$$

and $$V_P - V_Q = \frac{\triangle W_{Q \to P} \text{ (by agent)}}{\triangle q} \text{ volt (V).}$$

5. **Potential at a point due to a point charge** : The electric potential at a point due to a point charge q at a distance r from it, is

$$V = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q}{r} \text{ volt (V).}$$

6. **Relation of electric field and potential** : The electric field E and potential V at a point are related by the relation

$$E = -\frac{dV}{dr}.$$

This is why unit of intensity of electric field is **volt per metre.**

7. **Mutual potential energy (electrical)** : If q_1 and q_2 are two charges placed at $r_{1\,2}$ distance apart then $\frac{q_1 q_2}{4\pi\epsilon_0 r_{1\,2}}$ is the mutual electrical potential energy. If we have three charges q_1, q_2 and q_3 at the vertices of a triangle then electrical potential energy of the system is

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right) \text{ joule (J).}$$

If q_1, q_2, q_3, q_4 are the charges at the vertices of a quadrilateral then,

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_4}{r_{34}} + \frac{q_4q_1}{r_{41}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_4}{r_{24}}\right).$$

EXAMPLES

Ex. 1. *A point charge of* $+3 \times 10^{-6}$ *coulomb* (C) *is* 12 cm *distant from a second point charge of* $-1{\cdot}5 \times 10^{-6}$ C. *Calculate the force of attraction between them.*

Sol. We have, $$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq'}{r^2}.$$

$$\therefore \qquad F=9\times10^9\frac{(3\times10^{-6})(1\cdot5\times10^{-6})}{\cdot12^2}$$

$$\left(\because \quad \frac{1}{4\pi\,\epsilon_0}=9\times10^9\right)$$

or $$F=2\cdot81\text{ N.}$$ **Ans.**

Ex. 2. *Two small spheres are charged and they repel each other with a force* 2×10^{-3} N. *The charge on one sphere is twice on the other. When taken* 10 cm *farther apart, the force is* 5×10^{-4} N. *What are the charges and what was their original distance ?*

Sol. Let q and $2q$ be the charges and x be the distance between them.

Then, $$2\times10^{-3}=\frac{1}{4\pi\,\epsilon_0}\cdot\frac{2q^2}{x^2}.$$

Again $$5\times10^{-4}=\frac{1}{4\pi\,\epsilon_0}\cdot\frac{2q^2}{(x+\cdot1)^2}.$$

$$\therefore \qquad \frac{20}{5}=\frac{(x+\cdot1)^2}{x^2}$$

or $$2=\frac{x+\cdot1}{x}, \text{ or } 2x=x+\cdot1$$

or $$x=\cdot1\text{ m.}$$ **Ans.**

Putting value of x so obtained we have

$$2\times10^{-3}=9\times10^9\frac{2q^2}{\cdot1^2} \quad \left(\because \quad \frac{1}{4\pi\,\epsilon_0}=9\times10^9\right)$$

or $$q^2=\frac{10^{-14}}{9};\ \therefore\ q=\frac{10^{-7}}{3}=33\cdot33\times10^{-9}\text{C}$$

and $$2q=66\cdot66\times10^{-9}\text{C}.$$

So, the charges on the spheres are $33\cdot33\times10^{-9}$C and $66\cdot66\times10^{-9}$C. **Ans.**

Ex. 3. 5×10^{-5} *joule of work is done when a charge of* 2×10^{-9} C *is moved from a point of potential* (-3000 *volts*) *to another point P. Find the potential at the point P.*

Sol. Let V be the potential at P. Then potential difference between the two points $=V-(-3000)=(V+3000)$ volts.

This is equal to work done in taking 1 C of positive charge from P to the other point.

$$V+3000=\frac{5\times10^{-5}}{2\times10^{-9}}$$

or $$V+3000=25000$$

$$V=22000 \text{ volt.}$$ **Ans.**

Ex. 4. *A charge* 20×10^{-9} C *is placed at a point. What work will be required* (*a*) *to bring* 1 C *positive charge from infinity to a distance* 10 cm *from it ?* (*b*) *to carry a charge of* 5C *once completely round it in a circle of radius* 5 cm *? Give reasons.*

Sol. (a) By definition, the potential at any point is the work done in moving 1 C of positive charge from infinity up to the point. Hence *potential at a point is the required work done per coulomb of charge.*

Potential at a distance 10 cm $=\dfrac{1}{4\pi\epsilon_0}.\dfrac{20\times10^{-9}}{\cdot1}$

$=9\times10^9.20\times10^{-8}$ $\left(\because \dfrac{1}{4\pi\epsilon_0}=9\times10^9\right)$

$=1800$ volt.

Work to be done $=1800$ joule. **Ans.**

(b) By definition of potential difference.

$$\therefore \qquad V_P-V_Q=\frac{\triangle W_{Q\rightarrow P}\text{ (by agent)}}{\triangle q}.$$

Here P and Q are identical points.

$\therefore \qquad V_P-V_Q=0. \qquad \therefore \quad \triangle W=0.$ **Ans.**

Ex. 5. *A hollow spherical conductor of radius* 10 cm *is charged with* 5×10^{-9}C. *Find the potential* (*a*) *at a distance* 20 cm *from centre,* (*b*) *at the surface of the sphere,* (*c*) *at a distance* 5 cm *from centre.*

Sol. (a) *Charge on a sphere may be supposed to be concentrated at its centre for points lying outside.*

$\therefore$ Potential at a distance 20 cm from the centre

$$=\frac{1}{4\pi\epsilon_0}\cdot\frac{5\times10^{-9}}{\cdot2}$$

$$=9\times10^9\times\frac{5\times10^{-9}}{\cdot2}=225 \text{ V.}$$ **Ans.**

(b) Potential at a distance 10 cm

$$=\frac{1}{4\pi\epsilon_0}\cdot\frac{5\times10^{-9}}{\cdot1}$$

$$=9\times10^9.\frac{5\times10^{-9}}{\cdot1}=450 \text{ volt.}$$ **Ans.**

(c) *Potential inside a hollow conductor is the same at all points and is equal to the potential of the surface.*

∴ Potential at a distance 5 cm from the centre = 450 V. **Ans.**

Ex. 6. *Two pith balls, each of mass* 10 *mgm, are suspended from the same point, each by a silk fibre* 50 *cm long. When equal and similar charges are given to them, they are repelled by a distance* 10 cm *from each other. Calculate the charge on either ball.*

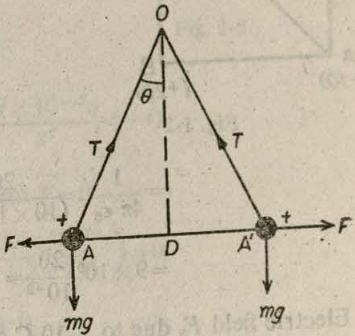

Fig. 1-1

Sol. Each pith ball is in equilibrium under the action of three forces—(i) weight mg of pith ball, (ii) tension T, (iii) electric force F.

So we can apply triangle of forces, or Lami's theorem or consider algebraic sum of moments of all forces about O and equate it to zero or equate resultant force along the vertical or horizontal direction to zero. The last one is the most convenient method.

We have $F=T\cos(90^\circ-\theta)=T\sin\theta$

where $\theta=\angle AOD$ (Fig. 1-1)

and $mg=T\cos\theta.$

Dividing, $\frac{F}{mg}=\tan\theta$

or $F=mg\tan\theta$

or $\frac{1}{4\pi\epsilon_0}\cdot\frac{q\,q}{d^2}=mg\tan\theta$

or $9\times10^9\frac{q^2}{\cdot1^2}=10\times10^{-6}\times9\cdot8.\frac{5}{\sqrt{50^2-5^2}}$

$$\left(\because\ \frac{1}{4\pi\epsilon_0}=9\times10^9\right)$$

or $$q=\sqrt{\frac{9\cdot8\times5}{9\times49\cdot45}}\times10^{-8}=3\cdot3\times10^{-9}\text{ C.}$$ **Ans.**

Ex. 7. *ABCD is a square of side* 10 cm. *Charges of* -10 C *and* $+20$ C *are placed at A and B respectively and* $+10$ C *at D. Find the electric field at C.*

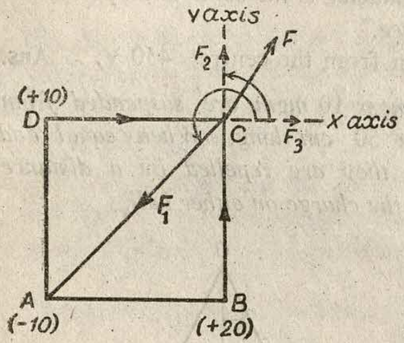

Fig. 1-2

Sol. Electric field F_1 at C due to -10 C at A

$$=\frac{1}{4\pi\epsilon_0}\cdot\frac{10}{(10\sqrt{2}\times10^{-2})^2}$$

along $\overrightarrow{CA}$ (attraction)

$$=9\times10^9\frac{10}{2\times10^{-2}}$$

$$=4\cdot5\times10^{12}\text{ Vm}^{-1}.$$

Electric field F_2 due to $+20$ C at B

$$=\frac{1}{4\pi\epsilon_0}\cdot\frac{20}{(10\times10^{-2})^2}\text{ along }\overrightarrow{BC}\text{ (repulsion)}$$

$$=9\times10^9\frac{20}{10^{-2}}=18\times10^{12}\text{ Vm}^{-1}.$$

Electric field F_3 due to $+10$ C at D

$$=\frac{1}{4\pi\epsilon_0}\cdot\frac{10}{(10\times10^{-2})^2}=9\times10^9\times\frac{10}{10^{-2}}$$

$$=9\times10^{12}\text{ Vm}^{-1}\text{ along }\overrightarrow{DC}\text{ (repulsion).}$$

$$\vec{F_1}=4\cdot5\times10^{12}\cos45^\circ\,(-\hat{i})+4\cdot5\times10^{12}\sin45^\circ\,(-\hat{j})$$

$$=-3\cdot18\times10^{12}\,\hat{i}-3\cdot18\times10^{12}\,\hat{j}$$

$$\vec{F_2}=18\times10^{12}\,\hat{j}$$

$$\vec{F_3}=9\times10^{12}\,\hat{i}$$

$$\vec{F}=\vec{F_1}+\vec{F_2}+\vec{F_3}$$

$$=[(-3\cdot18+9)\,\hat{i}+(-3\cdot18+18)\,\hat{j}]\times10^{12}$$

or $$\vec{F}=[5\cdot82\,\hat{i}+14\cdot82\,\hat{j}]\times10^{12}$$

$$\therefore\ |\vec{F}|=\sqrt{5\cdot82^2+14\cdot82^2}\times10^{12}=15\cdot92\times10^{12}\text{ Vm}^{-1}.$$

Let $\vec{F}$ make angle θ with x-axis, then

$$\tan\theta=\frac{14{\cdot}82}{5{\cdot}82}=2{\cdot}5464$$

or $$\theta=68^\circ 34'.$$ **Ans.**

Ex. 8. *The distance between two charges* 5×10^{-11} C *and* $-2{\cdot}7\times10^{-11}$ C *is* 20 cm. *Where should the third charge be placed on the line joining them so that it will not experience any force ?*

Sol. The charges are $+50$ and -27 pico coulomb (pC) (pico $=10^{-12}$).

Let x be the distance of the third charge 'q' from B to the right of it.

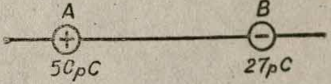

Fig. 1-3

$$\vec{F}_1=\frac{1}{4\pi\epsilon_0}\cdot\frac{50\times30^{-12}q}{({\cdot}2+x)^2}\,(-\hat{i})$$

and $$\vec{F}_2=\frac{1}{4\pi\epsilon_0}\cdot\frac{27\times10^{-12}q}{x^2}(+\hat{i})$$

$$\vec{F}=\vec{F}_1+\vec{F}_2.\ \text{But } \vec{F}=0$$

$$\therefore\quad \frac{1}{4\pi\epsilon_0}\left(\frac{27\times10^{-12}}{x^2}-\frac{50\times10^{-12}}{({\cdot}2+x)^2}\right)=0$$

or $$\frac{27\times10^{-12}}{x^2}=\frac{50\times10^{-12}}{({\cdot}2+x)^2}$$

$$27({\cdot}2+x)^2=50x^2$$

$$\frac{{\cdot}2+x}{x}=\sqrt{\frac{50}{27}}=\sqrt{1{\cdot}85}=1{\cdot}36$$

or $${\cdot}2+x=1{\cdot}36x,\quad\text{or}\quad {\cdot}36x={\cdot}2$$

or $$x=\frac{{\cdot}2}{{\cdot}36}={\cdot}556\text{ m}.$$ **Ans.**

Ex. 9. *A charge of mass* 50 *mgm and carrying* 5×10^{-9} C *is approaching a fixed charge* 10×10^{-9} C *with a velocity point* ·5 m/s. *Find the maximum approach of the charge.*

Sol. Let the charge come nearest to the fixed charge by x metre.

Potential at a distance x from the fixed charge

$$=\frac{1}{4\pi\epsilon_0}\cdot\frac{10\times10^{-9}}{x}=9\times10^{9}\frac{10\times10^{-9}}{x}=\frac{90}{x}\text{ volt.}$$

∴ Potential energy of the moving charge when it has reached the nearest point

$$=\frac{90}{x}\times 5\times 10^{-9} \text{ joule}$$ (∵ Potential at a point is also potential energy per unit charge)

$$=\frac{450\times 10^{-9}}{x}.$$

∴ Change in potential energy of the moving charge

$$=\frac{450\times 10^{-9}}{x}-0$$ (because electrical potential energy at infinity is zero)

$$=\frac{450\times 10^{-9}}{x} \text{ joule}$$

By the principle of conservation of energy,

gain in potential energy = loss in kinetic energy.

$$\therefore \quad \frac{450\times 10^{-9}}{x}=\frac{1}{2}\times(50\times 10^{-6})(\cdot 5)^2$$

or $$x=\frac{45}{625}=\cdot 072 \text{ m}=7\cdot 2 \text{ cm}.$$ **Ans.**

Ex. 10. *Show that when a dielectric slab of thickness t and permittivity ϵ_r is inserted between two fixed charges q and q', the force of repulsion between them is given by*

$$F=\frac{1}{4\pi\,\epsilon_0}\cdot\frac{qq'}{(d-t+\sqrt{\epsilon_r}t)^2}$$

and hence calculate the dielectric constant of glass if on interposing a glass slab of thickness half the distance between the charges force is reduced in the ratio 9 : 4.

Sol. Suppose that two charges q and q' are d distance apart and the space between them is completely filled with a dielectric of permittivity ϵ_r.

Then $$F=\frac{1}{4\pi\,\epsilon_0\epsilon_r}\cdot\frac{qq'}{d^2}.$$

Let us suppose that when the same two charges are d' distance apart in air the same force of repulsion arises between them. Then

$$F=\frac{1}{4\pi\,\epsilon_0}\cdot\frac{qq'}{d'^2}.$$

$$\therefore \quad \epsilon_r d^2 = d'^2, \quad \text{or } d' = \sqrt{\epsilon_r}\, d.$$

Thus equivalent air separation $= \sqrt{\epsilon_r}\, d$ so far the force of interaction is concerned.

Now let us suppose that in between above two charges a dielectric of thickness 't' and permittivity ϵ_r is inserted. Then effective distance of air between the charges $= (d-t) + \sqrt{\epsilon_r}\, t$.

$$\therefore \qquad F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq'}{(d-t+\sqrt{\epsilon_r}\, t)^2}.$$

Proved

Put $\qquad t = \frac{d}{2}.$

Then $$F' = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq'}{\left(d - \frac{d}{2} + \sqrt{\epsilon_r}\frac{d}{2}\right)^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{4qq'}{(1+\sqrt{\epsilon_r})^2 d^2}.$$

Force in air is given by

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq'}{d^2}.$$

$$\therefore \quad \frac{F'}{F} = \frac{4qq'}{4\pi\epsilon_0 (1+\sqrt{\epsilon_r})^2 d^2} \times \frac{4\pi\epsilon_0 d^2}{qq'} = \frac{4}{(1+\sqrt{\epsilon_r})^2}.$$

But $\qquad \frac{F'}{F} = \frac{4}{9}$ (given).

$$\therefore \quad \frac{4}{9} = \frac{4}{(1+\sqrt{\epsilon_r})^2}, \text{ or } \quad 1 + \sqrt{\epsilon_r} = 3, \quad \text{or } \sqrt{\epsilon_r} = 2$$

or $\quad \epsilon_r = 4.$

Ans.

Ex. 11. *A liquid drop hangs in equilibrium between two copper plates separated by a distance* 2 cm *and maintained at a potential difference of* 600 *volts. Calculate the charge on the drop if radius of the drop is* ·1 mm *and density of liquid is* 800 kg/m³.

Sol. There is a uniform electric field between the two charged plates. Electric field means force experienced by unit charge. Hence if q is the charge of drop, force experienced by it in the electric field of strength $E = qE$. When this force equals the weight of the drop, it hangs in equilibrium.

$$\therefore \quad qE = mg.$$

Again, $\qquad$ electric field = rate of charge of potential.

$\therefore \quad E=\frac{V}{d}$, where V = potential difference and d = distance between plates.

$$\therefore \quad q\times\frac{V}{d}=mg, \quad \text{or} \quad e=\frac{mgd}{V}$$

$$\text{or} \quad q=\frac{4\pi/3r^3\rho gd}{V}=\frac{4\pi}{3}.\frac{(\cdot 1\times10^{-3})^3.800\times9{\cdot}8\times2\times10^{-2}}{600}$$

$$\text{or} \quad q=\frac{4\pi\times8\times9{\cdot}8\times2}{3\times6}\times10^{-14}=1{\cdot}095\times10^{-12}\text{C}. \qquad \textbf{Ans.}$$

Ex. 12. *At the corner A of a square ABCD of side* 10 cm *is placed* 60 nC (*nano coulomb*). *Another charge* -40 nC *is located at the centre of the square. Find the work done in carrying a charge of* $+5$ nC *from the corner C to the corner B of the square.*

Sol. *Work done per unit charge is always equal to potential difference.* Hence we will calculate potential of C, potential of B and take potential difference between them. This will give work done per unit charge.

From geometry of figure,

$$OA=OB=OC=OD=5\sqrt{2}\text{ cm}=5\sqrt{2}\times10^{-2}\text{ m}.$$

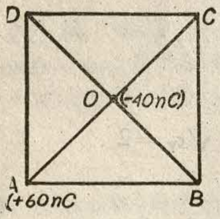

Fig. 1-4

Potential of C

$$=\frac{+60\times10^{-9}}{4\pi\epsilon_0 10\sqrt{2}\times10^{-2}}+\frac{-40\times10^{-9}}{4\pi\epsilon_0 5\sqrt{2}\times10^{-2}}$$

$$=\frac{9\times10^9\times60\times10^9}{10\sqrt{2}\times10^{-5}}-\frac{9\times10^9\times40\times10^9}{5\sqrt{2}\times10^{-2}}$$

$$\left(\because \quad \frac{1}{4\pi\epsilon_0}=9\times10^9\right)$$

$$=2700\sqrt{2}-3600\sqrt{2}=-900\sqrt{2}=-1272{\cdot}6\text{ volt}.$$

$$\text{Potential } B=\frac{60\times10^{-9}}{4\pi\epsilon_0\times10\times10^{-2}}+\frac{-40\times10^{-9}}{4\pi\epsilon_0\times5\sqrt{2}\times10^{-2}}$$

$$=5400-3600\sqrt{2}=5400-5090{\cdot}4=309{\cdot}6\text{ volt}.$$

Potential difference between B and $C=V_B-V_C$

$$=309{\cdot}6-(-1272{\cdot}6)=1582{\cdot}2\text{ volt}.$$

$\therefore$ Work done from C to $B=1582{\cdot}2$ joule.

$\therefore$ Work done in taking 5 nC from C to B

$$=1582{\cdot}2\times5\times10^{-9}=7{\cdot}91\times10^{-6}\text{ J}. \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. The force of attraction between two charges $+30\times10^{-9}$ C and -20×10^{-9} C separated by a distance ·20 m is found to be 5×10^{-5} N. Find the relative permittivity of the medium. (**Ans.** 2·7)

2. Two charges of $+4\times10^{-7}$ C and -8×10^{-7} C are 20 cm apart. Find the resultant intensity at a point *P* situated 20 cm from either charge.

(**Ans.** $15{\cdot}6\times10^{4}$ Vm^{-1} at 30° inclination with the line joining the two charges.)

3. The distance between $+50\times10^{-9}$ C and -32×10^{-9} C is 20 cm. Where a third charge be placed on the line joining them so that it will not experience any force ?

(**Ans.** at a point distant 1 m from the first and ·8 m from the second)

4. Two pith balls each weighing 10 mgm are suspended from same point by silk threads each of length ·25 m. When equal and similar charges are given on them, they repel each other and are ·10 m apart. Find the charge on each pith ball. (**Ans.** $4{\cdot}67\times10^{-9}$ C)

5. What is electric intensity in the field of a gold nucleus (atomic number of gold=79) at a distance of 10^{-14} m from the nucleus ?

(**Ans.** $1{\cdot}14\times10^{21}$ Vm^{-1})

6. An α-particle carrying electric charge $3{\cdot}2\times10^{-19}$ C is at a distance of $13{\cdot}8\times10^{-15}$ m from a gold nucleus (atomic number=79). Calculate the force exerted by gold nucleus on α-particle. (**Ans.** $1{\cdot}911\times10^{2}$ N)

7. A charge of 50×10^{-9} C is placed at a point *P*. What amount of work will we done to bring 2×10^{-9} C from infinity to a point 5 cm away from *P* ? What work will be done to move a unit charge around *P* in a circle of radius 8 cm ? (**Ans.** 18×10^{-6} J; nothing)

8. A small uncharged sphere is placed in contact with an equal and similar charged sphere and then removed to a distance of 4 cm. The force of repulsion between them is now 9×10^{-5} N. What was the original charge in the charged sphere ? (**Ans.** 8×10^{-9} C)

9. An electron is projected with an initial velocity 10^{7} ms^{-1} into an electric field between two parallel plates 1 cm apart. The electron enters the field at a point mid-way between the plates. If the electron just misses the positively charged plate as it emerges from the field, find the magnitude of the electric intensity between the plates. (Length of each plate is 2 cm; mass of electron= 9×10^{-31} kg and charge of electron= $1{\cdot}6\times10^{-19}$ C)

(**Ans.** $1{\cdot}406\times10^{4}$ Vm^{-1})

10. Two particles each of mass *m* kg and charged with *q* units of positive electricity are suspended by light insulating threads of length *l* from the same point. In the position of equilibrium each thread makes an angle of 45° with the horizon. Prove that $q=l\sqrt{8\pi\epsilon_0 mg}$.

11. Charges of $+q$, $+2q$, $+q$ and $-q$ are placed at the corners of a square *ABCD* respectively. Calculate the electric field at the intersection of diagonals of the square when $q=\frac{5}{3}\times10^{-9}$ C and length of each side is 10 cm.

(**Ans.** $\cdot9\times10^{4}$ Vm^{-1} or newton per coulomb)

12. Two small equal spheres carrying unlike and unequal charges are placed 8 cm apart and attract each other with a force of 4×10^{-5} N. After they have been connected for a moment by a thin conducting wire, they repel each other with a force of 2×10^{-5} N. Calculate their original charges.

(**Ans.** $10{\cdot}3\times10^{-9}$ C; $2{\cdot}76\times10^{-9}$ C)

13. Two small conductors ·3 m apart carry 10×10^{-9} C and 20×10^{-9} C of electric charges. Find the potential and intensity at a point exactly midway between them. (**Ans.** 1800 V; 4000 Vm^{-1})

14. An electron of mass 9×10^{-31} kg carry $1{\cdot}6\times10^{-19}$ C starts from rest from one point and reaches a second conductor with a velocity 10^{7} ms^{-1}. Calculate the potential difference between the points. (**Ans.** 281·25 V)

15. Calculate the radius of water drop which would just remain suspended in the earth's electric field of 300 Vm^{-1} when charged with one electronic charge. (One electronic charge= $1{\cdot}6\times10^{-19}$ C) (**Ans.** $1{\cdot}05\times10^{-7}$ m)

16. *ABCD* is a square of ·2 m side. Positive charges 2×10^{-9} C, 4×10^{-9} C and 8×10^{-9} C are placed at the points *A*, *B* and *C*. Calculate the work done to transfer one coulomb of charge from *D* to the centre of the square.

(**Ans.** 313·6 joule)

17. Two charges $+2\times10^{-9}$ C and -8×10^{-9} C are placed ·3 m apart in air. Find point or points where electric field and potential is zero.

(**Ans.** At a distance ·3 m from $+2\times10^{-9}$ C on the other side of it. There are many points where potential is zero. At those points potential is zero whose distances from the charges are in the ratio 1 : 4)

18. Two charges 4 cm apart repel each other with a force of 2×10^{-2} newton in air. When a dielectric slab of thickness 1 cm is placed between them, the force is reduced to $1{\cdot}5\times10^{-2}$ newton. Calculate permittivity of the dielectric. (**Ans.** 2·62)

19. Charges of magnitude $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$ and $\frac{4}{3}$ nanocoulomb are placed respectively at the corners *A*, *B*, *C*, *D* of a square of side 2 cm. Find the work done in taking $\frac{5}{3}$ nanocoulomb from mid-point of *AB* to the mid-point of *CD*.

(**Ans.** $1{\cdot}104\times10^{-6}$ joule)

20. Four small spheres carrying $+\frac{5}{3}$, $+\frac{10}{3}$, $+\frac{5}{3}$ and $-\frac{5}{3}$ nC are placed in order at the angular points of a square *ABCD* of 10 cm on each side. Calculate the force on a charge of $+\frac{2}{3}$ nC at the intersection *O* of diagonals.

(**Ans.** 6×10^{-6} newton from *O* to *D*)

21. An electron is liberated from a hot filament and attracted by anode at 1200 V. What is the speed when it strikes the anode ? ($e=1{\cdot}6\times10^{-19}$ C)

(Ans. $2{\cdot}1\times10^{7}$ ms^{-1})

(B)

22. A certain charge Q is to be divided into two parts q and $Q-q$. What is the relationship of Q with q if the two parts placed at a given distance apart are to have a maximum coulomb repulsion ?

$\left(\textbf{Ans. } q=\frac{Q}{2}\right)$

23. A uniform electric field exists in the region between two oppositely charged plane parallel plates. An electron is released from rest at the surface of negatively charged plate and strikes the surface of the opposite plate, 2 cm distant from the first in an interval of $1{\cdot}5\times10^{-8}$ s. (a) Find the electric intensity, (b) find the velocity of the electron when it strikes the second plate.

[**Ans.** (a) 1000 Vm^{-1}; (b) $2{\cdot}67\times10^{6}$ ms^{-1}]

24. Two brass plates are arranged horizontally one 2 cm above the other. The plates are charged to a potential difference of 6600 V. Now very fine liquid drops are sprayed into the space between the plates. Big drops collect at the bottom plates, some drops go up and some remain stationary. How do you explain this ? If the radius of stationary drops is 2×10^{-6} m and density of liquid 800 kgm^{-3}, calculate the charge carried by the stationary drops. Does this result confirm quantisation of charge ?

(**Ans.** $7{\cdot}962\times10^{-19}$ C; yes, each drop carries charge 5 times the quantum of charge which is $1{\cdot}6\times10^{-19}$ C)

25. Three charges -1μ C, 2μ C and 3μ C are respectively placed at the corners A, B, C of an equilateral triangle of 2 m on each side. Calculate the potential and electric field at mid-point of the side BC.

(**Ans.** 4×10^{4} volt; $9{\cdot}5\times10^{3}$ Vm^{-1} at inclination 18° 25′ away from the point)

26. Three small balls, each of mass 10 gm, are suspended separately from a common point by silk threads, each 1 m long. The balls are identically charged and hung at the corners of an equilateral triangle ·1 m long on a side. What is the charge on each ball ?

(**Ans.** 6×10^{-8} C)

27. In the Bohr model of hydrogen atom, an electron revolves in a circular orbit around the nucleus of a single proton. If the radius of the orbit is $5{\cdot}28\times10^{-11}$ m, find the number of revolutions of the electron per second. (Charge on electron$=1{\cdot}6\times10^{-19}$ C and charge on proton is same as on electron but positive.)

(*Hint* : Coulomb's force of attraction$=m\omega^2 r$.)

(**Ans.** $6{\cdot}6\times10^{15}$ revolutions/sec)

28. Two equal positive point charges are separated by a distance $2a$. A point test charge is located in a plane which is normal to the line joining these charges and mid-way between them. Calculate the radius r of the circle of symmetry in this plane for which the force on the test charge has a maximum value.

$\left(\text{Ans. } r=\frac{a}{\sqrt{2}}\right)$

29. An alpha particle of velocity $1{\cdot}6\times10^{7}$ ms^{-1} approaches gold nucleus ($Z=79$). Calculate the distance of the closest approach. Mass of an alpha particle$=6{\cdot}6\times10^{-27}$ kg. What is the significance of this 'closest approach'?

(**Ans.** $4{\cdot}3\times10^{-14}$ m; the size of the nucleus cannot be greater than $4{\cdot}3\times10^{-14}$ m)

30. Three charges $-q$, $+q$ and $-q$ are placed along a st. line at equal distances, say, a. Calculate the potential energy of this system of charges (i) when $+q$ charge is in the middle, and (ii) when this charge is at one end.

$$\left[\textbf{Ans.}\ \text{(i)}\ -\frac{3q^2}{8\pi\epsilon_0 a};\ \text{(ii)}\ \frac{-q^2}{8\pi\epsilon_0 a}\right]$$

31. Two concentric spherical shells of conducting material and of radii a and b ($b>a$) are given charges q_1 and q_2 respectively. Find out the potential at a point distant r from the centre, (i) outside the two shells, (ii) in between the two shells. What is the potential difference between the two shells and what will happen if they are joined by a wire?

$$\left[\textbf{Ans.}\ \text{(i)}\ \frac{q_1+q_2}{4\pi\epsilon_0 r};\ \text{(ii)}\ \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{b}\right);\ \text{(iii)}\ \frac{q_1}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right);\right.$$

(iv) they will come to a common potential]

32. Two equal charges, $+Q$ each, are at a distance r from each other. A third charge q is placed on the line joining the above two charges such that all these charges are in equilibrium. Find q, in terms of Q.

$$\left(\textbf{Ans.}\ q=-\frac{Q}{4}\right)$$

33. An infinite number of charges each equal to q are arranged in a line at distances 1, 2, 4, 8, 16,.........from a fixed point on the line. Find the potential and field at that fixed point. What will be the potential and electric field if, in the above set up, the consecutive charges have opposite sign?

$$\left(\textbf{Ans.}\ \frac{q}{2\pi\epsilon_0};\ \frac{q}{3\pi\epsilon_0};\ \frac{q}{6\pi\epsilon_0};\ \frac{q}{5\pi\epsilon_0}\right)$$

34. Four charges $+q$, $+2q$, $+3q$, and $+4q$ are placed at the four corners $ABCD$ of a square of side 'a'. Calculate the field and potential at the centre of the square.

$\Big($**Ans.** $\dfrac{\sqrt{2}q}{\pi\epsilon_0 a^2}$ along the perpendicular bisector of line joining $+3q$ and $+4q$ and away from the centre; potential$=\dfrac{5q}{\sqrt{2}\pi\epsilon_0 a}\Big)$

35. Two identical positive charges are placed at the diagonally opposite corners of a square and two more identical but negative charges are placed at the remaining two corners. What is the work done in putting all charges together at the centre if q is the magnitude of each charge and $2a$ is the length of each side of the square?

$$\left(\textbf{Ans.}\ \frac{q^2(4-\sqrt{2})}{8\pi\epsilon_0 a}\ \text{joule}\right)$$

36. A charge Q is placed at each of two opposite corners of a square of side a. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is zero, how are Q and q related? (b) What is the poten-

tial energy of the system of charges ? What is the significance of its negative potential energy ?

$\left(\textbf{Ans. } Q=-2\sqrt{2}q;\ \frac{1}{4\pi\epsilon_0}\cdot\frac{-7q^2}{\sqrt{2}a}\right.$ when a is side of square.
They form a closed stable system.)

37. A thin copper ring of radius 'a' is charged with q units of electricity. Calculate the electric field at a point on the axis of the ring.

$\left[\textbf{Ans. } E=\frac{1}{4\pi\epsilon_0}\cdot\frac{qx}{(a^2+x^2)^{3/2}}\right.$ where x is the distance of point from the centre of ring.]

38. An electron is placed at the centre of the copper ring of the problem 37. Show that if the electron is displaced a little, it will perform simple harmonic motion of frequency $f=\frac{1}{2\pi}\sqrt{\frac{eq}{4\pi\epsilon_0 ma^3}}$. (I. I. T. 1982]

39. A thin half ring of radius 20 cm is uniformly charged with a total charge ·7 nC. Find the magnitude of the electric field at the centre of the half-ring. (**Ans.** 100 Vm^{-1})

40. A thin non-conducting ring of radius r has a linear charge density $q=q_0\cos\theta$, where q_0 is a constant and θ is the angle at the centre from the diameter of maximum charge density in the anticlockwise direction. Find the electric field at the centre of the ring. $\left(\textbf{Ans. } \frac{q_0}{4\epsilon_0 r}\right)$

41. A thin wire ring of radius r has an electric charge q. What is the tension of the wire if a point charge q_0 is placed at the ring's centre ?

$\left(\textbf{Ans. } \frac{qq_0}{8\pi^2\epsilon_0 r^2}\right)$

42. A charge $+Q$ is uniformly distributed over a thin ring of radius R. Find the velocity of a negative point charge $-Q$ at the moment it passes through the centre of the ring if the charge $-Q$ was initially at rest at infinity on the axis of the ring. The mass of the charge $-Q$ is equal to m.

$\left(\textbf{Ans. } v=\sqrt{\frac{1}{4\pi\epsilon_0}\cdot\frac{2Q^2}{Rm}}\right)$

43. Three identical particles, each of charge $Q=0{\cdot}5\ \mu\text{C}$ and mass $m=7{\cdot}5\times10^{-10}$ kg are placed at the vertices of an equilateral triangle of side $r_0=1$ m and released. The particles repel each other by coulomb forces to larger equilateral triangles. Find the relative velocity between any two of them when the triangle has grown to a side of $r=2$ m. $\left[\textbf{Ans. } v=\sqrt{\frac{1}{4\pi\epsilon_0}\cdot\frac{6Q^2}{m}\left(\frac{1}{r_0}-\frac{1}{r}\right)}\right.$
$=3\times10^3\ \text{ms}^{-1}$]

□□□

CHAPTER 2

POTENTIAL AND FIELD DUE TO A DIPOLE

1. Dipole : A pair of two equal and unlike charges separated by a small distance (small in comparison to distances of physical interest) is called an electric dipole.

If $+q$ and $-q$ charges are separated by a small distance $2l$ then dipole moment is given by $p=2ql$ coulomb metre (Cm).

The potential 'V' at a distance r in a direction θ from the axis of a dipole is

$$V=\frac{1}{4\pi\epsilon_0}\cdot\frac{p\cos\theta}{r^2}\text{ volt (V).}$$

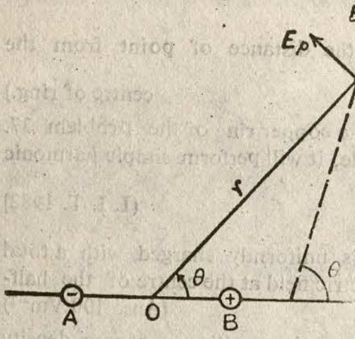

Fig. 2-1

The electric field at the same point is :

$$E=\frac{1}{4\pi\epsilon_0}\cdot\frac{p}{r^3}\sqrt{1+3\cos^2\theta}\text{ volt per metre (Vm}^{-1}\text{).}$$

The components of the field along and perpendicular to r are

$$E_r=\frac{1}{4\pi\epsilon_0}\cdot\frac{2p\cos\theta}{r^3}\quad\text{and}\quad E_p=\frac{1}{4\pi\epsilon_0}\cdot\frac{p\sin\theta}{r^3}.$$

Direction of the field with r is given by

$$\tan\alpha=\frac{\tan\theta}{2}.$$

2. Dipole in an electric field : When a dipole of moment p is placed in a uniform electric field E, it experiences torque (τ) given by $\tau=p\,E\sin\theta$ newton metre (Nm) where θ is the angle between the dipole axis ($-$charge to $+$charge direction) and the electric field.

In vector notation, $\vec{\tau}=\vec{p}\times\vec{E}$.

3. Energy of a dipole in an electric field : The work done in turning the dipole by external agent through θ from its zero potential energy position ($\theta=0$) is stored as potential energy of the dipole.

$$U=pE(1-\cos\theta)\text{ joule (J).}$$

EXAMPLES

Ex. 1. *ABC is a small isosceles right angled triangle of hypotenuse 1 cm. A charge of* $+40$ *pC (pico coulomb) is placed at right angled corner A and* -20 *pC and* -20 *pC at B and C respectively. Show that this system of charge may be treated as a dipole for all external points at large distances. Calculate the potential due to this system of charges at a point on the prolongation of the side AC at a distance 40 cm from A.*

Sol. The two charges -20 pC and -20 pC at B and C may be supposed to be concentrated at the mid-point of the hypotenuse. Thus the whole system may be treated as a pair of two charges $+40$ pC at A and -40 pC at the mid-point of BC. Hence they are equivalent to a dipole of length $\cdot 5 \times 10^{-2}$ m.

$$\therefore \quad p \text{ (dipole moment)} = 2ql$$

$$= 40 \times 10^{-12} \times \cdot 5 \times 10^{-2}$$

$$(\because \quad 1 \text{ pC} = 10^{-12} \text{ C})$$

$$= 2 \times 10^{-13} \text{ Cm.}$$

We know, potential at a point due to a dipole

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos\theta}{r^2}.$$

$$\therefore \quad V = \frac{1}{4\pi\epsilon_0} \cdot \frac{2 \times 10^{-13} \cos 45^\circ}{(\cdot 4)^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-13} \times \cdot 707}{\cdot 16}$$

$$\left(\because \quad \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}\right)$$

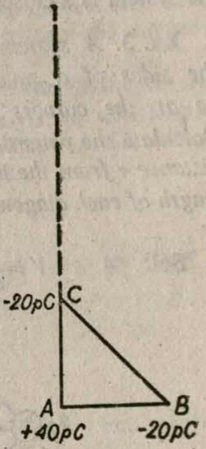

Fig. 2-2

$$\text{or} \qquad V = \frac{18 \times 707}{16} \times 10^{-5} = 7 \cdot 95 \times 10^{-3} \text{ volt.}$$ **Ans.**

Ex. 2. *Calculate the binding energy of a dipole consisting of two charges* $+4$ *pC and* -4 *pC separated by a distance 20μ (micron).*

Sol. $1 \text{ pC} = 10^{-12}$ C and $1\ \mu = 10^{-6}$ m.

The binding energy of any system is the energy required to take the charges to infinity from the present configuration.

The energy required is the work to be done to bring the charges to the present configuration from infinity and this is exactly what is self-electrical potential energy.

$$\text{Electrical potential energy} = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq'}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{(-4\times10^{-12})\times4\times10^{-12}}{20\times10^{-6}}$$

$$= -9\times10^{9}\frac{16\times10^{-24}}{20\times10^{-6}} \qquad \left(\because \frac{1}{4\pi\epsilon_0} = 9\times10^{9}\ \text{Nm}^2\text{C}^{-2}\right)$$

$$= -72\times10^{-10}\ \text{J}.$$

$\therefore$ Binding energy of a dipole $= -7{\cdot}2\times10^{-9}$ joule (J). **Ans.**

The minus sign shows that work is done by the force itself and the system is a closed one.

Ex. 3. *A system consists of two identical dipoles placed along the sides of a square ABCD in such a way that +q and +q of them lie at the corners A and C and −q and −q at the corner B. Calculate the potential due to the system on the diagonal BD at a distance r from the intersection of the diagonals of the square. The length of each diagonal is 2a.*

Sol. $$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{AP} + \frac{q}{CP} - \frac{2q}{BP}\right)$$

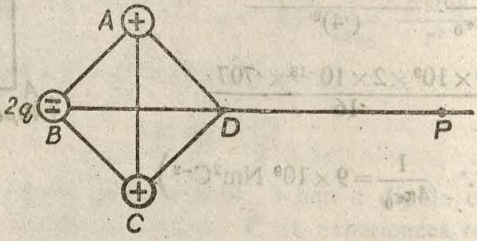

Fig. 2-3

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{2}{AP} - \frac{2}{BP}\right) \qquad (\because AP = CP)$$

$$= \frac{2q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{r^2+a^2}} - \frac{1}{r+a}\right)$$

$$= \frac{2q}{4\pi\epsilon_0}\left[\frac{1}{r(1+a^2/r^2)^{\frac{1}{2}}} - \frac{1}{r(1+a/r)}\right]$$

or $$V=\frac{q}{2\pi\epsilon_0 r}\left[\left(1+a^2/r^2\right)^{-\frac{1}{2}}-\left(1+a/r\right)^{-1}\right].$$

We expand and neglect powers greater than 2. Then

$$V=\frac{q}{2\pi\epsilon_0 r}\left[\left(1-\frac{1}{2}\cdot\frac{a^2}{r^2}\right)-\left(1-\frac{a}{r}+\frac{a^2}{r^2}\right)\right]=\frac{qa(2r-3a)}{4\pi\epsilon_0 r^3}.\quad \textbf{Ans.}$$

EXERCISES

1. *ABC* is a very small equilateral triangle of side 5×10^{-3} m. A charge of $+20$ aC (atto coulomb) is placed at the corner *A* and two charges, each of -10 aC, at *B* and *C*. Calculate the potential at a point on the prolongation of *AC* distant 2 cm from *A*.

[*Hint* : 1 aC (atto coulomb)$=10^{-18}$ coulomb] (Ans. $1{\cdot}687\times10^{-7}$ volt)

2. An electric dipole consists of two opposite charges of magnitude 1 μC (micro coulomb) separated by a distance of 2 cm. The dipole is placed in an electric field of 10^5 Vm^{-1}. (a) What maximum torque does the field exert on the dipole ? (b) How much work must an external agent do to turn the dipole end for end, starting from a position of alignment $\theta=0$?

[Ans. (a) 2×10^{-3} Nm; (b) 4×10^{-3} joule]

3. Show that the potential at a point of co-ordinates (x, y) with reference to the axis of the dipole as x-axis and the line perpendicular to the axis and passing through the centre of the dipole as y-axis is

$$V=\frac{1}{4\pi\epsilon_0}\cdot\frac{px}{(x^2+y^2)^{3/2}}$$

and hence show that the components of fields along x and y axes are given by,

$$E_x=\frac{q}{4\pi\epsilon_0}\cdot\frac{2x^2-y^2}{(x^2+y^2)^{5/2}}$$

$$E_y=-\frac{p}{4\pi\epsilon_0}\cdot\frac{3xy}{(x^2+y^2)^{5/2}}.$$

[*Hint* : Find the value of $\cos\theta$ and r in terms of x and y and substitute their values in the standard formula $E_x=-\frac{\partial V}{\partial x}$ and $E_y=\frac{\partial V}{\partial y}$.]

4. A dipole of moment 4×10^{-14} Cm is placed with its centre at one corner of a cube of 20 cm on each side and its axis coinciding with one of the edges at that corner. Calculate the potential and field at the diagonally opposite corner.

(Ans. $1{\cdot}73\times10^{-3}$ volt; $1{\cdot}225\times10^{-2}$ Vm^{-1})

5. Two dipoles of moment 5×10^{-12} Cm form a cross with their axes ($-$ to $+$) along the co-ordinate axes. Calculate the potential at a point 20 cm away in a direction making angle 30° with the x-axis. (Ans. $1{\cdot}536$ volt)

6. Find the locus of the points where the electric field due to a dipole is (i) perpendicular to its axis, (ii) anti-parallel to the axis.

$\left[\textit{Hint} : \tan\alpha=\frac{\tan\theta}{2}\right]$ (Ans. a st. line at an inclination of 54°44′ with the axis; a line perpendicular to the axis)

7. Find the locus of points where the electric field (resultant) will have always a bearing of 45° with the axis of the dipole.

(Ans. A straight line at an inclination of 29°18′ with the axis)

8. A dipole consisting of +10 nC (nano coulomb) and −10 nC separated by 2 cm oscillates in an electric field of strength 60000 $\mathrm{Vm^{-1}}$. Calculate the frequency of vibration of the dipole if its moment of inertia about the axis of oscillation is 3×10^{-10} $\mathrm{kgm^2}$.

$$\left[\textit{Hint}: t=2\pi\sqrt{\frac{I}{pE}} \text{ a formula similar to } t=2\pi\sqrt{\frac{I}{mB}} \text{ in magnetism.}\right]$$

(Ans. 31·83 Hz)

9. Suppose that the orientation of four charges is such that $+q$ and $+q$ of it lie at the opposite corners of a square and $-q$ and $-q$ at the other two remaining corners. Calculate the potential and field due to this system at a point on the diagonal of the square at a distance r from the centre of the diagonals. The length of each diagonal is $2a$. Assume a to be small.

$$\left(\text{Ans. } V=\frac{1}{4\pi\epsilon_0}\cdot\frac{3qa^2}{r^3};\ E=\frac{1}{4\pi\epsilon_0}\cdot\frac{9qa^2}{r^4}\right)$$

10. Two identical dipoles have their axes at right angles to each other and also bisecting each other. Calculate the field at a distance r from the point of intersection of their axes in a direction θ with the axis of one of the dipoles.

$$\left[\text{Ans. } E=\frac{1}{4\pi\epsilon_0}\cdot\frac{p}{r^3}\sqrt{5+3\sin 2\theta} \text{ where } p \text{ is dipole moment of each dipole.}\right]$$

11. A system consists of charges $+q$ and $+q$ at the opposite corners of a square of side $2a$ and $-q$ and $-q$ at the other two remaining corners. Calculate the potential and field at a distance r from the centre of the square along a line parallel to the two sides of the square. Assume $a \ll r$.

$$\left(\text{Ans. } V=0 \text{ and } E=\frac{3qa^2}{\pi\epsilon_0 r^4}\right)$$

12. Two electric dipoles, each of dipole moment $p=6{\cdot}2\times10^{-30}$ Cm, are placed with their axes along the same line with their centres at a distance $d=10^{-8}$ m apart. Calculate the force of attraction between the dipoles.

(Ans. $2{\cdot}1\times10^{-16}$ N)

13. An electric dipole of dipole moment p is placed in a uniform electric field E_0 with its axis parallel to E_0. In this case one of the equipotential surfaces enclosing the dipole forms a sphere. Find the radius of this sphere.

$$\left(\text{Ans. } r=\sqrt[3]{\frac{p}{4\pi\epsilon_0 E_0}}\right)$$

14. Calculate the energy released in the formation of 1 kg of hydrogen chloride, it is given that the dipole moment of hydrogen chloride molecules is $3{\cdot}44\times10^{-30}$ Cm and the separation between hydrogen and chlorine atoms in the equilibrium position is $1{\cdot}01\times10^{-10}$ m. (Ans. $1{\cdot}65\times10^{6}$ J)

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CHAPTER 3

GAUSS'S THEOREM AND ITS APPLICATIONS

Gauss's theorem : The total electric flux through a closed surface drawn in an electric field is equal to $\frac{1}{\epsilon_0}$ times the charge enclosed by that surface. Mathematically,

$$\Sigma \vec{E}.\triangle\vec{S} = \frac{q}{\epsilon_0}.$$

Applications :

(i) *Intensity of field due to an isolated uniformly charged spherical conductor :* Suppose P is a point situated outside a uniformly charged sphere having $+q$ charge. Draw a concentric spherical Gaussian surface through P.

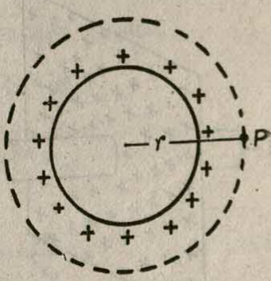

Fig. 3-1

$$\text{Now} \quad \Sigma\vec{E}.\triangle\vec{S} = \Sigma E\triangle S \cos 0^\circ$$
$$= E\Sigma\triangle S = E.4\pi r^2.$$

$$\therefore \quad E\, 4\pi r^3 = \frac{q}{\epsilon_0}$$

or $\quad E = \frac{1}{4\pi\epsilon_0}.\frac{q}{r^2}$; vectorially, $\vec{E} = \frac{1}{4\pi\epsilon_0}.\frac{q\hat{r}}{r^2}$.

If the point lies inside then the charge inside the Gaussian surface is zero.

$$\therefore \quad E.4\pi r^2 = 0, \qquad \therefore \quad E = 0.$$

(ii) *Intensity near a long cylindrical conductor* :

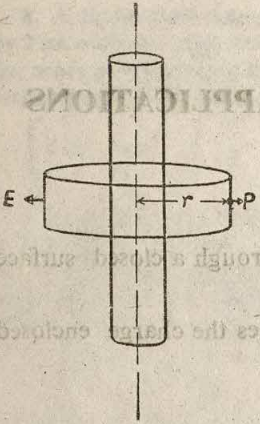

Fig. 3-2

Suppose P is a point situated outside an infinitely long charged cylindrical conductor. Draw a co-axial cylindrical Gaussian surface through P of length l.

For curved surface

$$\Sigma\vec{E}.\triangle\vec{S}=\Sigma E\triangle S\cos 0°=E\Sigma\triangle S$$
$$=E.2\pi rl.$$

For flat surfaces, $\Sigma\vec{E}.\triangle\vec{S}=0$.

$\therefore$ $(\Sigma\vec{E}.\triangle\vec{S})$ for the entire surface $=E2\pi rl$.

If q=linear density of charge then charge inside the Gaussian surface is ql.

$$\therefore\quad E.2\pi rl=\frac{1}{\epsilon_0}.ql$$

or $$E=\frac{1}{4\pi\epsilon_0}.\frac{2q}{r}\quad\text{or}\quad\vec{E}=\frac{1}{4\pi\epsilon_0}.\frac{2q\hat{r}}{r}.$$

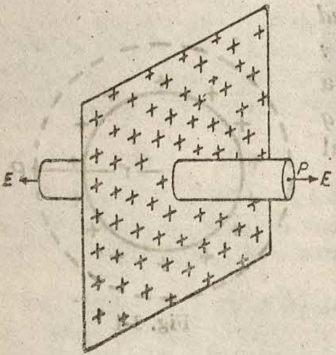

Fig. 3-3

(iii) *Electric field due to an infinity sheet of charge* :

Suppose P is a point close to an infinite sheet of charge. σ= surface density of charge. Draw a cylindrical surface with axis perpendicular to the sheet and having the point on its end face.

The cylinder is extended to the same extent on the other side of the sheet. If S=area of each end face then $(\Sigma\vec{E}.\triangle\vec{S})$ of the entire Gaussian surface$=2ES$.

$$\therefore\quad\text{By Gauss's theorem } 2ES=\frac{1}{\epsilon_0}S\sigma$$

or $E=\dfrac{\sigma}{2\epsilon_0}$, or $\vec{E}=\dfrac{\sigma}{2\epsilon_0}\hat{n}$ where $\hat{n}$ is the unit vector along the normal to the plane of sheet.

If it is a conducting sheet, charge will be just double because a conducting sheet will have charge on both the surface.

$$\therefore \quad E=\frac{\sigma}{\epsilon_0}, \quad \text{or} \quad \vec{E}=\frac{\sigma}{\epsilon_0}\hat{n}.$$

(iv) *Mechanical force on the surface of a charged conductor* : The mechanical force per unit area of a charged conductor of surface density σ is given by Gauss's theorem :

$$F=\frac{\sigma^2}{2\epsilon_0}.$$

(v) *Electrostatic energy in a medium* : The density of energy of electric field in a medium is given by :

$$u=\tfrac{1}{2}\epsilon_0 E^2 \textit{ joule per cubic metre } (\text{Jm}^{-3}).$$

EXAMPLES

Ex. 1. *A sphere of a radius 'a' is uniformly charged throughout its volume with a volume charge density of σ. Calculate electric field at a distance r from the centre of the sphere when (i) $r<a$, (ii) when $r>a$.*

Sol. Consider a concentric spherical Gaussian surface.

$$\text{Electric flux}=\Sigma\vec{E}.\triangle\vec{S}=4\pi r^2 E.$$

(i) Charge inside the Gaussian surface $=\frac{4\pi}{3}r^3.\sigma$.

$$\therefore \quad 4\pi r^2 E=\frac{1}{\epsilon_0}.\frac{4\pi}{3}r^3\sigma, \quad \text{or} \quad E=\frac{\sigma r}{3\epsilon_0}.$$ **Ans.**

(ii) Charge inside the Gaussian surface $=\frac{4\pi}{3}a^3\sigma$.

$$\therefore \quad 4\pi r^2 E=\frac{1}{\epsilon_0}.\frac{4\pi}{3}a^3\sigma;$$

$$E=\frac{a^3\sigma}{3\epsilon_0 r^2}.$$ **Ans.**

Ex. 2. *A piece of metal foil* 2×10^{-4} m^2 *in area and one centigramme in mass lies on a horizontal metal plate of area* 2 m^2. *Calculate the charge to be given to the plate so as to just lift the foil.*

Sol. The mechanical force per unit area of foil $=\dfrac{\sigma^2}{2\epsilon_0}$.

$\therefore$ Mechanical force on foil $=\dfrac{\sigma^2}{2\epsilon_0}\times S$ where $S=$ area of foil.

When this force will be equal to the weight of the foil, it will be just lifted.

$$\therefore \quad \frac{\sigma^2 S}{2\,\epsilon_0}=mg$$

$$\therefore \quad \sigma=\sqrt{\frac{2\epsilon_0\, mg}{S}}.$$

Putting the standard value of $\epsilon_0=8{\cdot}85\times10^{-12}$ and the given values of m and S we have,

$$\sigma=\sqrt{\frac{2\times8{\cdot}85\times10^{-12}\times{\cdot}01\times10^{-3}\times9{\cdot}8}{2\times10^{-4}}}.$$

$\therefore$ Charge to be given to the plate $=$ area $\times$ surface density

$$=2\times\sqrt{\frac{2\times8{\cdot}85\times10^{-12}\times{\cdot}01\times10^{-3}\times9{\cdot}8}{2\times10^{-4}}}$$

$$=\sqrt{4\times8{\cdot}85\times9{\cdot}8\times10^{-13}}=5{\cdot}89\times10^{-6}\text{ C.}$$ **Ans.**

Ex. 3. *If the field near the earth's surface* 300 Vm^{-1} *directed downwards, what is the surface density of charge on the surface of the earth ?*

Sol. We have, $E=\dfrac{\sigma}{\epsilon_0}$. Rate of fall of potential is equal to electric intensity.

$\therefore$ Here $E=300$ Vm^{-1}; $\therefore$ $\sigma=300\times8{\cdot}85\times10^{-12}$

$=2{\cdot}655\times10^{-9}$ Cm^{-2}. **Ans.**

Ex. 4. *A soap bubble of radius* 3 cm *is charged with* 9 nC (*nonacoulomb*). *Find the excess of pressure inside the bubble. Surface tension of soap solution* $=3\times10^{-3}$ Nm^{-1}.

Sol. There is excess of pressure on account of surface tension by an amount $\frac{4T}{r}$. This pressure tends to contract the bubble. There is an outward pull due to charge on it by an amount $\frac{\sigma^2}{2\epsilon_0}$. This helps the bubble to stretch out. Hence this partly balances the excess of pressure.

$\therefore$ Excess of pressure inside a charged soap bubble

$$=\frac{4T}{r}-\frac{\sigma^2}{2\epsilon_0}.$$

Now, q (total charge) $=4\pi r^2\sigma$

$$\therefore \qquad \sigma=\frac{q}{4\pi r^2}.$$

$$\therefore \qquad \text{Excess of pressure}=\frac{4T}{r}-\frac{q^2}{32\pi^2 r^4\epsilon_0}$$

$$=\frac{4\times3\times10^{-3}}{3\times10^{-2}}-\frac{(9\times10^{-9})^2}{32\pi^2(3\times10^{-2})^4\times8{\cdot}85\times10^{-12}}={\cdot}4-\frac{100}{32\pi^2\times8{\cdot}85}$$

$$={\cdot}4-{\cdot}036={\cdot}364\ \text{Nm}^{-2}.$$ **Ans.**

Ex. 5. *Air gives way when there is a gradient of potential by* 3×10^6 *volt per metre. What is the maximum voltage to which a sphere of radius* 1 cm *can be charged? What is the charge required?*

Sol. *Rate of fall of potential is the electric intensity.* Hence a spherical conductor can be charged to that potential when electric intensity on its surface is 3×10^6 Vm^{-1}.

Let $V=$ Potential to which a sphere of radius of r can be raised.

But $E=\frac{1}{4\pi\epsilon_0}.\frac{Q}{r^2}$, and $V=\frac{1}{4\pi\epsilon_0}.\frac{Q}{r}$ (assuming *the charge to be concentrated at the centre which we can do for points on the surface and outside the surface*).

$$\therefore \quad \frac{E}{V}=\frac{Q}{4\pi\epsilon_0 r^2}\times\frac{4\pi\epsilon_0 r}{Q}=\frac{1}{r}.$$

$$\therefore \quad E=\frac{V}{r}.$$

$$\therefore \quad V=Er=3\times10^6\times(1\times10^{-2})=3\times10^4 \text{ volt.} \qquad \textbf{Ans.}$$

Now $Q=4\pi\epsilon_0 rV$

$$=\frac{1}{9\times10^9}\times(1\times10^{-2})\times3\times10^4 \qquad \left(\because \quad 4\pi\epsilon_0=\frac{1}{9\times10^9}\right)$$

$$=3{\cdot}33\times10^{-8}\text{ C.} \qquad \textbf{Ans.}$$

Ex. 6. *A sphere of radius* 10 cm *is charged with* 1 nC (*nona-coulomb*). *Calculate the energy density of the medium at a distance* 20 cm *from its centre.*

Sol. We have, $E=\frac{Q}{4\pi\epsilon_0 r^2}$, (assuming charge to be concentrated at the centre which we can always do for points on the surface and outside of the sphere).

Now, $u_E=\frac{1}{2}\epsilon_0 E^2$; $\therefore \quad u_E=\frac{1}{2}\epsilon_0 . \frac{Q^2}{16\pi^2\epsilon_0^2 r^4}=\frac{Q^2}{32\pi^2\epsilon_0 r^4}$;

$$\therefore \quad u_E=\frac{(10^{-9})^2}{32\pi^2\times8{\cdot}85\times10^{-12}(20\times10^{-2})^4}=\frac{10^{-2}}{32\pi^2\times8{\cdot}85\times16}$$

$$=2{\cdot}24\times10^{-7}\text{ Jm}^{-3}. \qquad \textbf{Ans.}$$

Ex. 7. *Calculate mutual force of attraction between two parallel plates* 1 cm *apart and maintained at a potential difference of* 1000 *volt. Area of each plate* $=10^{-2}$ m^2.

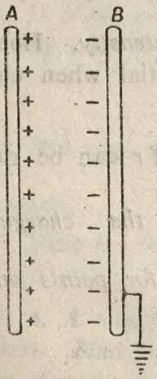

Fig. 3-4

Sol. Electric intensity E at a point near a plane conductor having charge only on one side $=\frac{\sigma}{2\epsilon}$ where ϵ = permittivity of the medium between plates.

$\therefore$ Force of attraction per unit area

$=\frac{\sigma}{2\epsilon}\times(-\sigma)$, ($\because$ $-\sigma$ is the charge per unit area of the other plate and force per unit charge is the intensity of electric field)

$=-\frac{\sigma^2}{2\epsilon}$. (minus sign simply shows that it is a force of attraction.)

$\therefore$ Force of attraction between plates $=\dfrac{\sigma^2}{2\epsilon}.A$ where $A=$ area of each plate.

Now, electric intensity at any point $=\dfrac{\sigma}{2\epsilon}+\dfrac{\sigma}{2\epsilon}=\dfrac{\sigma}{\epsilon}$, because $\dfrac{\sigma}{2\epsilon}$ is the electric intensity at any point due to charge on plate A from A to B and electric intensity at the same point due to charge on other plate (B) is also $\dfrac{\sigma}{2\epsilon}$ in the same direction. Recall that direction of field is the direction of force experienced by a unit positive charge.

$$\therefore \quad E=\frac{\sigma}{\epsilon}, \quad \text{or} \quad \sigma=\epsilon E.$$

$$\therefore \quad \text{Mutual force of attraction}=\frac{A}{2\epsilon}.\ \epsilon^2E^2=\frac{1}{2}\epsilon E^2A.$$

For air or vacuum, $\epsilon=\epsilon_0$.

$\therefore$ Mutual force of attraction $=\frac{1}{2}\epsilon_0E^2A$.

We know, intensity $=$ potential gradient (numerically).

$$\therefore \quad E=\frac{V}{d}=\frac{1000}{1\times10^{-2}}=10^5\ \text{Vm}^{-1}.$$

$\epsilon_0=8{\cdot}85\times10^{-12}$ and $A=10^{-2}\ \text{m}^2$.

$$\therefore \quad \text{Force of attraction}=\tfrac{1}{2}\times8{\cdot}85\times10^{-12}(10^5)^2.10^{-2}$$
$$=4{\cdot}43\times10^{-4}\ \text{newton}.$$ **Ans.**

EXERCISES

(A)

1. A gold foil weighing 100 mgm per square centimetre is placed on a horizontal conducting plate. What should be the density of charge so that the foil may just rise ? (Ans. $9{\cdot}3\times10^{-6}\ \text{Cm}^{-2}$)

2. Calculate the surface density of charge at a place on the earth's surface where the rate of fall of potential is 250 volt per metre.

[*Hint* : Rate of fall of potential $=$ electric field $=E=\dfrac{\sigma}{\epsilon_0}$ by Gauss's theorem.]

(Ans. $2{\cdot}21\times10^{-9}\ \text{Cm}^{-2}$)

3. A spark passes in air when the potential gradient is 3×10^6 volt per metre. What must be the radius of an isolated metal sphere which can be charged to a potential of 3 million volt before sparking into air ? (Ans. 1 m)

$\left[\textit{Hint} : E=\dfrac{Q}{4\pi\epsilon_0r^2} \text{ and } V=\dfrac{Q}{4\pi\epsilon_0r};\ \therefore\ E=\dfrac{V}{r}\right]$

4. Calculate the charge which must be placed on a sphere of radius 10 cm in order that the repulsion per square metre of the surface may just balance the atmospheric pressure which is $1{\cdot}013\times10^5$ Nm^{-2}.

$\left[\textit{Hint}: P=\frac{\sigma^2}{2\epsilon_0} \text{ and } Q=4\pi r^2\sigma\right]$ (Ans. $1{\cdot}683\times10^{-4}$ C)

5. An infinite charged sheet has a surface density of charge 10^{-7} Cm^{-2}. How far apart are the equipotential surfaces differing by 5 volts ?

[*Hint* : Electric intensity=rate of fall of potential.]

(Ans. $\cdot885\times10^{-3}$ m or $\cdot885$ mm)

6. An isolated metal sphere whose diameter is 10 cm has a potential of 8000 volts. What is the energy density at the surface of the sphere ?

(Ans. $\cdot11$ Jm^{-3})

7. What charge must be there upon a soap bubble of radius $1{\cdot}5$ cm, if the air pressure is the same inside and outside the bubble ? Assume the surface tension to be 27×10^{-3} Nm^{-1}.

[*Hint* : The excess of pressure inside a soap bubble due to surface tension $=\frac{4T}{r}$; the mechanical pull due to charge$=\frac{\sigma^2}{2\epsilon_0}$.

$$\therefore\quad \frac{4T}{r}=\frac{\sigma^2}{2\epsilon_0},\quad \text{or}\quad \sigma=\sqrt{\frac{8\epsilon_0 T}{r}}\ \Bigg]$$

(Ans. $31{\cdot}29\times10^{-9}$ C)

8. An isolated sphere of radius 5 cm is charged to a potential 159 kV (kilo volt). Find the electrostatic force per unit area of the surface.

$\left[\textit{Hint}: \text{Force per unit area}=\frac{\sigma^2}{2\epsilon_0} \text{ and } V=\frac{Q}{4\pi\epsilon_0 r}=\frac{4\pi r^2\sigma}{4\pi\epsilon_0 r}=\frac{r.\sigma}{\epsilon_0}\right]$

(Ans. $44{\cdot}75$ Nm^{-2})

9. A metal plate of radius 20 cm is charged positively to a potential of 6000 volts and placed at a distance of 5 cm from a parallel earth connected plate. Find the total pull.

(Ans. 8×10^{-3} N)

10. Two parallel plates at a distance $\cdot2$ cm apart are raised to a potential difference of 1200 volts. If the space between them is filled with air, calculate the mutual pull per unit area

(Ans.. $1{\cdot}59$ Nm^{-2})

11. The air pressure is the same inside and outside a charged soap bubble of radius 1 cm. If the surface tension is $\cdot03$ Nm^{-1}, calculate the potential in volts

(Ans. $1{\cdot}65\times10^4$ volts)

12. A spark passes into air when the potential gradient at the surface of a charged conductor is 3×10^6 Vm^{-1}. What must be the radius of a metal sphere (insulated) which may be raised to a potential of 2×10^6 volt before sparking into the air ? How much energy will be stored just before a spark occurs ?

(Ans. $\cdot67$ m; $148{\cdot}3$ J)

(B)

13. Two concentric spherical shells of radii a and b $(b>a)$ are uniformly charged and they carry equal charges q. Find the electric field at a distance r

from their common centre when (i) $r<a$, (ii) $a<r<b$, (iii) $r>b$.

[*Hint* : Draw Gaussian surface through the point and apply Gauss's theorem.]

$$\left[\textbf{Ans.}\ \text{(i) zero, (ii) } \frac{q}{4\pi\epsilon_0 r^2},\ \text{(iii) } \frac{2q}{4\pi\epsilon_0 r^2}\right]$$

14. Two co-axial cylinders of infinite length of radii a and b have linear charge densities ρ_1 and ρ_2 respectively. Calculate the electric field at a distance r when (i) $r<a$, (ii) $a<r<b$, (iii) $r>b$.

[*Hint*: Draw co-axial cylindrical Gaussian surface and apply Gauss's theorem.]

$$\left[\text{Ans. (i) zero; (ii) } \frac{\rho_1}{2\pi\epsilon_0 r};\ \text{(iii) } \frac{\rho_1+\rho_2}{2\pi\epsilon_0 r}\right]$$

15. Calculate the electric flux through a hemisphere of radius R. The electric field E is uniform and is (a) parallel, (b) perpendicular to the axis of the hemisphere. [**Ans.** (a) $\pi R^2 E$; (b) zero]

16. Two point charges $+q$ and $-q$ are separated by a distance $2l$. Imagine a mid-way plane perpendicular to the line joining the charges. Calculate the electric flux across a circle of radius R in this plane. The circle has its centre on the line joining the two charges.

$$\left[\text{Ans. } \frac{q}{\epsilon_0}\left(1-\frac{l}{\sqrt{l^2+R^2}}\right)\right]$$

17. Show by the principle of superposition concept that if a cavity is removed from a uniformly charged sphere the field inside the cavity is $(\rho\,\vec{a})/3\epsilon_0$ where ρ is the density of charge and $\vec{a}$ is the vector from the centre of the sphere to the centre of the cavity.

18. A point charge 60 nC is placed at the centre of a thick insulated metallic spherical shell. The shell has radii 10 and 12 cm. Find the electric field at distances 5, 11 and 15 cm from the centre. What is the force between the point charge and the shell ? (**Ans.** $2{\cdot}16\times10^5$ Vm^{-1}; zero; $2{\cdot}4\times10^4$ Vm^{-1}; zero)

19. Show that the electric field at a point between two coaxial cylinders of radii a and b is $E=\dfrac{V_0}{r\log b/a}$ where V_0 is the potential difference between the cylinders and r is the distance of the point from the common axis.

20. The electric fields of the earth at heights 200 m and 300 m are 100 Vm^{-1} and 60 Vm^{-1} respectively in the downward direction. Calculate the net amount of charge contained in a cube 100 m on edge between 200 m and 300 m. Neglect the curvature of the earth. (**Ans.** 3·54 μC)

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CHAPTER 4

CAPACITORS

1. Capacitance : If Q is the charge given to a conductor and V is the potential to which it is raised by this amount of charge, then it is found that $Q \propto V$

or $Q = CV$ where C is a constant called capacitance of the conductor.

Its unit is farad (F).

$$\therefore \quad C = \frac{Q}{V} \text{ farad (F).}$$

2. Capacitor : A pair of conductors separated by some insulating medium is called a capacitor. The insulating medium plays a very important role and in fact it is this medium which is responsible for formation of capacitor, the conductors only serving as means of applying electric field in the medium. This medium is called 'dielectric' of the capacitor.

Capacitance of a Capacitor : When Q units of electricity is given to one of the conductors, and thereby a potential difference V is set up between the conductors, the capacitance is then defined as

$$C = \frac{Q}{V} \text{ farad (F).}$$

3. (i) Capacitance of a sphere : $C = 4\pi\epsilon_0\epsilon_r R$ where R is the radius of sphere and ϵ_r is the permittivity of the surrounding medium.

(ii) Spherical capacitor : $C = 4\pi\epsilon_0\epsilon_r \dfrac{ab}{b-a}$ where a = radius of inner sphere, b = radius of outer sphere.

(iii) Cylindrical capacitor : $C = \dfrac{2\pi\epsilon_0\epsilon_r l}{\log_e b/a}$ where l = length of either cylinder, a = radius of inner cylinder, b = radius of outer cylinder.

(iv) Parallel plate capacitor : $C = \dfrac{\epsilon_0\epsilon_r A}{d}$ where A = area of either plate, d = distance between plates.

Note : ϵ_r in all the above expressions stands for the relative permittivity of the dielectric of the capacitor.

4. **Energy of a charged conductor or capacitor** : If C is the capacity of a conductor or capacitor and V is the potential of conductor and P.D. in case of a capacitor, then energy of conductor or capacitor is given by

$$U=\frac{Q^2}{2C} \text{ joule (J)}$$

or $$U=\tfrac{1}{2}CV^2 \text{ joule (J).}$$

EXAMPLES

Ex. 1. *Two spheres of* 3 *and* 8 cm *radii are charged respectively with* 4×10^{-9} *C and* 16×10^{-9} *C of electricity. If they are joined by a fine wire, find how much charge will flow from one sphere to the other ?*

Sol. Capacity of the first sphere $=4\pi\epsilon_0 3\times10^{-2}$ F

,, ,, the second ,, $=4\pi\epsilon_0 8\times10^{2}$ F.

Capacity of the two after they are joined $=4\pi\epsilon_0\,(3+8)\times10^{-2}$ F

$=4\pi\epsilon_0 .11\times10^{-2}$ F.

Total charge $=20\times10^{-9}$ C.

Let V be the common potential.

Then, $$V=\frac{Q}{C}=\frac{20\times10^{\ 9}}{4\pi\epsilon_0\ 11\times10^{-2}} \text{ volt.}$$

Now, Q_1 (charge on 1st) $=VC=\dfrac{20\times10^{-9}}{4\pi\epsilon_0 11\times10^{-2}}\cdot 4\pi\epsilon_0 3\times10^{-2}$

$=5{\cdot}45\times10^{-9}$ C.

Q_2 (charge on the second) $=VC=\dfrac{20\times10^{-9}}{4\pi\epsilon_0 11\times10^{-2}}\cdot 4\pi\epsilon_0 8\times10^{-2}$

$=14{\cdot}55\times10^{-9}$ C.

Hence $1{\cdot}45\times10^{-9}$ C will flow from the second conductor to the first. **Ans.**

Ex. 2. *Eight globules of mercury, each having a diameter of* 1 *mm and a charge of* 5×10^{-9} *C unite to form a large globule. What would be the potential energy of the new globule ?*

Sol. Let $r=$ radius of the new globule.

$$\therefore\quad \frac{4\pi}{3}r^3=8.\frac{4\pi}{3}\left(\frac{10^{-3}}{2}\right)^3, \quad\text{or}\quad r=2.\frac{10^{-3}}{2}=10^{-3}\text{ m.}$$

Total charge $=8\times5\times10^{-9}=40\times10^{-9}$ C.

Capacitance of the new globule $=4\pi\epsilon_0\,10^{-3}$ F.

$$U=\frac{1}{2}\cdot\frac{Q^2}{C}.$$

$$\therefore\quad U=\frac{(40\times10^{-9})^2}{2\times4\pi\epsilon_0\,10^{-3}}=9\times10^9.8\times10^{-13}\qquad\left(\because\ \frac{1}{4\pi\epsilon_0}=9\times10^9\right)$$

$$=72\times10^{-4}\text{ joule.}$$

Ans.

Ex. 3. *Show that if a dielectric of thickness t and having same area as the plates of a parallel plate capacitor is introduced, the capacitance of the capacitor is given by*

$$C=\frac{\epsilon_0 A}{\left(d-t+\frac{t}{\epsilon_r}\right)}$$

Sol. Suppose we have a parallel plate air capacitor of capacitance C. Then $C=\frac{\epsilon_0 A}{d}$ where d = distance between the plates.

If we fill up the entire space by another dielectric of permittivity ϵ_r then

$$C'=\frac{\epsilon_0\,\epsilon_r A}{d}=\frac{\epsilon_0 A}{\frac{d}{\epsilon_r}}.$$

Now if the two capacitances are supposed to be equal then we find that $\frac{\epsilon_r}{d}$ occurs in place of d.

Thus, 'd' distance of dielectric $\equiv\frac{d}{\epsilon_r}$ distance of air, so far the capacitance of a parallel plate capacitor is concerned.

Therefore, if a dielectric of thickness t is introduced then it being equivalent to t/ϵ_r of air, the effective air distance between plates is $(d-t)+t/\epsilon_r$.

$$\therefore\qquad C=\frac{\epsilon_0 A}{(d-t+t/\epsilon_r)}.$$

Proved.

Ex. 4. *Two equal drops of water having same and similar charge coalesce to form a large single drop. Show how the surface density, capacity, potential and energy change ?*

Sol. In the process of coalescence two things remain constant, *mass* and *charge*. We will make use of this fact.

Let r = radius of each drop

R = radius of drop formed.

Then since mass remains constant,

$$\frac{4\pi}{3}R^3\rho = 2\times\frac{4\pi}{3}r^3\rho$$

or
$$R = 2^{\frac{1}{3}}r.$$

Now, σ_1 (surface density before) $= \frac{q}{4\pi r^2}$, where $q =$ charge on each,

$$\sigma_2 \text{ (surface density after)} = \frac{2q}{4\pi R^2} = \frac{2q}{4\pi 2^{\frac{2}{3}} r^2}.$$

$$\therefore \qquad \frac{\sigma_1}{\sigma_2} = \frac{2^{\frac{2}{3}}}{2} = \frac{1}{2^{\frac{1}{3}}} = \frac{1}{1{\cdot}26}$$

$$\therefore \qquad \sigma_1 : \sigma_2 = 1 : 1{\cdot}26. \qquad \textbf{Ans.}$$

C_1 (capacity before) $= 4\pi\epsilon_0 r$.

C_2 (capacity after) $= 4\pi\epsilon_0 R = 4\pi\epsilon_0 2^{\frac{1}{3}} r$.

$$C_1 : C_2 = 1 : 2^{\frac{1}{3}} = 1 : 1{\cdot}26. \qquad \textbf{Ans.}$$

V_1 (potential of each before) $= \frac{q}{4\pi\epsilon_0 r}$.

$$V_2 \text{ (potential after)} = \frac{2q}{4\pi\epsilon_0 R} = \frac{2q}{4\pi\epsilon_0 2^{\frac{1}{3}} r}.$$

$$\frac{V_1}{V_2} = \frac{1}{2^{\frac{2}{3}}}, \quad \text{or} \quad V_1 : V_2 = 1 : 1{\cdot}59. \qquad \textbf{Ans.}$$

$$U_1 \text{ (energy of each)} = \frac{q^2}{2C_1} = \frac{q^2}{8\pi\epsilon_0 r}. \qquad (\because \; C_1 = 4\pi\epsilon_0 r)$$

$$U_2 \text{ (energy of combination)} = \frac{(2q)^2}{2C_2} = \frac{4q^2}{8\pi\epsilon_0 R}$$

$$= \frac{4q^2}{8\pi\epsilon_0 2^{\frac{1}{3}} r}.$$

$$\therefore \qquad U_1 : U_2 = 1 : 2^{\frac{5}{3}} = 1 : 3{\cdot}18. \qquad \textbf{Ans.}$$

Ex. 5. *Two spheres of radii* 3 cm *and* 5 cm *are charged to potentials* 3000 *and* 4500 *volt respectively. They are then connected by a thin metallic wire. Calculate the loss of electric energy in this process. What happens to this energy ?*

Sol. C_1 (capacitance of the first) $=4\pi\epsilon_0 r=4\pi\epsilon_0(3\times10^{-2})$ farad,

C_2 (,, ,, second) $=4\pi\epsilon_0\,(5\times10^{-2})$ farad.

Q_1 (charge of the first) $=V_1C_1=3000\times4\pi\epsilon_0\times3\times10^{-2}$
$=4\pi\epsilon_0\times90$ C,

Q_2 (charge of the second) $=V_2C_2=4500\times4\pi\epsilon_0\times5\times10^{-2}$
$=4\pi\epsilon_0\times225$ C.

When joined together by a wire two things get added, charge and capacitance. Potential becomes common.

$\therefore$ Capacitance of the combination

$$=4\pi\epsilon_0(3\times10^{-2})+4\pi\epsilon_0\times(5\times10^{-2})=4\pi\epsilon_0\times(8\times10^{-2})\text{ F},$$

Charge of the combination $=Q_1+Q_2=4\pi\epsilon_0\times315$ C.

Now, U_1 (energy of 1st) $=\dfrac{Q_1^2}{2C_1}=\dfrac{(4\pi\epsilon_0)^2.8100}{2(4\pi\epsilon_0)(3\times10^{-2})}$

$$=4\pi\epsilon_0.(13{\cdot}5\times10^4)\text{ J}.$$

U_2 (energy of the second) $=\dfrac{Q_2^2}{2C_2}=\dfrac{(4\pi\epsilon_0)^2.225^2}{2.4\pi\epsilon_0(5\times10^{-2})}$

$$=4\pi\epsilon_0(50{\cdot}63\times10^4)\text{ J}.$$

$$\therefore\qquad U_1+U_2=4\pi\epsilon_0\times64{\cdot}13\times10^4\text{ J}.$$

U (energy of the combination) $=\dfrac{Q^2}{2C}=\dfrac{(4\pi\epsilon_0)^2\times315^2}{2\times4\pi\epsilon_0(8\times10^{-2})}$

$$=4\pi\epsilon_0\times(62\times10^4)\text{ J}.$$

$\therefore$ Loss in energy $=4\pi\epsilon_0(64{\cdot}13-62)\times10^4=4\pi\epsilon_0\times2{\cdot}13\times10^4$

$$=\frac{1}{9\times10^9}\times2{\cdot}13\times10^4=2{\cdot}36\times10^{-7}\text{ joule.}$$ **Ans.**

The loss appears as heat in the wire or appears as spark.

Ex. 6. *There are three concentric hollow spheres of radii a, b and c and they are charged with charges* q_1, q_2 *and* q_3 *respectively. Calculate the potentials of the spheres.*

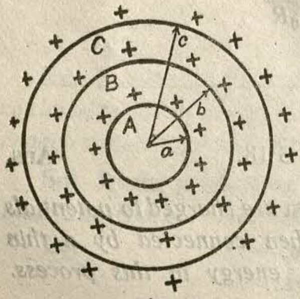


Fig. 4-1

Sol. In solving this problem and similar other problems we have to make note of the fact *potential inside a hollow conductor is the same at all points and is equal to that of its surface.* This is true for all hollow conductors—spherical, cylindrical or of any arbitrary shape. Another point which we have to note is that *for points on the surface or outside of a spherical conductor we can assume the charges to be concentrated at its centre.*

$\therefore$ Potential of A = potential due to charge on A + potential of interior of B and potential of interior of C

$$=\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right).\quad \textbf{Ans.}$$

Potential of B = exterior potential due to A on B + potential due to charge on B + potential of interior of C

$$=\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{b}+\frac{q_2}{b}+\frac{q_3}{c}\right)=\frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right).\quad \textbf{Ans.}$$

Potential of C = exterior potential due to A on C + exterior potential due to B on C + potential due to charge on C

$$=\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{c}+\frac{q_2}{c}+\frac{q_3}{c}\right)=\frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right).\quad \textbf{Ans.}$$

Ex. 7. *A condenser is made of two thin plates with a glass slab of 2 mm thickness between them. The tin plate is 5 cm × 10 cm and the relative permittivity of glass is 7·5. Find the capacitance of the capacitor.*

Sol. We have $C=\frac{\epsilon_0\,\epsilon_r\,A}{d}$.

Here $A = 5\text{ cm}\times 10\text{ m} = 50\text{ cm}^2 = 50\times 10^{-4}\text{ m}^2$.

$d = 2\text{ mm} = 2\times 10^{-3}\text{ m}$.

$$\therefore\quad C=\frac{8{\cdot}85\times 10^{-12}\times 7{\cdot}5\times 50\times 10^{-4}}{2{\cdot}2\times 10^{-3}}=16{\cdot}5\times 10^{-12}\text{ farad}=16{\cdot}5\text{ pF}.\quad \textbf{Ans.}$$

[$\because$ pico (p) = 10^{-12}]

Ex. 8. *Show that capacitance of a spherical capacitor is* $\frac{4\pi\epsilon_0 b^2}{(b-a)}$ *when outer is charged and inner is earthed.*

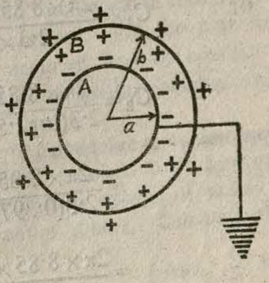


Fig. 4-2

Sol. Let us give Q amount of electricity to outer sphere B. Part of this will lie on the outer surface and the rest on the inner surface because B is surrounded by earthed bodies both from inside and outside.

If q_1 = charge on outer surface of B

q_2 = charge on inner surface of B

then $Q = q_1 + q_2$ and induced charge on $A = -q_2$.

Potential of $B=\frac{1}{4\pi\epsilon_0}\left(-\frac{q_2}{b}+\frac{q_2}{b}+\frac{q_1}{b}\right)=\frac{q_1}{4\pi\epsilon_0 b}$. (consult Ex. 6)

Potential of $A=\frac{1}{4\pi\epsilon_0}\left(-\frac{q_2}{a}+\frac{q_1+q_2}{b}\right)=0$. (as A is earthed)

$$\therefore \quad \frac{q_2}{a}=\frac{q_1+q_2}{b}, \text{ or } q_2\left(\frac{b-a}{ab}\right)=\frac{q_1}{b}, \text{ or } q_2=q_1\frac{a}{b-a}.$$

$$\therefore \quad \text{Capacity of condenser}=\frac{\text{Charge}}{\text{P.D.}}=\frac{Q}{V_B}=\frac{q_1+q_2}{\dfrac{q_1}{4\pi\epsilon_0 b}}$$

$$=4\pi\epsilon_0 b\left(1+\frac{q_2}{q_1}\right)=4\pi\epsilon_0 b\left(1+\frac{a}{b-a}\right)=4\pi\epsilon_0\,\frac{b^2}{b-a}. \qquad \textbf{Proved.}$$

Ex. 9. *A cylindrical capacitor consists of two coaxial conducting surfaces of radii* 2 cm *and* 2·5 cm *separated by two layers of insulating material of dielectric coefficient* 3 *and* 5. *The radius of common surface of the insulating layers is* 2·3 cm. *Calculate the capacitance of the capacitor if its length is* 15 cm.

Sol. There are in fact two cylindrical capacitors : one of inner and outer radii 2 cm and 2·3 cm and dielectric of permittivity 3 and the other of inner and outer radii 2·3 cm and 2·5 cm and dielectric of permittivity 5.

$$C_1=\frac{2\pi\,\epsilon_0\,\epsilon_r l}{2{\cdot}3\,\log_{10}\dfrac{b}{a}}=\frac{2\pi\times 8{\cdot}85\times 10^{-12}\times 3\times{\cdot}15}{2{\cdot}3\,\log\dfrac{2{\cdot}3}{2}}$$

$$=\frac{2\pi\times 8{\cdot}85\times 3\times{\cdot}15\times 10^{-12}}{2{\cdot}3\,(0{\cdot}3617-0{\cdot}3010)}$$

or $$C_1=\frac{2\pi\times 8{\cdot}85\times{\cdot}15\times 3}{2{\cdot}3\times{\cdot}0607}\times 10^{-12}=179{\cdot}25\times 10^{-12}\text{ farad}$$

$$C_2=\frac{2\pi\times 8{\cdot}85\times{\cdot}15\times 5}{2{\cdot}3(\log 2{\cdot}5-\log 2{\cdot}3)}\times 10^{-12}$$

$$=\frac{2\pi\times 8{\cdot}85\times{\cdot}15\times 5}{2{\cdot}3(0{\cdot}3979-0{\cdot}3617)}$$

$$=\frac{2\pi\times 8{\cdot}85\times{\cdot}15\times 5}{2{\cdot}3\times{\cdot}0362}\times 10^{-12}=500{\cdot}96\times 10^{-12}\text{ farad.}$$

The two are in series. The effective capacitance in series connection is given by $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$.

$$\therefore \quad C=\frac{C_1 C_2}{C_1+C_2}=\frac{179{\cdot}25\times 500{\cdot}96}{179{\cdot}25+500{\cdot}96}\times 10^{-12}=132\times 10^{-12} \text{ farad.}$$

Ans.

EXERCISES

(A)

1. A charge of 15 nC (nano-coulomb) raises the potential of a spherical conductor from 500 volt to 1500 volt. Find the radius of the conductor.
(**Ans.** ·135 m or 13·5 cm)

2. A hollow spherical conductor of radius 10 cm is surrounded concentrically by another bigger shell of radius 12 cm and thickness 1 cm. The inner one is charged with 39 nC (nano coulomb). Calculate the potential of inner sphere when (i) outer one is not there, (ii) when outer one is there, (iii) when outer one is there and earthed. [**Ans.** (i) 3510 V, (ii) 3285 V, (iii) 585 volts]

3. A brass sphere of radius 10 cm is electrified to a potential of 24 kV (kilo volt). It is then made to share charge with another uncharged brass sphere and the potential is found to fall to 6 kV. What is the radius of the second sphere ?
(**Ans.** ·3 m)

4. 64 droplets of water of equal size unite to form a large drop. If each droplet had equal and similar charge, how will the potential, energy and capacitance change ?
(**Ans.** potential will increase 16 times, capacitance 4 times and energy 16 times)

5. A charged sphere of radius a is put into conducting communication with an uncharged sphere of radius b. Show that the ratio of the original distribution to that of the final distribution is $(a+b):a$.

6. A sphere of radius 10 cm is concentrically surrounded by a thin spherical shell of radius of 11 cm. A charge of 100 μC is imparted to the outer sphere and inner is earthed. Calculate charge on the outter surface of inner sphere and on the two surfaces of outer sphere.
(**Ans.** $-90{\cdot}9\ \mu C$; $+9{\cdot}09\ \mu C$)
[*Hint* : Ex. 9.]

7. Two spherical rain drops of radii 1 mm and 2 mm come in contact of a lightning conductor at potential 500 volts and then coalesce. Find the potential of the resulting drop and the electrical energy they acquired from the conductor and gained or lost subsequently due to coalescence after getting detached from the conductor.
(**Ans.** 721 V; $4{\cdot}172\times10^{-8}$ J; $1{\cdot}8\times10^{-8}$ J gained due to coalescence)

8. Water from a tap maintained at a potential of V is allowed to fall by drops of radius 'r' through a small hole into a hollow conducting sphere of radius R standing on an insulating stand until it fills the entire sphere. Calculate the potential of the hollow conductor.
$\left(\textbf{Ans.}\ \frac{R^2}{r^2}V\right)$

9. An insulated sphere of radius 10 cm has a positive charge of 20 nC and another of radius 5 cm has a positive charge of 15 nC. If the two are connected, find the direction in which charge will flow. Calculate their common potential and loss in electrical energy.

(**Ans.** Charge will flow from 5 cm sphere to 10 cm sphere; 2100 V; $1{\cdot}5\times10^{-6}$ joule)

10. Calculate the potential energy of a nano coulomb charge when placed at a distance of 10 cm from a micro coulomb charge. (**Ans.** 8.99×10^{-5} joule)

11 A parallel plate capacitor is made up of 15 strips each of 6 cm × 4 cm, separated by a sheet of mica of dielectric coeffcient 6 and constant thickness of ·15 mm. Calculate its capacitance. (**Ans.** ·0119 μF)

12. A parallel plate capacitor is made up of 11 metal plates each of area 15 cm² separated by sheets of mica of relative permittivity 6 and thickness ·2 mm, the alternate plates being joined together. Calculate the capacitance of the capacitor in microfarad.

[*Hint* : There will be $(n-1)$ simlar parallel plate capacitor of capacitance $\frac{\epsilon_0\epsilon_r A}{d}$. $\therefore\ C=(n-1)\frac{\epsilon_0\epsilon_r A}{d}$.] (**Ans.** ·00398 μF)

13. A slab of copper of thickness t is thrust into a parallel plate capacitor of area A and plate separation d. What is the capacitance before and after the slab is introduced ?

[*Hint* : There is no electric field inside a copper slab or we may say that metals are insulators of $\epsilon_r=\infty$.]

[*Hint* : Ex. 3.] [**Ans.** (1) $C=\frac{\epsilon_0 A}{d}$; (ii) $C=\frac{\epsilon_0 A}{d-t}$]

14. A parallel plate capacitor has a plate area A and plate separation d. It is charged to a potential V and then the battery is disconnected. Now a dielectric slab of dielectric coefficient ϵ_r and of thickness d is inserted in between the plates. Calculate the energy stored before and after the insertion of the slab. Account for the change in energy, if any.

[**Ans.** Decrease in energy is $\frac{1}{2}\cdot\frac{\epsilon_0 AV^2}{d}\left(\frac{\epsilon_r-1}{\epsilon_r}\right)$; Energy is transferred to the agent introducing the slab.]

15. Assuming that the energy of a charged parallel plate capacitor is $\frac{1}{2}CV^2$ and this energy is stored in the medium between the plates, show that energy density in the medium is

$$u_E=\tfrac{1}{2}\epsilon E^2.$$

16. A spherical capacitor is made of two spherical conductors of radii 4 cm and 6 cm respectively. A spherical shell of permittivity 4 and thickness 1 cm surrounds the inner sphere, the remainder of the space between the spheres being filled with an insulator of permittivity 6. Calculate the capacitance of the capacitor. (**Ans.** 61·54 μμF)

17. Calculate the capacitance per metre of a capacitor formed by two long co-axial cylinders of radii 5 and 5·2 cm filled with dielectric of permittivity 1·2.

[*Hint* : $C=\frac{2\pi\epsilon_0\epsilon_r l}{\log_e b/a}$.] (**Ans.** $1{\cdot}7\times10^{-9}$ farad)

18. A submarine cable consists of a wire 3 mm in diameter and insulated with 3 mm of guttapercha (permittivity=4·26) is placed in water. Calculate the capacitance of 1 kilometre of the cable in micro-farad.

(*Hint* : Consider submarine cable as a cylindrical capacitor of inner radius 1·5 mm and outer radius of 4·5 mm.) (**Ans.** ·215 μF)

19. Find the capacitance of unit length of a cylindrical capacitor of which the conductors have radii 2·5 cm and 4·5 cm respectively and the dielectric consists of two layers whose cylinder of contact is 3·5 cm in radius; the inner layer having a dielectric coefficient 4 and the outer layer 6. (**Ans.** $\cdot 442\times10^{-9}$ farad)

20. A parallel plate capacitor is charged to certain potential difference. A slab of 3 mm thickness is inserted between the plates, and it becomes necessary to increase the distance between them by 2·4 mm to maintain the same p.d. Find the dielectric coefficient of the slab. (**Ans.** $\epsilon_r=5$)

[*Hint* : See Ex. 3.]

21. A parallel plate condenser of area 100 cm², plate separation 5 mm is charged to a p.d. of 100 volt when air is used between plates. If now air is replaced by glass ($\epsilon_r=6$), calculate (a) new capacity, (b) the new p.d. and (c) the loss of energy. Account for the loss of energy.

[**Ans.** (a) $1\cdot062\times10^{-10}$ F; (b) 16·7 volts; (c) $7\cdot37\times10^{-8}$J]

(B)

22. Two equal metal spheres of radius 10 cm are joined together by a long fine wire. They are insulated and electrified by a battery of e. m. f. 10 volts. Connection with battery is not disturbed. If now one of the spheres contracts to one-fourth of its original surface, will the system consisting of the two spheres draw or give charge to the battery ? By how much ? What amount of charge was taken by the system in the beginning ?

(**Ans.** it will give ·056 nC to the battery; ·222 nC)

23. Calculate the self potential energy of charge q distributed over the surface of a hollow sphere of radius R. What about this self energy when $R=0$, when the charge is an ideal point charge ? $\left(\textbf{Ans. } \dfrac{q^2}{8\pi\epsilon_0 R}\right)$

24. A charge q is distributed uniformly throughout a non-conducting spherical volume of radius R. (a) Show that the potential at a distance x from the centre when $x<R$ is given by $V=\dfrac{q(3R^2-x^2)}{8\pi\epsilon R^3}$ and the self potential energy of the charge distributed is $U=\dfrac{3q^2}{20\pi\epsilon R}$.

[*Hint* : $V=V_1+V_2$ where V_1=potential due to inner charge, V_2=potential due to charge lying outside. Calculate V_1 assuming charge inside of sphere of radius x concentrated at its centre. To calculate V_2 consider a shell of radius z and thickness dz and calculate potential at the point taking note of the fact stated in Ex. 7. Integrate from $z=x$ to $z=R$. For calculation of self potential energy consider a shell of radius z and thickness dz, potential of shell$=\frac{4\pi/3z^2\sigma}{4\pi\epsilon z}=\frac{z^2\sigma}{3\epsilon}$ where σ=density of charge. To increase thickness of shell by dz, charge to be added$=4\pi z^2dz\sigma$. Work to be done$=\frac{z^2\sigma}{3\epsilon}.4\pi z^2dz\sigma$ $=\frac{4\pi\sigma^2}{3\epsilon}.z^4dz$. This is self potential energy of shell. Integrate from $z=0$ to $z=R$.)

25. Calculate the capacitance of a spherical capacitor consisting of two concentric spherical conductors of radii a and b $(b>a)$, when outer one is charged and inner is earthed. The space between the spheres is filled by insulating material of relative permittivity ϵ_r.

[*Hint* : See Ex. 9.] $\left(\textbf{Ans. } 4\pi\epsilon_0\frac{(\epsilon_r-1)\,ab+b^2}{b-a}\right)$

26. A parallel plate capacitor is filled half-half with two dielectrics of dielectric constants ϵ_1 and ϵ_2 respectively (i) one dielectric placed over the other, (ii) one completely filling one half and the other filling the other half. Calculate their capacitances. The distance between the plates is d and area of each plate is A.

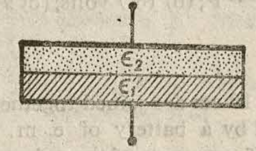


Fig. 4-3

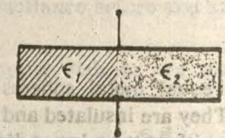


Fig. 4-4

[*Hint :* (i) In fact there are two capacitors of equal area but of different dielectrics and thickness, they are in series. (ii) There are two capacitors of the same thickness but of different area and dielectrics, they are in parallel.]

$$\left[\textbf{Ans. (i) } \frac{2\epsilon_0A}{d}\left(\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}\right);\text{ (ii) } C=\frac{\epsilon_0A}{d}\left(\frac{\epsilon_1+\epsilon_2}{2}\right)\right]$$

27. A solid silver sphere of radius 1 cm is charged to a potential of 10 kV. Calculate the fraction of the atoms ionised assuming that one atom contributes only one free electron. Density of silver$=10{\cdot}5\times10^3$ kgm^{-3} and its atomic weight is 107·9. (Avogadro's number$=6\times10^{26}$ kg mol^{-1} and charge of an electron $=1{\cdot}6\times10^{-19}$ C.) (**Ans.** $2{\cdot}84\times10^{-13}$)

28. Find the capacitance of a system consisting of a metal sphere of radius a placed at a large distance from an identical metallic sphere. (**Ans.** $2\pi\epsilon_0\epsilon_r\,a$)

[*Hint :* When the distance is large, the charges on the spheres may be assumed to be distributed practically uniformly over the surfaces of the spheres.

Consider their potential difference and apply $C=Q/V$.)

29. Two long straight wires with equal cross-sectional radii a are located parallel to each other in air at a large distance b apart. Find the capacitance per unit length of the wires.

$\left(\textbf{Ans. } \frac{\pi\epsilon_0}{\log b/a}\right)$

[*Hint* : Take charges to be uniformly distributed. Consider field at any point and hence find p.d. between wires.]

30. A parallel plate capacitor contains a mica sheet (thickness 10^{-3} m) and a sheet of fibre (thickness $0{\cdot}5\times10^{-3}$ m). The dielectric constant of mica is 8 and that of the fibre is 2·5. Assuming that the fibre breaks down when subjected to the electric field of $6{\cdot}4\times10^{-6}$ Vm^{-1}, find the maximum safe voltage that can be applied to the capacitor. (Ans. 5200 volt)

31. A parallel plate capacitor consists of two square plates of effective area A, length L and plate separation d. If one of the plates is turned through a small angle θ, what is the change in the capacitance of the capacitor ?

$$\left(\textbf{Ans. } \frac{\epsilon_0 AL\theta}{2d^2}\right)$$

□□□

CHAPTER 5

POLARISATION, ELECTRIC DISPLACEMENT AND ELECTRIC FIELD

1. Polarisation : A dielectric in a capacitor may be depicted as an assemblage of large number of electric dipoles oriented in random directions. When an electric field is applied these dipoles are aligned in the direction of the applied field. This phenomenon of alignment of electric dipoles is known as Polarisation (cf. Magnetisation) and is measured by *dipole moment per unit volume of the dielectric.* The intermediate positive and negative charges neutralise each other except the charges on the extreme faces of the dielectric. These charges are called the polarised charges. It can be shown that polarisation is equal to the surface density of polarised charge.

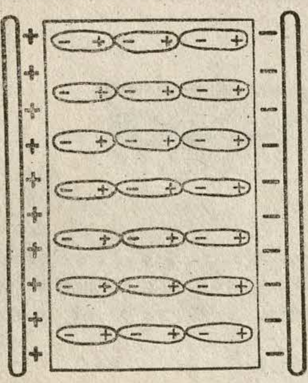


Fig. 5-1

$\therefore$ $P=\sigma'$ *coulomb per square-metre* (Cm^{-2}). This is a vector and is a characteristic of the medium. For an electric field in a dielectric two vectors are defined.

(i) **The E-vector :** This is the vector defined as the force experienced by unit positive charge.

(ii) **The D-vector** : This in an auxiliary vector which is given by the electric vector multiplied by the scalar ϵ_0 in a vacuum or $\epsilon_0\epsilon_r$ in any dielectric. Since it is given by the electric vector multiplied by a scalar, it is also a vector having the same direction as the *E*-vector but of different magnitude and dimension.

$\vec{D}=\epsilon_0\,\vec{E}_0=\epsilon\,\vec{E}$ where $\vec{E}_0$ is the *E*-vector in a vacuum and $\vec{E}$ is the same in a dielectric.

By Gauss's theorem, $E=\dfrac{\sigma}{\epsilon}=\dfrac{\sigma}{\epsilon_0\,\epsilon_r}$.

$\therefore\quad D=\epsilon E=\epsilon_0\,\epsilon_r.\dfrac{\sigma}{\epsilon_0\,\epsilon_r}=\sigma.$ Thus *D* is equal to surface density of free charge. Since D is independent of medium, D must also be equal to $\epsilon_0 E_0$ where E_0 is the electric intensity in air or vacuum and ϵ_0 is the permittivity of vacuum.

$$\therefore \quad \epsilon_0 E_0 = \epsilon E, \quad \text{or} \quad E_0 = \frac{\epsilon_0 \epsilon_r E}{\epsilon_0} = \epsilon_r E, \quad \text{or} \quad E_0 = \epsilon_r E.$$

2. Relation of D, P and E : The three vectors D, P and E of an electric field inside a dielectric are related by the relation,

$$D = \epsilon_0 E + P. \quad \text{Vectorially, } \vec{D} = \epsilon_0 \vec{E} + \vec{P}.$$

3. Relation of polarised charge with free charge :

$$\therefore \qquad D = \sigma \quad \text{and} \quad P = \sigma'.$$

$\therefore$ From the relation $D = \epsilon_0 E + P$, we have,

$$\sigma = \epsilon_0 \frac{\sigma}{\epsilon_0 \epsilon_r} + \sigma' \quad \left(\because \quad E = \frac{\sigma}{\epsilon_0 \epsilon_r} \right)$$

or
$$\sigma' = \sigma\left(1 - \frac{1}{\epsilon_r}\right).$$

4. Relation of P and E :

We have, $\qquad D = \epsilon_0 E + P.$

But $\qquad D = \epsilon_0 \epsilon_r E. \quad \therefore \quad \epsilon_0 \epsilon_r E = \epsilon_0 E + P$

or $\qquad P = \epsilon_0 (\epsilon_r - 1) E.$

5. Electric susceptibility (χ_e). The electric susceptibility of a dielectric is defined as

$$\chi_e = \frac{P}{\epsilon_0 E}.$$

χ_e, defined in this way, has no unit.

6. Relation of χ_e and ϵ_r : $\chi_e = \dfrac{P}{\epsilon_0 E} = \dfrac{\epsilon_0(\epsilon_r - 1)E}{\epsilon_0 E}$

$$= (\epsilon_r - 1)$$

or
$$\chi_e = \epsilon_r - 1.$$

EXAMPLES

Ex. 1. *A parallel plate capacitor having glass as dielectric has surface density of surface* ·01 μC/m² *on it. The area of each plate is* ·01 m², *and the distance between the plates is* 1 cm. *Calculate the* *three vectors namely polarisation vector, E-vector and D-vector in glass. The dielectric constant of glass is* 10.

Sol. We have, $E=\dfrac{\sigma}{\epsilon_0\ \epsilon_r}=\dfrac{\cdot 01\times 10^{-6}}{8\cdot 85\times 10^{-12}\times 10}$

$=1\cdot 13\times 10^{2}\ \mathrm{Vm^{-1}}$. **Ans.**

We have, $D=\sigma$.

$\therefore$ $D=\cdot 01\times 10^{-6}=10^{-8}\ \mathrm{Cm^{2}}$. **Ans.**

Further we have, $D=\epsilon_0\ E+P$.

$$\therefore\quad P=D-\epsilon_0 E=10^{-8}-8\cdot 85\times 10^{-12}\ \frac{\cdot 01\times 10^{-6}}{8\cdot 85\times 10^{-12}\times 10}$$

$$=10^{-8}-\cdot 1\times 10^{-8}.$$

or $P=\cdot 9\times 10^{-8}\ \mathrm{Cm^{-2}}$. **Ans.**

Ex. 2. *The dielectric coefficient of a certain material is* 3·5. *Compute its permittivity and electric susceptibility.*

Sol. $\epsilon=\epsilon_0\epsilon_r$.

$\therefore$ $\epsilon=8\cdot 85\times 10^{-12}\times 3\cdot 5=31\times 10^{-12}\ \mathrm{Fm^{-1}}$. **Ans.**

We have the relation between χ_e and ϵ_r :

$\chi_e=\epsilon_r-1=3\cdot 5-1=2\cdot 5$. **Ans.**

Ex. 3. *A uniform electric field of intensity* $2\times 10^{6}\ \mathrm{Vm^{-1}}$ *is set up within a large block of material of dielectric coefficient* 3. *A cavity is cut out in the form of a short cylinder whose ends are perpendicular to the field. Find the electric intensity within the cavity and surface density of induced charges on its end surfaces.*

Sol. Within the cavity there is vacuum. So we have to find E_0. Relation of E_0 and E is $E_0=\epsilon_r E$.

$$\therefore\quad E_0=3\times 2\times 10^{6}=6\times 10^{6}\ \mathrm{Vm^{-1}}.$$

We have, $\sigma'=\sigma\left(1-\dfrac{1}{\epsilon_r}\right);\ E=\dfrac{\sigma}{\epsilon_0\ \epsilon_r}$.

$$\therefore\quad \sigma'=\epsilon_0\epsilon_r E\left(1-\frac{1}{\epsilon_r}\right)=\epsilon_0 E(\epsilon_r-1).$$

$$\therefore\quad \sigma'=8\cdot 85\times 10^{-12}\times 2\times 10^{6}(3-1)$$

$=35\cdot 4\times 10^{-6}\ \mathrm{Cm^{-2}}$. **Ans.**

Ex. 4. *A parallel plate capacitor having plate separation* 5 mm *is charged up by a battery. On introducing a mica sheet of* 2 mm *maintaining the connections of the plates with the terminals of battery, it is found that it draws* ·25% *more charge from the battery. Calculate permittivity and susceptibility of mica.*

Sol. We have, $Q=VC=V.\dfrac{\epsilon_0 A}{d}.$

Again $Q'=V.C'=V.\dfrac{\epsilon_0 A}{d-t+t/\epsilon_r}$

($\because$ V remains same as we have not disconnected the battery.)

$$\therefore \qquad \frac{Q'}{Q}=\frac{V\epsilon_0 A}{(d-t+t/\epsilon_r)}\times\frac{d}{V\epsilon_0 A}$$

or $$\frac{Q'}{Q}=\frac{d}{d-t+t/\epsilon_r}.$$

Here, $$\frac{Q'}{Q}=\frac{125}{100}=\frac{5}{4}.$$

$$\therefore \qquad \frac{5}{4}=\frac{5\times10^{-23}}{\left(5-2+\dfrac{2}{\epsilon_r}\right)\times10^{-3}}$$

or $$\frac{5}{4}=\frac{5}{3+2/\epsilon_r}, \quad \text{or} \quad 4=3+\frac{2}{\epsilon_r}, \quad \text{or} \quad 1=\frac{2}{\epsilon_r}$$

or $\epsilon_r=2.$ **Ans.**

$\therefore$ $\epsilon=\epsilon_0\epsilon_r=8{\cdot}85\times10^{-12}\times2=17{\cdot}7\times10^{-12}\text{ Fm}^{-1}.$ **Ans.**

$\chi_e=(\epsilon_r-1)=2-1=1.$

Ex. 5. *A dielectric of dielectric coefficient 3 fills up three-fourth of the space between the plates of a parallel plate capacitor. What per cent of the energy is stored in the dielectric ?*

Sol. Energy of the capacitor $=\dfrac{Q^2}{2C}$

$$=\frac{Q^2}{2.\dfrac{\epsilon_0 A}{\left(d-\dfrac{3}{4}d+\dfrac{3/4d}{3}\right)}}=\frac{Q^2 d}{4\epsilon_0 A}$$

Energy in the dielectric = energy of condenser of same area, charge and having plate separation equal to the thickness of dielectric

$$=\frac{Q^2}{2.C'}=\frac{Q^2}{2\frac{\epsilon_0\epsilon_r A}{d}}=\frac{Q^2}{2\frac{\epsilon_0.3A}{3d/4}}=\frac{Q^2d}{8\epsilon_0 A}$$

($\because$ here thickness of dielectric $=\frac{3}{4}$ of plate separation).

$$\therefore \quad \% \text{ of energy stored}=\frac{\frac{Q^2d}{8\epsilon_0 A}}{\frac{Q^2d}{4\epsilon_0 A}}\times 100=50\%. \qquad \textbf{Ans.}$$

EXERCISES

(A)

1. A parallel plate capacitor has a capacitance of 100 pf (pico farad), a plate area of 100 cm ^{2} and a mica dielectric of permittivity 5·4. At 50 V potential difference, calculate (a) E in the mica, (b) the free charge on the plates and (c) the polarised charge.

[*Hint :* Use $Q=VC$ to know the free charge.

Use the relation $Q'=Q\left(1-\frac{1}{\epsilon_r}\right)$ to know the polarised charge.]

[Ans. (a) $1{\cdot}046\times10^4$ Vm^{-1}; (b) 5×10^{-9} C; (c) $4{\cdot}1\times10^{-9}$ C]

2. Two parallel plates of area 100 cm^2 each are given equal and opposite charges of $8{\cdot}9\times10^{-7}$ C. Within the dielectric material filling the space between plates the electric field strength is $1{\cdot}4\times10^6$ Vm^{-1}. Find (a) the dielectric constant (ϵ_r) of the material, (b) the magnitude of polarised charge on each face of dielectric. [**Ans.** (a) 7·18; (b) $7{\cdot}66\times10^{-7}$ C]

3. A parallel plate capacitor has a surface area 100 cm^2 and surface density of charge 10 nC/m^2 on it. The distance between the plates is 1 cm. A glass slab ($\epsilon_r=10$) of thickness 5 mm is kept midway between the plates. Calculate (a) the electric field in air, (b) the electric field in slab, (c) electric susceptibility of glass.

[**Ans.** (a) $1{\cdot}13\times10^3$ Vm^{-1}; (b) $\cdot113\times10^3$ Vm^{-1}; (c) 9]

4. Show that metal can be considered as dielectric of infinite dielectric constant.

(*Hint* : $E=\frac{\sigma}{\epsilon_0\epsilon_r}$. Inside a metal electric field is zero.

$\therefore \quad \frac{\sigma}{\epsilon_0\epsilon_r}=0;\ \sigma\neq0,\ \epsilon_0\neq0.\quad\therefore$ The only possibility is that $\epsilon_r=\infty$.)

5. In a parallel plate capacitor a mica sheet of dielectric coefficient 7 fills half the space between the plates. What per cent of the energy is stored in the air gap ? (**Ans.** 87·5%)

6. Two oppositely charged conducting plates, having numerically equal surface densities, are separated by a dielectric 5 mm thick of dielectric coefficient 3. The resultant electric intensity in the dielectric is 10^6 Vm^{-1}. Compute (a) the electric displacement, (b) the polarisation in the dielectric, (c) components of the electric intensity in the dielectric due to free charge and (d) the component of electric intensity due to polarised charge.

[**Ans.** (a) $2{\cdot}66\times10^{-5}$ Cm^{-2}; (b) $1{\cdot}77\times10^{-5}$ Cm^{-2}; (c) 3×10^6 Vm^{-1}; (d) 2×10^6 Vm^{-1}]

7. Two parallel conducting plates 5 mm apart are given equal and opposite surface densities of charge of 20 μC/m^2. The region between the plates is occupied by two sheets of dielectric, one 2 mm thick, of dielectric coefficient 3, the other 3 mm thick and of dielectric coefficient 4. Calculate (a) the electric intensities in each dielectric, (b) the displacement in each dielectric, (c) the surface density of induced charge on each dielectric.

[**Ans.** (a) $7{\cdot}5\times10^5$ Vm^{-1}; $5{\cdot}6\times10^5$ Vm^{-1}; (b) 20 μC/m^2, (c) 13·3 μC/m^2; 15·0 μC/m^2]

(B)

8. The space between the plates of a parallel plate capacitor is filled consecutively with two dielectric layers 1 and 2 having thickness d_1 and d_2 and the relative permittivities ϵ_1 and ϵ_2 respectively. The area of each plate is equal to S. Find the density of charge σ' of the bound charges on the boundary plane if the voltage across the capacitor equals to V and the electric field is directed from layer 1 to layer 2.

$$\left[\textit{Hint}: \sigma'=\sigma_+'+\sigma_-';\ \sigma_+'=\sigma\left(1-\frac{1}{\epsilon_1}\right);\ \sigma_-'=-\sigma\left(1-\frac{1}{\epsilon_2}\right)\right]$$

$$\left[\textbf{Ans. } \sigma'=\frac{\epsilon_0(\epsilon_1-\epsilon_2)V}{\epsilon_1 d_2+\epsilon_2 d_1}\right]$$

9. Four metal plates are located at a small distance d from one another as shown in Fig. 6-14 of the next chapter. If V is the potential difference between A and B, find the electric field strength between the plates 2 and 3, between 1 and 2 and between 3 and 4.

Also find the density of charges on the plates 1, 2, 3 and 4.

$$\left(\textbf{Ans. } E_{1\,2}=E_{3\,4}=\frac{V}{2d};\ E_{2\,3}=\frac{V}{d};\ |\sigma_1|=|\sigma_4|=\frac{1}{2}\epsilon_0\frac{V}{d};\ |\sigma_2|=|\sigma_3|=\frac{3}{2}\epsilon_0\frac{V}{d}\right)$$

[*Hint* : Work done round a closed path drawn through static electric fields is zero, that is, algebraic sum of p.d's round a closed loop is zero and electric field at any point is vector sum of the fields due to the six layers of charges on the plates.]

10. An electric field of strength $E=100$ kVm^{-1} produces polarisation in water equivalent to the correct orientation of only one out of N molecules. Find N. The electric moment of a water molecule is $6{\cdot}2\times10^{-30}$ Cm. Relative permittivity of water$=81$ and concentration of water molecules n_0 $=3{\cdot}4\times10^{28}$ m^{-3}.

[*Hint* : $P=\epsilon_0(\epsilon_r-1)E$ and Polarisation=electric dipole moment per unit volume.]

$$\left[\textbf{Ans. } N=\frac{n_0\,p}{\epsilon_0(\epsilon_r-1)E}=3\times10^3\right]$$

11. A parallel air capacitor having plate separation by d is connected to a cell of emf E. A dielectric slab of relative permittivity ϵ_r and thickness d is introduced between the plates at a constant velocity v. Calculate the current produced in the circuit. The plates are square in shape and of length a on each side.

$$\left[\textbf{Ans. } \frac{E\epsilon_0(\epsilon_r-1)av}{d}\right]$$

□□□

CHAPTER 6

GROUPING OF CAPACITORS

1. Capacitors in series : When a number of capacitors are joined in series the capacitance of the equivalent capacitor is given by

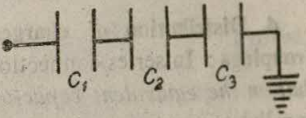


Fig. 6-1.

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots.$$

When there are two capacitors,

$$C=\frac{C_1C_2}{C_1+C_2}\left(\frac{\text{product}}{\text{sum}}\right).$$

If V_1, V_2, V_3 .... are the p.d's across C_1, C_2, C_3...., then

Q (charge on each capacitor) $=V_1C_1=V_2C_2=V_3C_3=\ldots.$

$$\therefore \quad V_1 : V_2 : V_3 \ldots. = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3} : \ldots.$$

that is, *p.d's are divided in the inverse ratio of capacitances of the capacitors.*

2. Capacitors in parallel : When a number of capacitors are joined in parallel, the capacitance of the equivalent capacitor is given by

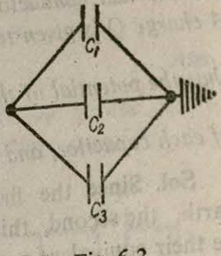


Fig. 6-2.

$$C=C_1+C_2+C_3+\ldots.$$

If Q_1, Q_2, Q_3 .. are the charges on C_1, C_2, C_3 ...., then

V (common potential difference) $=Q_1/C_1=Q_2/C_2=Q_3/C_3=\ldots\ldots$

$$\therefore \quad Q_1 : Q_2 : Q_3 \ldots. = C_1 : C_2 : C_3 : \ldots.$$

that is, *charges are divided in proportion to the capacitances of the capacitors.*

3. Combination of capacitors having cross-connection : To find equivalent capacitance of a number of capacitors joined in a complex way (complexity is always due to cross-connection), we have to make use to Kirchhoff's point rule and loop rule.

(i) *Point rule* : The algebraic sum of induced charges on conductors meeting at a point is zero.

(ii) *Loop rule* : The algebraic sum of p.d's across the capacitors in a loop is equal to zero if there is no seat of emf's in the loop; otherwise it is equal to algebraic sum of emf's.

4. Distribution of charges on capacitors in series and parallel groupings : In series connection charges on *capacitors are the same as that on the equivalent capacitor.* The charges on the capacitors in parallel connection *are in the ratio of their capacitances.*

EXAMPLES

Ex. 1. *Three capacitors of capacitance* 4, 5 *and* 8 μF *are connected in such a way that the first two are in parallel and the third is in series with them. Find the capacity of the combination.*

Sol. Let C be the capacitance of the first two. Then

$$C=4+5=9\ \mu\text{F}.$$

Now 9 μF is in series with 8 μF.

$$\therefore \qquad \frac{1}{C'}=\frac{1}{9}+\frac{1}{8}=\frac{17}{72} \quad \text{or} \quad C'=\frac{72}{17}=4\cdot24\ \mu\text{F}.$$

Ex. 2. *Four indentical capacitors are connected in series. The first and the last conductors of the combination are connected to earth. A charge Q is given to the second plate of the first capacitor. Prove that the potential of this conductor is* $\frac{3Q}{4C}$, *where C is the capacitance of each capacitor, and that of second is* $Q/2C$.

Sol. Since the first and the last conductors are connected to the earth, the second, third and fourth capacitors are in series. Let C' be their equivalent capacitance.

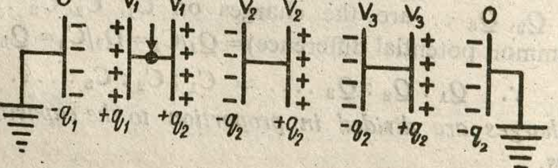


Fig. 6-3.

Then $$\frac{1}{C'}=\frac{1}{C}+\frac{1}{C}+\frac{1}{C}=\frac{3}{C} \quad \text{or} \quad C'=\frac{C}{3}.$$

The first is in parallel with C'.

$\therefore$ C_{equ} (equivalent capacitance of the system) $=C+\frac{C}{3}=\frac{4C}{3}$.

The potential difference across the equivalent capacitor

$$=V_1-0=V_1.$$

$$\therefore \quad Q=V_1\times\frac{4C}{3} \quad \text{or} \quad V_1=\frac{3Q}{4C}.$$ **Proved.**

Applying $Q=VC$ formula to the first capacitor and the second capacitor we have,

$$q_1=(V_1-0)\times C \text{ and } q_2=(V_1-V_2)\times C.$$

But $Q=q_1+q_2$.

$\therefore$ $Q=V_1C+(V_1-V_2)C=2V_1C-V_2C$

or $V_2C=2V_1C-Q=2.\frac{3Q}{4C}.\ C-Q=\frac{Q}{2C}$. **Proved.**

Ex. 3. *Three capacitors of capacitances* 2 μF, 3 μF *and* 5 μF *are connected in parallel and the combination is charged up by a battery of* 110 *volts. Calculate the total charge taken from the battery and the charges on the capacitors.*

Sol. $$Q_1=V_1C_1=110\times2\times10^{-6}=220\times10^{-6}\text{ C},$$
$$Q_2=V_2C_2=110\times3\times10^{-6}=330\times10^{-6}\text{ C},$$
$$Q_3=V_3C_3=110\times5\times10^{-6}=550\times10^{-6}\text{ C},$$

$\therefore$ Total charge taken $=(220+330+550)\times10^{-6}$
$=1100\times10^{-6}$ C. **Ans.**

Ex. 4. *Five capacitors are connected as shown in the figure* (Fig. 6-4). *Calculate the equivalent capacitance between a and b.*

Sol. We have shown distribution of charge in compliance with point rule (Fig. 6-5).

Applying loop rule to loop *acda* (see fig. 6-5),

$$-\frac{q_1}{\frac{1}{2}}+\frac{q_3-q_1}{\frac{1}{4}}+\frac{q_2}{\frac{1}{3}}=0$$

or $2q_1-4(q_3-q_1)-3q_2=0$

or $6q_1-3q_2-4q_3=0$ .. (i)

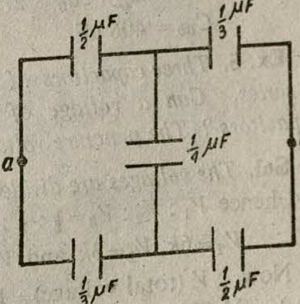


Fig. 6-4

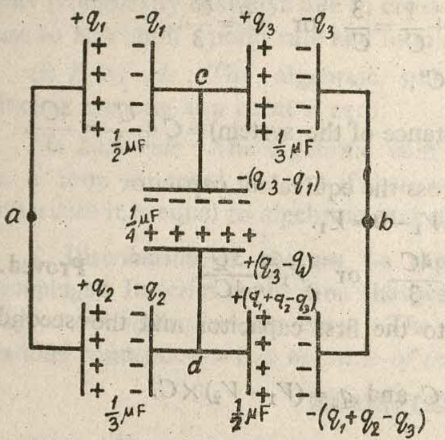


Fig. 6-5

From loop *cdbc*,

$$\frac{q_3-q_1}{\frac{1}{4}}-\frac{q_1+q_2-q_3}{\frac{1}{2}}+\frac{q_3}{\frac{1}{3}}=0$$

or $6q_1+2q_2-9q_3=0$ .. (ii).

Writing the equations together and applying cross-multiplication principle we have,

$$6q_1-3q_2-4q_3=0,$$
$$6q_1+2q_2-9q_3=0.$$

$$\frac{q_1}{27-(-8)}=\frac{q_2}{(-24)-(-54)}$$
$$=\frac{q_3}{12-(-18)}$$

or $\frac{q_1}{35}=\frac{q_2}{30}=\frac{q_3}{30}$, or $\frac{q_1}{7}=\frac{q_2}{6}=\frac{q_3}{6}=k$ (say).

Then, $q_1=7k;\ \ q_2=6k;\ \ q_3=6k.$

Now, equivalent capacitance between any points *a* and *b* is given by,

$$C_{ab}=\frac{Q\ \text{(charge given to } a)}{V_{ab}}.$$

Now, $V_{ab}=V_a-V_b=V_a-V_c+V_c-V_b=\frac{q_1}{\frac{1}{2}}+\frac{q_3}{\frac{1}{3}}=2q_1+3q_3$

$Q=q_1+q_2.$

$\therefore\quad C_{ab}=\frac{q_1+q_2}{2q_1+3q_3}=\frac{7k+6k}{2.7k+3.6k}=\frac{13k}{14k+18k}=\frac{13}{32}.$

$C_{ab}=\cdot 406\ \mu\text{F}.$

Ex. 5. *Three capacitors of capacitance* 2, 4 *and* 6 μF *are connected in series. Can a voltage of* 11,000 V *be applied to this battery of capacitors ? The puncture voltage of each capacitor is* 4000 V.

Sol. *The voltages are divided in the inverse ratio of the capacitances* and hence $V_1 : V_2 : V_3=\frac{1}{2} : \frac{1}{4} : \frac{1}{6}=6 : 3 : 2.$

$\therefore\ V_1=6k,\ V_2=3k$ and $V_3=2k$ where k is the common ratio.

Now, V (total voltage) $=V_1+V_2+V_3$

$\therefore\ 11{,}000=6k+3k+2k$ or $k=1000$

$\therefore\ V_1=6000$ V, $V_2=3000$ V and $V_3=2000$ V.

Since the voltage across the first capacitor is greater than the puncture voltage (4000 V), 11,000 V cannot be applied to the combination.

Ex. 6. *A capacitor of* 3 μF *is charged to a potential difference of* 300 V, *and a capacitor of* 2 μF *to* 200 V. *Both charged capacitors are connected in parallel with* (i) *plates of like charges together and* (ii) *with plates of unlike charges together. What amount of charge will flow through the connecting wires ?*

Sol. (i) $Q_1 = V_1C_1$ and $Q_2 = V_2C_2$.

After connection total charge $= Q_1 + Q_2$, common potential difference $= V$ (say) and total capacitance $= C_1 + C_2$.

$$\therefore \quad V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}$$

$$\therefore \quad \text{Charge on the first} = VC_1 = \frac{V_1C_1 + V_2C_2}{C_1 + C_2} \times C_1.$$

$\therefore$ Charge that flows through the wire

$$= V_1C_1 - \frac{V_1C_1 + V_2C_2}{C_1 + C_2} \times C_1 = \frac{(V_1 - V_2)C_1C_2}{C_1 + C_2}$$

$$= \frac{(300 - 200) \times 6 \times 10^{-12}}{5 \times 10^{-6}}$$

$$= 120\ \mu\text{C}. \qquad \textbf{Ans.}$$

(ii) Here total charge after connection $= Q_1 - Q_2$ and total capacitance $= C_1 + C_2$.

$$\therefore \quad V\ \text{(common potential)} = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{V_1C_1 - V_2C_2}{C_1 + C_2}.$$

$$\therefore \quad \text{Charge on the first} = \frac{V_1C_1 - V_2C_2}{C_1 + C_2} \times C_1.$$

$\therefore$ Charge that flows through the wire

$$= V_1C_1 - \frac{V_1C_1 - V_2C_2}{C_1 + C_2} = \frac{(V_1 + V_2)C_1C_2}{C_1 + C_2}$$

$$= \frac{(300 + 200) \times 6 \times 10^{-12}}{5 \times 10^{-6}} \qquad \textbf{Ans.}$$

$$= 600\ \mu\text{C}.$$

EXERCISES

(A)

1. Three capacitors of capacitance 1, 2 and 3 μF are connected in series. Find the resultant capacitance.

(**Ans.** ·55 μF)

2. Three capacitors of capacitance 4, 3 and 2 μF are connected in such a way that first and second are in series and the third in parallel with them. Find the equivalent capacitance.

(**Ans.** 3·71 μF)

3. Three identical capacitors are first connected in series and then the first and the last conductors of the combination are connected to earth. A charge Q is communicated to the second conductor of the first capacitor. Prove that the potential of this conductor is $\frac{2Q}{3C}$ where C is the capacitance of each capacitor.

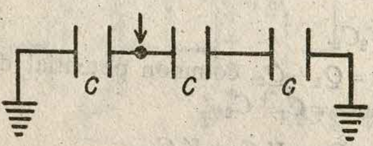


Fig.6-6

4. Four capacitors C_1, C_2, C_3 and C_4 are connected as shown in the figure (Fig. 6-7). Calculate the equivalent capacitance when, (i) switch S is open, (ii) switch S is closed. Take capacitances of the capacitors 1, 2, 3 and 4 μF respectively.

[**Ans.** (i) 2·38 μF; (ii) 2·4 μF]

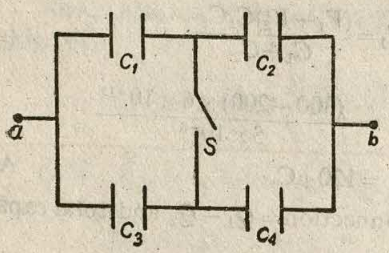


Fig. 6-7

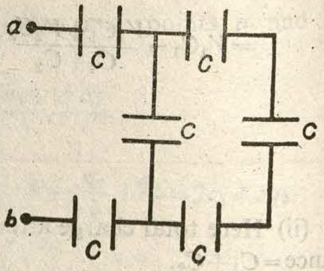


Fig. 6-8

5. Show that in problem 3 the potential of the second conductor of the second capacitor is $\frac{Q}{3C}$.

6. In the network of identical capacitors shown in the fig. 6-8, find the equivalent capacitance between a and b.

$\left(\textbf{Ans. } \frac{4C}{11}\right)$

7. Three capacitors of capacitances 1, 2 and 3 microfarad are connected in such a way that the first and the second are in parallel and the third is in series with the combination of the other two. Calculate the equivalent capacitance. If a potential difference of 200 volt is applied across the combination, what is the charge on each capacitor ?

(**Ans.** 1·5 μF; 100 μC on 1 μF; 200 μC on 2 μF and 300 μC on 3 μF)

8. In the network of capacitors equal capacitance shown in the figure 6.9 find the equivalent capacitance between *a* and *b*. Take $C=2\ \mu F$. (Ans. ·75 μF)

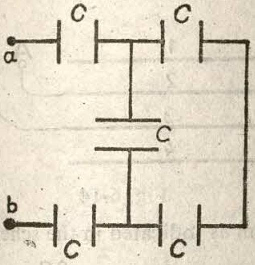


Fig. 6·9

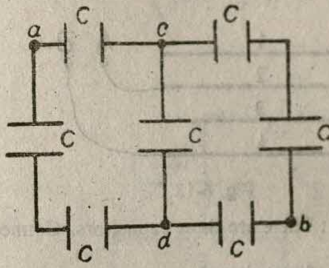


Fig. 6-10

(B)

9. Two batteries of e.m.fs 12 volt and 13 volt are joined in series through two capacitors; the positive of 12 volt battery is joined to the negative of 13 volt battery through a capacitor of 3 μF and the negative of 12 volt battery is joined to the positive of the 13 volt battery through 7 μF capacitor. Calculate the steady p.d.'s across the capacitors.

[*Hint* : Apply loop rule to the circuit and equate the net charge drawn by the capacitors from the batteries to zero.] (Ans. 17·5 volt; 7·5 volt)

10. Calculate the steady p.d. across the capacitors of the above problem when the two batteries are in parallel. (Ans. ·7 volt; ·3 volt)

11. In the network of identical capacitors (Fig. 6-10), find the capacitance between (i) *a* and *b*, (ii) *c* and *d*.

12. Find the capacitance of the infinite circuit formed by repetition of the same link consisting of identical capacitors as shown in Fig. 6-11.

[Ans. $1/2(\sqrt{5}-1)$ C]

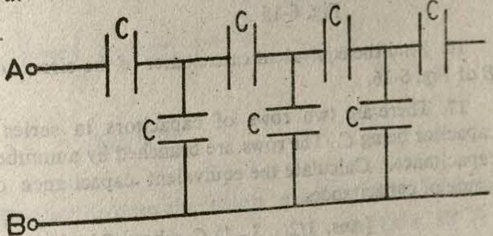


Fig. 6-11

13. In the circuit shown in Fig. 6-12 the emf of each battery is equal to 60 V, and the capacitances of the capacitors are $C_1=2\ \mu F$ and $C_2=3\ \mu F$. Find the charges which will flow after the switch is closed through the wires 1, 2 and 3 in the directions indicated by the arrows.

(Ans. $q_1=-24\ \mu C$, $q_2=-36\ \mu C$, $q_3=+60\ \mu C$)

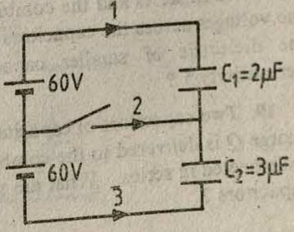


Fig. 6-12

14. Four identical plates are located in air at equal distances apart by d. The area of each plate is equal to S. Find the capacitance of the system between A and B if the plates are connected as shown in Fig. 6-13 and Fig. 6-14.

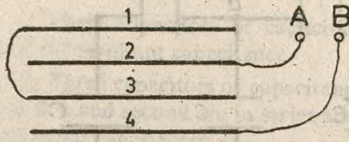


Fig. 6-13

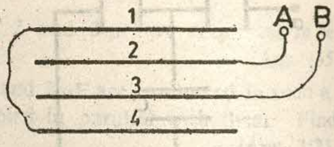


Fig. 6-14

[*Hint* : There are three capacitors. Connect them as indicated in the schematic diagram of capacitors.] $\left[\textbf{Ans. } (a)\ \frac{2C}{3},\ (b)\ \frac{3C}{2}\right]$

15. A circuit has a section AB shown in Fig. 6-15. The emf of the cell is 10 V and the capacitors have capacitances $C_1 = 1\ \mu\text{F}$ and $C_2 = 2\ \mu\text{F}$. The potential difference $V_{AB} = 5$ V. Find the charges on the capacitor. (**Ans.** each 10 μC)

[*Hint* : Imagine a battery of 5 volt between A and B.]

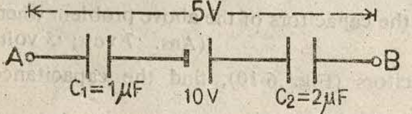


Fig. 6-15

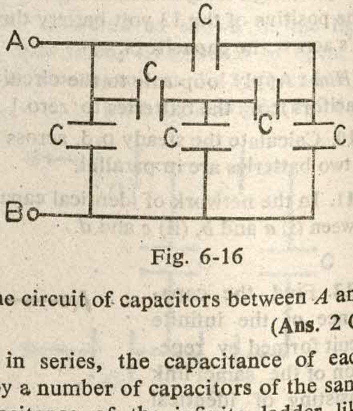


Fig. 6-16

16. Find the equivalent capacitance of the circuit of capacitors between A and B of Fig. 6-16. (**Ans.** 2 C)

17. There are two rows of capacitors in series, the capacitance of each capacitor being C. The rows are branched by a number of capacitors of the same capacitance. Calculate the equivalent capacitance of the infinite ladder like frame of capacitances.

[**Ans.** $1/2(\sqrt{3}-1)\,C$, where C is the capacitance of each capacitor.]

18. Two capacitors having dielectric of common material ($\epsilon_r = 2$) are connected in series and the combination is put across a steady p.d. of 220 V. If the voltages across the capacitors are 120 V and 100 V, what will be their p.d's if the dielectric of smaller capacitor is replaced by a dielectric of relative permittivity 5 ? (**Ans.** 71V; 149V)

19. Two capacitors of capacitances C_1 and C_2 are connected in parallel and a charge Q is delivered to the combination. The two are then disconnected and reconnected in series. What are new potential difference and charges on the capacitors ?

$$\left(\textbf{Ans. } V' = \frac{Q}{\sqrt{C_1 C_2}};\ Q' = Q\,\frac{\sqrt{C_1 C_2}}{C_1 + C_2}\right)$$

20. Calculate the equivalent capacitance between the points indicated in the figures (Fig. 6-17, 6-18 and 6-19). $\left(\text{Ans. } \frac{13C}{9}, \frac{6C}{5}, \frac{29C}{17}\right)$

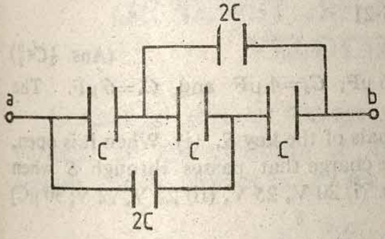


Fig. 6-17

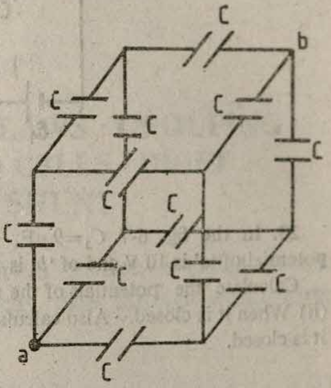


Fig. 6-18

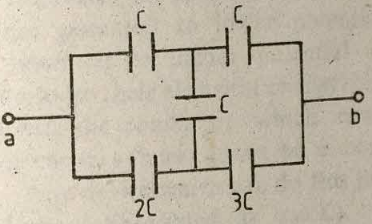


Fig. 6-19

21. Find the potential difference between the points a and b of the circuit (Fig. 6-20) when the battery has 100 V emf. (Ans. 20 V)

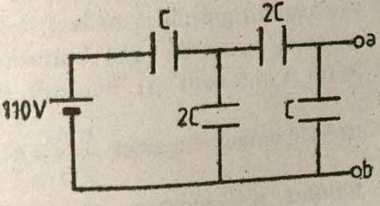


Fig. 6-20

22. What amount of heat will be generated in the circuit shown in the figure (Fig. 6-21) after the switch is shifted from the position 1 to position 2 ?

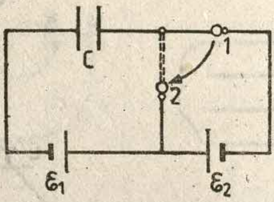


Fig. 6-21

(**Ans.** $\frac{1}{2}C\varepsilon_2^2$)

23. In the fig. 6-7 $C_1=9\ \mu F$, $C_2=6\ \mu F$, $C_3=4\ \mu F$ and $C_4=6\ \mu F$. The potential of '*a*' is 10 V and of '*b*' is 35 V.

Calculate the potentials of the terminals of the key *S*, (i) When it is open, (ii) When it is closed. Also calculate the charge that passes through *S* when it is closed. [**Ans.** (i) 20 V, 25 V, (ii) 22 V, 22 V; 30 μC]

□□□

CURRENT ELECTRICITY

CHAPTER 1

TERMS : KIRCHHOFF'S LAWS : GROUPING OF RESISTORS AND CELLS : DRIFT VELOCITY : SHUNT

1. Electric current : Charge carriers in motion is called electric current. If $\triangle q$ coulomb of electricity pass through a transverse section of a conductor in $\triangle t$ sec then

$$I = \frac{\triangle q}{\triangle t} \text{ ampere (A).} \qquad \ldots (1)$$

2. Electromotive Force and Potential Difference : When an electric field is set up in a conductor the mobile charge carriers (electrons) in it flow from higher potential to lower potential end (electrons flow from lower potential to higher potential end as they carry negative charge) and loose their electrical energy. So unless we have a device connected with the conductor which can supply the lost energy to the charge carriers there cannot be a continuous flow of charge through it. Any device which can do this job is called a seat of electromotive force (abbreviated as e.m.f.). A voltaic cell, a thermocouple and a coil rotating in a magnetic field are some common examples of seat of e.m.f. Thus seat of e.m.f. is that device in which energy in non-electrical form is converted into electrical form and in which charge carriers are passed on from lower potential to higher potential. The amount of e.m.f. of a seat is measured by rate at which energy in non-electrical form is converted into electrical form during the passage of unit charge. If $\triangle W$ = *energy converted into electrical form* when $\triangle q$ amount of electricity passes through it, then e.m.f. (*e*) of the seat is given by

$$e = \frac{\triangle W}{\triangle q} \text{ joule per coulomb or volt.} \qquad \ldots (2)$$

Thus charge carriers carrying $\triangle q$ amount of charge receive $\triangle W = e\triangle q$ joule of energy while passing through seat of e.m.f.

from lower (negative) potential plate to higher (positive) potential plate.

The potential difference between any two points is defined as *the energy given up by charge positive carriers* carrying unit charge and is, therefore, equal to the work done by the charge carriers carrying unit charge from one point to the other.

If $\triangle W$ is the *energy given up* by charge carriers carrying $\triangle q$ charge, then the potential difference between the ends of a current-carrying conductor is

$$V=\frac{\triangle W}{\triangle q} \text{ joule per coulomb or volt.} \qquad .. \quad (3)$$

3. Resistance : *An atomic view* : When an electric field is set up in a conductor the charge carriers (electrons) are set in motion. But they do not have a smooth sailing through the conductor as there are fixed 'ionic cores' in the conductor. The electrons constantly collide with these ionic cores often suffering changes in speed and direction. The constant on slaught of electrons makes ionic cores vibrate more vigorously and so the conductor appear hot. When the whole of electrical energy received by the charge carriers from the seat of e.m.f. is converted into heat, the conductor is said to be purely resistive. If R is the resistance of the conductor, the rate at which heat is produced is i^2R. This is known as Joule's law of generation of heat. If H is the rate of generation of heat in a conductor by a current i, then the resistance R of the conductor is given by

$$H=i^2R. \qquad .. \quad (4)$$

Unit, *watt/ampere*^{2} or *ohm* (Ω).

4. Circuit equation : Let us consider a seat of e. m. f. (say a cell) connected to a resistor R. It is a fact that every seat of e.m.f. also has some resistance. This is called internal resistance (r) of the seat. Let us consider flow of current through the circuit for $\triangle t$ second. Then the amount of charge ($\triangle q$) that will flow in $\triangle t$ is given by,

$$i=\frac{\triangle q}{\triangle t}, \text{ or } \triangle q=i\triangle t.$$

During the time $\triangle t$ the energy received by charge carriers which is also the work done by the seat on the carriers in forcing them from lower to higher potential $=e\triangle q=ei\triangle t$.

Heat produced in the resistance $= i^2(R+r)\triangle t$.

From the principle of conservation of energy $i^2(R+r)\triangle t = e.i\triangle t$

or

$$i = \frac{e}{R+r}. \qquad \text{.. (5)}$$

5. Potential Difference between any two points : Suppose a and b are any two points in a circuit. There are resistances as well as cells (seats of e. m. f.'s) between a and b. Let us suppose that current is flowing from a to b and the strength of the current is i. Consider flow of current for $\triangle t$ sec. Then $\triangle q$ (charge that flows down in $\triangle t$ sec) $= i\triangle t$.

$\therefore$ Energy given up by charge carriers in passing from a to b $= V_{ab}\, i\triangle t =$ energy received by charge carriers from the remainder of the circuit (not shown in the figure).

As there are seats of e. m. f.'s also in between a and b, charge carriers also receive or give up energy as they pass through them. They receive energy (or we may say work is done on them) when they pass from negative to positive and give up energy (or we may say work is done by them) when they pass through seat from positive to negative.

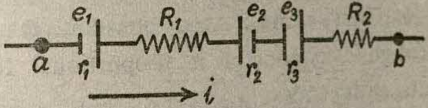


Fig. 1-1

Net energy received $= V_{ab}\, i\triangle t + e_1\, i\triangle t - e_2\, i\triangle t + e_3\, i\triangle t$

$= V_{ab}\, i\triangle t + (\Sigma e)\, i\triangle t$ where Σe stands for algebraic sum of e. m. f.'s in the direction of current.

$$\text{Heat produced} = i^2(r_1 + R_1 + r_2 + r_3 + R_2)\triangle t$$
$$= \Sigma i^2 R\triangle t.$$

Principle of conservation of energy requires

$$V_{ab}\, i\triangle t + (\Sigma e) i\triangle t = (\Sigma i^2 R)\triangle t$$

or

$$V_{ab} = \Sigma iR - \Sigma e. \qquad \text{.. (6)}$$

This is the most general expression for potential difference between two points in a circuit. Special cases of this general expression are Ohm's law, Kirchhoff's loop rule and the above circuit equation.

If we have only one resistance and no cell in between then

$$\Sigma e=0,\ \Sigma iR=iR.$$

$$\therefore\quad V_{ab}=iR \quad \text{or} \quad i=\frac{V_{ab}}{R}\text{—well known Ohm's Law.}$$

If a and b are two coincident points we get a loop. Then $V_{ab}=0$. Hence for a loop,

$$\Sigma iR-\Sigma e=0 \text{ or } \Sigma iR=\Sigma e \text{ —This is Kirchhoff's loop rule.}$$

In a simple circuit consisting of a cell and resistance,

$$V_{ab}=0;\ \Sigma e=e \text{ and } \Sigma ir=ir+iR.$$

$$\therefore\qquad e=i(r+R)$$

or

$$i=\frac{e}{r+R}\quad\text{—circuit equation.}$$

6. Ohm's Law : At a constant temperature current flowing through a conductor is proportional to the potenital difference between its ends.

$V \propto i$, or $V=iR$ where R is a constant of the conductor called resistance of conductor.

Its unit is ohm symbolised as Ω (capital omega).

7. Kirchhoff's law : (i) The algebraic sum of currents meeting at a point is zero or $\Sigma i=0$. This is called point rule.

(ii) The algebraic sum of product of current and resistance in a 'loop' is equal to algebraic sum of e.m.f's in that 'loop', the two sums being considered in the same sense. That is, $\Sigma ir=\Sigma e$. This is called 'loop rule'.

8. Terminal voltage (or P. D.) of a cell : Suppose a and b are two points at the terminals of a cell (a seat of e.m.f.). Then V_{ab} is called terminal voltage or P. D. of the seat. Taking the passage from a to b via external resistance, we have $\Sigma e=0$, $\Sigma iR=iR$.

$$\therefore\quad V_{ab}=iR, \text{ or } i=\frac{V_{ab}}{R}.$$

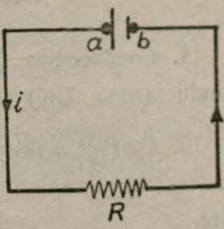


Fig. 1-2

Taking the passage from a to b via seat of e.m.f. we have $\Sigma e=-e$ (minus because we are taking passage from +ve to −ve inside the cell); $\Sigma iR=-ir$.

$\therefore \quad V_{ab}=-ir-(-e)=e-ir$

or $\quad e=V_{ab}+ir$.

The difference $(e-V_{ab})$ which is not indicated by a voltmeter is called '*Lost volt*'.

9. Law of resistance of a conductor : The resistance of a conductor is proportional to its length and inversely proportional to its area of cross-section.

i.e.
$$R=\rho\frac{l}{S} \quad .. \quad (7)$$

where ρ is a constant called resistivity of the material of wire (previously it was called specific resistance). Unit of resistivity is ohm metre (Ωm).

10. Grouping of resistances : (i) When conductors are joined end to end and the *same current passes through all of them*, they are said to be in series. The equivalent resistance is given by,

$$R=r_1+r_2+r_3+\cdots \quad .. \quad (8)$$

The total voltage across the combination is divided among the resistors in the ratio of their resistances.

(ii) When conductors are joined in such a way that one end of all is joined to one common point and the remaining end of all to another common point and there is division of current among the conductors, they are said to be in parallel. The equivalent resistance is given by

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\ldots\text{and } i_1r_1=i_2r_2=i_3r_3=V. \quad .. \quad (9)$$

That is, here *voltage across each resistor is the same and is equal to the voltage across the combination but the total current is divided among the resistors in the inverse ratio of their resistances.*

When there are two conductors in parallel, the effective resistance is $\frac{r_1r_2}{r_1+r_2}$ $\left(\text{i.e., }\frac{\text{product of resistances}}{\text{sum of resistances}}\right)$.

11. **Conductance (G)** : The reciprocal of resistance is called conductance. Unit, Ω^{-1} or siemens (S).

12. **Current density (J)** : The value of current through unit area of transverse section of a conductor is called current density of wire.

$$J=\frac{i}{A}\,\text{Am}^{-2}. \qquad \text{.. (10)}$$

It is to be noted here that current density is a vector but not the current.

13. **Electrical conductivity (σ)** : The reciprocal of resistivity is called electrical conductivity.

$$\sigma=\frac{1}{\rho}\,\Omega^{-1}\,\text{m}^{-1}, \text{ or } \sigma=\frac{1}{\rho} \text{ siemens per metre (Sm}^{-1}). \qquad \text{.. (11)}$$

14. The resistance of metals generally changes with temperature according to the formula

$$R_t=R_0(1+\alpha t),$$

where R_t = Resistance of wire at t°C

R_0 = Resistance of same wire at 0°C

and α = a constant of the material of wire and is called temperature coefficient of resistance.

More accurately, $$R_t=R_0(1+\alpha t+\beta t^2) \qquad \text{.. (12)}$$

where β is another constant called the second temperature coefficient. This is generally very small.

15. **Drift velocity** : When an electric field is set up in a conductor the charge carriers in it are set in motion. But they do not have very smooth sailing through the conductor as there are fixed 'ionic cores' in the conductor which put obstruction on their passage. The electrons (charge carriers) collide constantly with these ionic cores often suffering changes in magnitude and direction. Thus the motion of

charge carriers through a conductor is not a smooth one rather it is alternately accelerated and stopped. Such motion is called drifting motion. Thus electrons drift through a conductor when a p.d. is set up between the ends of the conductor. The velocity with which electrons drift through a conductor is called its drift velocity (u).

Drift velocity $(u)=\frac{JM}{nN\rho q}$ where J = current density, M = atomic weight, N = *Avogadro's number* and ρ = *density of conductor*, q = charge carried by each carrier and n = number of charge carriers available from each atom.

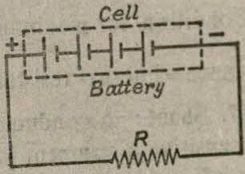


Fig. 1-3

16. Grouping of cells : When n cells are joined in the sequence of positive terminal of one with negative of the next, they are said to be in series. The e.m.f. of the battery formed is ne where e = e. m. f. of each cell and its internal resistance nr, where r = resistance of each cell.

The current delivered by the battery to a resistor R is given by circuit equation,

$$i=\frac{ne}{nr+R}.$$

When m cells are joined with the positive poles of all together at a point and negative poles together at some other point, they are in parallel. The e.m.f. of the battery is same as that of each cell but its internal resistance $1/m$ th of each cell. The current delivered by the battery to a resistor R is

$$i=\frac{e}{R+\frac{r}{m}}=\frac{me}{mR+r}.$$

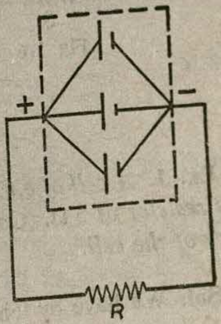


Fig. 1-4

When cells are arranged in rows and in each row several cells are in series, they are said to be in mixed grouping. Suppose there are m rows and in each row there are n cells in series.

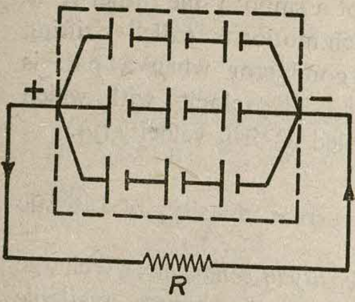


Fig. 1-5

E. m. f. of battery $=ne$, internal resistance of battery $=nr/m$. Current delivered by the battery to a resistor R is

$$i=\frac{ne}{R+\dfrac{nr}{m}}=\frac{mne}{mR+nr};$$

$mn=N$, total number of cells,

$$\therefore \quad i=\frac{Ne}{mR+nr}.$$

Condition for maximum current is $mR=nr$, or $R=\frac{nr}{m}$ i.e. external resistance = internal resistance of battery.

17. Shunt : A conductor of low resistance connected in parallel to a sensitive instrument is called shunt. Shunt works on the principle of 'current division' in parallel connection of conductors. When conductors are joined in parallel, current is divided among them in inverse ratio of their resistances i.e.

$$i_1r_1=i_2r_2=i_3r_3\ldots\ldots$$

Suppose a galvanometer of resistance G is shunted by S. C is the main current. C will be divided among G and S such that

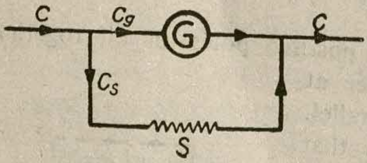


Fig. 1-6

$$C=C_g+C_s \text{ and } C_gG=C_sS.$$

Hence, $$C_g=\frac{C}{G+S}\times S$$

and $$C_s=\frac{C}{G+S}\times G.$$

EXAMPLES

Ex. 1. *A cell of e.m.f. 2 V produces ·3 A current when connected to a resistor of 5 Ω. Calculate the terminal voltage and internal resistance of the cell.*

Sol. We have by 'circuit equation', $i=\frac{e}{R+r}$.

$\therefore \quad {\cdot}3=\dfrac{2}{5+r}$ or $1{\cdot}5+{\cdot}3r=2$, or $r=1{\cdot}67\ \Omega$. **Ans.**

We have applying Ohm's law to 5 Ω,

$$V=iR={\cdot}3\times5=1{\cdot}5\text{ V}. \qquad \textbf{Ans.}$$

Ex. 2. *Two batteries of e.m.f.'s* 12 *V and* 6 *V and internal resistance* ·2 Ω *and* ·1 Ω *respectively are connected in opposition through a resistor* 3·7 Ω. *Calculate the terminal voltage of the first battery following either path.*

Sol. We have from 'circuit equation'

$$i=\frac{\Sigma e}{\Sigma R}=\frac{12+(-6)}{3{\cdot}7+{\cdot}2+{\cdot}1}=\frac{6}{4}=1{\cdot}5\text{ A.}$$

The p.d. between any two points a and b is given by

$$V_{ab}=\Sigma ir-\Sigma e.$$

In applying this formula *consider p.d. between points in the direction of the current.* Here current is flowing from b to a through the cell. Hence we consider V_{ba}.

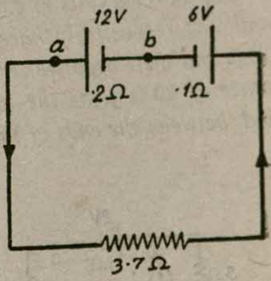


Fig. 1-7

$\Sigma ir=1{\cdot}5\times{\cdot}2={\cdot}3$ and $\Sigma e=12$

$\therefore \quad V_{ba}={\cdot}3-12=-11{\cdot}7.$

$\because \quad V_{ba}=-V_{ab},$

$\therefore \quad V_{ab}=-(-11{\cdot}7)=11{\cdot}7$ volts.

We may also consider V_{ba} directly following the path of current from a to b through the external resistance.

Now $\Sigma ir=3{\cdot}7\times1{\cdot}5+{\cdot}1\times1{\cdot}5=5{\cdot}55+{\cdot}15=5{\cdot}70$ volt

$\Sigma e=-6$ (minus because from +ve to negative inside the cell, emf is −ve)

$\therefore \quad V_{ab}=5{\cdot}7-(-6)=11{\cdot}7$ volts. **Ans.**

Illustration of Kirchhoff's rules :

General note : Whenever there is 'cross-connection', the problem can be solved by Kirchhoff's rules.

The first step in applying Kirchhoff's rules is to assign values of currents in compliance with 'point rule' in all the parts without applying intuition about the direction of current and remaining completely unmindful about the presence of cells. Then apply 'loop rule' to any number of loops and set up as many equations as required. Solve them and obtain unknown values of currents.

Calculation of 'equivalent resistance' of a net work of conductors : First carefully see whether there is any 'cross-connection'. If there is none, equivalent resistance can be calculated by applying 'law of series and parallel' connection of resistors. If there is cross-connection, one has to proceed with Kirchhoff's rules. Assign currents in the different parts in compliance with 'point rule'. Along identical paths from 'entrance' to 'exit' assume same current. Then use the formula $V=RI$ where $V=$ p. d. between 'entrance' and 'exit' points; $I=$current entering and leaving the network and $R=$required equivalent resistance.

Ex. 3. *Two cells of e.m.f.* 2 *V and* 1 *V and internal resistances* 1 Ω *and* 2 Ω *respectively have their positive terminals joined by a wire of* 10 Ω *and their negative terminals by a wire of* 4 Ω *resistance. A resistance of* 10 Ω *joins the mid point of these two wires. Calculate the p.d. between the ends of this wire.*

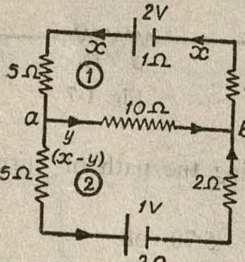


Fig. 1-8

Sol. Distribution of current in accordance with point rule is shown in the figure 1-8.

Applying loop rule ($\Sigma e=\Sigma ir$) to loop (1) and (2) we have,

$$5x+10y+2x+1x=2$$

or $$4x+5y=1 \quad \text{.. (i)}$$

and $$5(x-y)+2(x-y)+2(x-y)-10y=-1$$

or $$9x-19y=-1 \quad \text{.. (ii).}$$

Solving (i) and (ii) for x and y we have $x=14/121$ and $y=13/121$.

Now, P.D. across 10 Ω$=10y=10\times13/121=130/121=1{\cdot}08$ V. **Ans.**

Ex. 4. *In a Wheatstone bridge the four resistances in the arms of the bridge are* $AB=2$ Ω, $BC=4$ Ω, $AD=1$ Ω *and* $DC=3$ Ω. *The terminals of a cell of e.m.f.* 2 *V and* 1 Ω *internal resistance are connected to A and C. If a galvanometer of resistance* 10 Ω *is connected between B and D, find the current through the galvanometer.*

Sol. The distribution of currents is shown in the figure (Fig. 1-9) in compliance with 'point rule'.

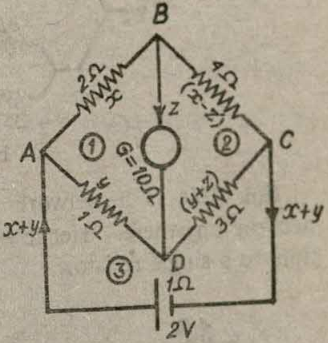


Fig. 1-9

Applying loop rule ($\Sigma ir=\Sigma e$) to loop (1), (2) and (3) we have, from (1),

$$2x+10z-y=0$$

(sum is zero because there is no seat of e.m.f. in this loop)

or $\quad 2x-y+10z=0 \quad .. \quad$ (i).

From (2),

$$10z+3(y+z)-4(x-z)=0$$

or $\quad 4x-3y-17z=0 \quad .. \quad$ (ii).

From (3),

$$1y+3(y+z)+(x+y)=2$$

or $\quad x+5y+3z=2 \quad .. \quad$ (iii).

Writing (i) and (ii) together and applying 'cross-multiplication' principle,

$$2x-y+10z=0$$
$$4x-3y-17z=0$$

$$\frac{x}{17-(-30)}=\frac{y}{40-(-34)}=\frac{z}{(-6)-(-4)}$$

or $$\frac{x}{47}=\frac{y}{74}=\frac{z}{-2}=k \text{ (say).}$$

$\therefore$ $$x=47k;\ y=74k;\ z=-2k.$$

Putting x, y and z in (iii),

$$47k+5\times 74k+3(-2k)=2, \text{ or } k=\frac{2}{411}.$$

$\therefore$ $$z=-\frac{4}{411}=-\cdot 00973 \text{ A.}$$

The minus sign shows that current is opposite to the assumed direction .

$\therefore$ Current through galvanometer is ·00973 A from D to B. **Ans.**

Ex. 5. *Calculate the equivalent resistance between the points a and b of the following network of conductors.*

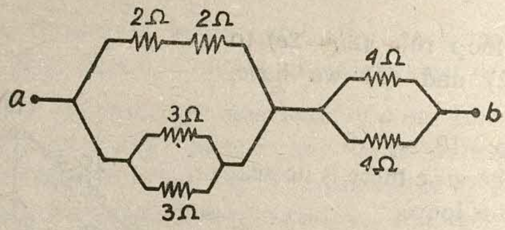


Fig. 1-10 (a)

Sol. This is a network of six conductors having no 'cross-connection' anywhere. Hence we can solve them by 'successive reduction' to a single resistor.

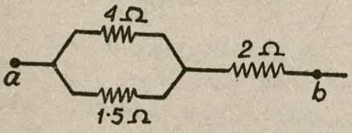


Fig. 1-10 (b)

First reduction : Resistors 2 Ω and 2 Ω being in series, their equivalent resistance is given by,

$$r=2\,\Omega+2\,\Omega=4\,\Omega.$$

Resistors 3 Ω and 3 Ω being in parallel, their equivalent resistance is given by,

$$\frac{1}{r}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

or

$$r=1{\cdot}5\,\Omega.$$

Resistors 4 Ω and 4 Ω being in parallel their equivalent resistance is given by,

$$\frac{1}{r}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4},\text{ or } r=2\,\Omega.$$

Second reduction : 4 Ω and 1·5 Ω are in parallel, their equivalent resistance is given by,

$$\frac{1}{r}=\frac{1}{4}+\frac{1}{1{\cdot}5}=\frac{1}{4}+\frac{2}{3}=\frac{3+8}{12}=\frac{11}{12}.$$

$$r=\frac{12}{11}=1{\cdot}09\,\Omega.$$

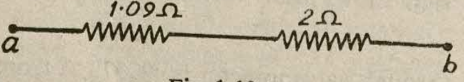


Fig. 1-10 (c)

Third and final : 1·09 Ω and 2 Ω being in series, their equivalent resistance is given by, $r = 1{\cdot}09 + 2 = 3{\cdot}09\ \Omega$.

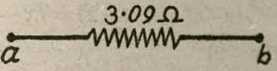


Fig. 1-10 (d)

3·09 Ω **Ans.**

Ex. 6. *Calculate the equivalent resistance between a and b of the following network of conductors.*

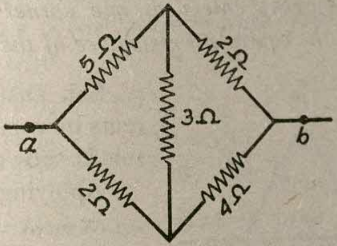


Fig. 1-11

Sol. This is a network of five conductors having a 'cross-connection' (cross-connection 3 Ω resistor).

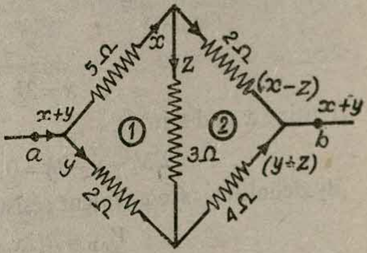


Fig. 1-12

Distribution of currents in compliance with point rule is shown in the figure.

Applying loop rule to (1) and (2),

$$5x + 3z - 2y = 0$$

or $$5x - 2y + 3z = 0 \quad \text{.. (i)}$$

and $$3z + 4(y+z) - 2(x-z) = 0$$

or $$9z + 4y - 2x = 0$$

or $$2x - 4y - 9z = 0 \quad \text{.. (ii).}$$

Writing (i) and (ii) together and cross-multiplying,

$$\begin{array}{c} 5x - 2y + 3z = 0 \\ 2x - 4y - 9z = 0 \\ \hline \end{array}$$

$$\frac{x}{18-(-12)} = \frac{y}{6-(-45)} = \frac{z}{(-20)-(-4)}$$

or $$\frac{x}{30} = \frac{y}{51} = \frac{z}{-16} = k \text{ (say)}$$

or $$x = 30k;\ y = 51k \text{ and } z = -16k.$$

Now equivalent resistance between a and b is given by

$$V_{ab}=R(x+y).$$

Now $\quad V_{ab}=V_{ac}+V_{cb}=5x+2(x-z)=7x-2z.$

$\therefore \quad 7x-2z=R(x+y)$

or $\quad R=\dfrac{7x-2z}{x+y}=\dfrac{7\times 30k-2(-16k)}{30k+51k}=\dfrac{242}{81}=3\ \Omega.$ **Ans.**

Ex. 7. *A regular hexagon is formed by six wires each of resistance 'r' ohm and the corners are joined to the centre by wires of the same resistance. If the current enters at one corner and leaves at the opposite corner, find the equivalent resistance of the conductor.*

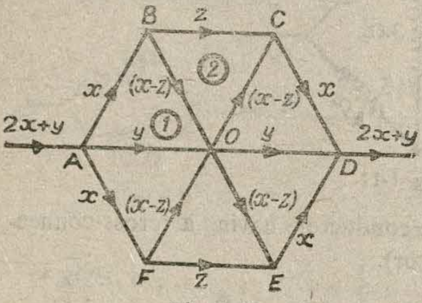


Fig. 1-13

Sol. Distribution of currents in compliance with point rule is shown in the figure.

Applying loop rule to (1),

$rx+r(x-z)-ry=0$

or $\quad 2x-y-z=0 \quad \ldots$ (i).

Applying loop rule to (2),

$r(x-z)+r(x-z)-rz=0$

or $\quad 2x-3z=0$

or $\quad \dfrac{x}{3}=\dfrac{z}{2}=k$ (say);

$\therefore \quad x=3k$ and $z=2k.$

Putting x and z in (i),

$$2.3k-y-2k=0, \quad \text{or} \quad y=4k.$$

By definition of equivalent resistance between A and D,

$$V_{AD}=R(2x+y).$$

But $\quad V_{AD}=V_{AO}+V_{OD}=ry+ry=2ry$

$\therefore \quad 2ry=R(2x+y);$

$\therefore \quad R=\dfrac{2ry}{2x+y}=\dfrac{2.4k}{2.3k+4k}\cdot r=\dfrac{4}{5}r.$ **Ans.**

Ex. 8. *A wire of nicrome of length* 100 cm *and radius* ·36 mm *has* 1·5 Ω *resistance. Calculate resistivity of nicrome and also its conductivity.*

Sol. We have for a conductor, $R=\rho\dfrac{l}{S}.$

$$\therefore \qquad \rho=\frac{RS}{l}=\frac{R\pi r^2}{l} \qquad (\because\ S=\pi r^2 \text{ for a wire})$$

$$=\frac{1{\cdot}5\times\pi\times({\cdot}36\times10^{-3})^2}{100\times10^{-2}}\ \Omega\text{m}$$

$$=6{\cdot}12\times10^{-7}=61{\cdot}2\times10^{-8}\ \Omega\text{m}.$$

$$\theta \text{ (electrical conductivity)}=\frac{1}{\rho}=\frac{1}{61{\cdot}2\times10^{-8}}=1{\cdot}64\times10^{6}\ \text{Sm}^{-1}. \quad \textbf{Ans.}$$

Ex. 9. *The resistance of a wire at* 0°C *and* 100°C *is* 5·2Ω *and* 7·85Ω *respectively. Calculate the mean temperature coefficient of resistance of the material of the wire.*

Sol. We have, $R_t=R_0\ (1+\alpha t)$

$$\text{or} \qquad \alpha=\frac{R_t-R_0}{R_0 t}=\frac{7{\cdot}85-5{\cdot}20}{5{\cdot}20\times100}=5{\cdot}1\times10^{-3}/\text{K}. \qquad \textbf{Ans.}$$

Ex. 10. *Calculate the drift velocity of electrons in an aluminium wire of radius* 2 *mm when a current of* 5 *A passes through it.* (*Atomic weight*=27; *Avogadro's number*=6×10^{26} *kg mol*$^{-1}$ *and density of aluminium*=2710 *kg m*$^{-3}$.)

Sol. We have, u, drift velocity$=\dfrac{JM}{nN\rho q}$ where J=current density, M=molecular wt. or atomic weight, N=Avogadro's number and ρ=density.

$$\text{Here,} \qquad J=\frac{5}{\pi(2\times10^{-3})^2}=\frac{5\times10^6}{4\pi}\ \text{Am}^{-2}.$$

$n=3$, because aluminium is trivalent,

q=charge of electron$=1{\cdot}6\times10^{-19}$ C.

$$\therefore\ u=\frac{\dfrac{5\times10^6}{4\pi}\times27}{3\times6\times10^{26}\times2710\times1{\cdot}6\times10^{-19}}=\frac{5\times27\times10^{-2}}{4\pi\times18\times271\times1{\cdot}6}$$

$$=1{\cdot}38\times10^{-5}\ \text{ms}^{-1}.$$

EXERCISES

(A)

1. A wire carries a constant current of 10 A. How many coulomb pass a cross-section of the wire in 20 seconds ? How many charge carriers ? (charge of electron=$1{\cdot}6\times10^{-19}$ C)

$\left[\textit{Hint} : I=\frac{\Delta q}{\Delta t} \right]$ (Ans. 200 C; $1{\cdot}25\times10^{21}$)

2. Two batteries of e. m. f. e_1 and e_2 produce current of 1·5 A when they are in opposition to each other and 2·5 A when they are in series. Find the ratio of the two e. m. f's.

(Ans. $e_1 : e_2=4:1$)

3. In a battery of 10 cells supposed to be in series some are wrongly connected. The cells are sealed in a box When a similar cell is connected in opposition to the battery, the current produced by the combination through an enternal resistor of 14·5 Ω is ·075 A. How many cells are wrongly connected ? What is the e. m. f. of the battery and what is its internal resistance if e. m. f. of each cell is 1·5 V and internal resistance ·5 Ω ?

(Ans. 4 cells are wrongly connected; 3 V; 5 Ω)

4. How you would arrange 24 cells each of e. m. f. 1·5 V and internal resistance ·5 Ω to deliver maximum current to a resistor of 4 Ω ? What is the maximum current ? (Ans. In two rows each containing 12 cells; 2·6 A)

5. An English letter *A* is constructed of a uniform wire of resistance 1 ohm per metre. The sides of the latter are each 2 m long and the cross-piece in the middle is 1 m while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs.

(Ans. $2\frac{2}{3}$ ohm)

6. Calculate resistivity of the material of a wire whose resistance is 5 Ω, length 1 m and radius ·6 mm.

(Ans. $5{\cdot}656\times10^{-3}$ Ωm)

7. The resistance of 100 m of copper wire of diameter $\cdot056\times10^{-2}$ m is 6·62 Ω. Calculate (i) resistivity, (ii) electrical conductivity of copper.

[Ans. (i) $1{\cdot}63\times10^{-8}$ Ωm, (ii) $6{\cdot}13\times10^{7}$ Sm^{-1}]

8. Calculate the resistance of a steel wire of length 10 m and diameter ·6 mm at 0°C and 100°C. (Electrical resistivity of steel at 0°C is 15×10^{-8} Ωm and mean temperature coefficient of resistance of steel is 50×10^{-4} K^{-1})

(Ans. 5·3 Ω and 7·96 Ω)

9. Three wires whose lengths are in the ratio 3 : 8 : 18 and radii in the ratio 1 : 2 : 3 are connected in parallel. Find the ratio of the currents through them.

(Ans. 2 : 3 : 3)

10. A moving coil galvanometer of resistance 50 Ω is connected in series with a fixed resistance 50 Ω and the combination is connected to a cell of steady e. m. f. and negligible internal resistance. What is the shunt required to produce current through galvanometer half of its previous value ? (Ans. 25 Ω)

11. A resistance box, a battery and a galvanometer of resistance *g* ohm are connected in series. If the galvanometer is shunted by resistance of *S* ohm, find the change in resistance in the box required to maintain the current from the battery unchanged.

$\left(\text{Ans. } \frac{g^2}{g+S}\right)$

12. A shunt is to be applied to a galvanometer of 99 Ω so that only 10% of the total current passes through galvanometer. Find the shunt required. What additional shunt will be required to send 1% of the total current through

the galvanometer ? (Ans. 11 Ω; 1·1 Ω)

13. Two identical cells of e. m. f. 1·5 volt and internal resistence 1 Ω each are joined in series and then the combination is connected in parallel with a third cell of same e. m. f. 1·5 V and internal resistance 1 Ω. Calculate the terminal voltage of the cells. (Ans. 1 V, 1 V and 2 V)

14. A battery of e. m. f. 6 V and internal resistance 5 Ω is joined in parallel with another cell of e. m. f. 10 V and internal resistence 1 Ω and the combination is used to send current through a resistor of 12 Ω. Calculate the current through each battery.

(Ans. $\frac{6}{11}$ A through 6 V battery in opposition to its e.m.f. and $1\frac{8}{11}$ A through 10 V battery in the direction of its e. m. f.)

15. A battery of e. m. f. 6 volt and internal resistence 1 ohm is connected in parallel to another battery of e.m.f. 8 volt and internal resistance 2 ohm. The combination is then used to send current through an external resistance of 10 ohm. Find the current through the external resistance. (Ans. $\frac{5}{8}$ A)

16. Two batteries of e.m.fs 24 volt and 6 volt and internal resistances 2 ohm and 1 ohm respectively are joined in parallel; negative to negative by a resistor of 2 ohm and positive to positive by another resistor of 4 ohm. Calculate the terminal voltages of the two batteries. (Ans. 20 V; 8 V)

17. The sides of a square *ABCD* are formed of wires each of resistance *r* and *B* is joined to *D* by a wire of resistance 2*r*. Find the equivalent resistance of the skeleton (i) between *B* and *D*, (ii) between *A* and *C* and, (iii) between *A* and *B*. [Ans. (i) $\frac{2}{2}r$, (ii) r, (iii) $\frac{2}{8}r$]

18. In a Wheatstone bridge the four resistance of the arms of the bridge are $AB=2\ \Omega$, $BC=4\ \Omega$, $AD=1\ \Omega$ and $DC=3\ \Omega$. The terminals of a cell of e.m.f. 2 V and negligible internal resistance are connected by wires of negligible resistance to *A* and *C*. If a galvanometer of resistance 100 Ω is connected between *B* and *D*, find the current in the galvanometer. (Ans. ·0016 A)

19. *ABCD* is a square of side '*a*' metre and is made up of wires of resistance *x* ohm per metre. Similar wire is connected across the diagonals *AC* and *BD*. Show that the effective resistance of the frame work between the corners *A* and *C* is $(2-\sqrt{2})\,ax$ ohm.

20. In a Wheatstone bridge a battery of 2 volt and interal resistance 2 Ω is used. Find the current through the galvanometer in the unbalanced position of the bridge when $P=1\ \Omega$, $Q=2\ \Omega$, $R=2\ \Omega$, $S=3\ \Omega$ and $G=4\ \Omega$. Ans. ·011 A)

(B)

21. When one mole of zinc combines with sulphuric acid 445 kJ of heat is liberated and 235 kJ of heat is absorbed when a mole of copper is liberated from blue vitriol. Avogadro's number$=6{\cdot}02\times10^{23}$: electronic charge $=1{\cdot}6\times10^{-19}$ C and zinc or copper is divalent. Use these data to find the emf of a Daniell cell.

[*Hint* : Energy liberated in the form of heat is converted into electrical energy in a Daniell cell.]

(**Ans.** 1·09 V)

22. Two resistors with temperature coefficients α_1 and α_2 have resistances R_{01} and R_{02} at 0°C. Find the temperature coefficient of the compound resistor consisting of these two resistors in series. $\left(\textbf{Ans. } \frac{\alpha_1 R_{01}+\alpha_0 R_{02}}{R_{01}+R_{02}}\right)$

23. Show that if E is the electric intensity inside a conductor of electrical conductivity (σ), Ohm's law may be conveniently put in the form, $\vec{J}=\sigma\,\vec{E}$ where J=current density in the conductor.

24. Show that 'drift velocity' of electrons in a conductor is given by $u=\frac{J}{ne}$ where J=current density, n-number of mobile electrons per unit volume and e=charge carried by each electron. Calculate the same in a copper wire of radius ·6 mm when a current of 1 A flows through it assumin , that one mobile electron is available from one atom of copper. Atomic weight of copper=63·55; Density of copper=8930 kg m^{-3}, Avogadro's number=$6{\cdot}02\times10^{26}$ kg mol^{-1} and $e=1{\cdot}6\times10^{-19}$ C.

(**An.** $6{\cdot}5\times10^{-5}$ ms^{-1})

25. A spherical silver electrode 8 cm in radius is concentric with a spherical metal shell of radius 10 cm. The inter space contains an insulating material of resistivity 10^6 Ωm. Compute the resistance of the insulator.

[*Hint* : resistance of a shell of radius $z=\rho\frac{dz}{4\pi z^2}$. Integrate from $z=8$ to $e=10$.]

(**Ans.** $1{\cdot}99\times10^5$ Ω)

26. Two batteries of e.m.f. 12 V and 6 V and internal resistance 2 Ω and 1 Ω are connected in parallel and the combination is connected to resistor 2 Ω through a key. Find the potential difference between positive and negative terminals of batteries (a) when key is open, (b) when key is closed. Find the e. m. f. and internal resistance of a single cell which would be equivalent to the combination.

(**Ans.** (a) 8 V, (b) 6 V, (c) 8 V; ·67 Ω)

27. Find the potential difference between the points a and b of the circuit (Fig. 1-15). If a and b are joined by a wire what is the current through 12-volt cell ?

(**Ans.** ·22 volt, ·464 ampere)

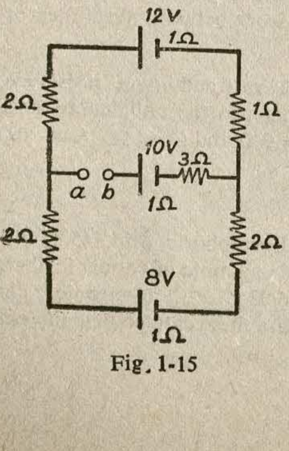
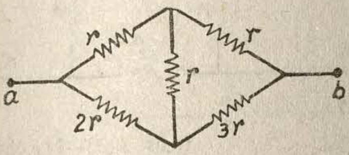


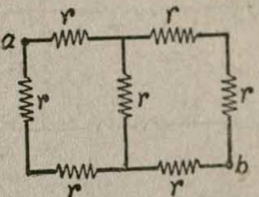
Fig. 1-15

28. Twelve equal wires each of resistance r ohm are connected in the form of a cube. Find the equivalent resistance of the cube (i) between diagonally opposite corners, (ii) between corners of the same edge, (iii) between opposite corners of a face. (**Ans.** (i) $\frac{5}{6}r$, (ii) $\frac{7}{12}r$, (iii) $\frac{3}{4}r$)

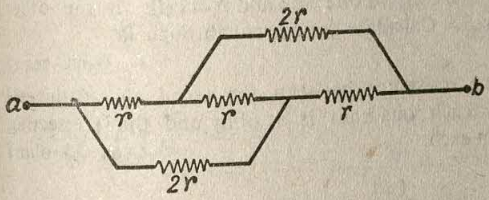
29. Find the equivalent resistance between the terminal points a and b of the circuits 1, 2, 3 and 4.



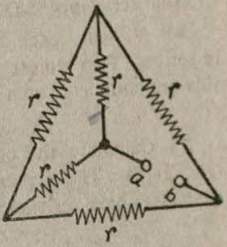
circuit 1



circuit 2



circuit 3



circuit 4

[Ans. (1) $1\frac{8}{19}r$, (2) $\frac{7}{5}r$, (3) $\frac{7}{5}r$, (4) r]

30. A net work of conductors is in the form of a triangle ABC in which $AB=1\ \Omega$, $BC=2\ \Omega$ and $CA=3\ \Omega$ and the angular points A, B and C are joined to any point O inside the triangle by resistances $1\ \Omega$, $2\ \Omega$ and $3\ \Omega$ respectively. Find the equivalent resistance between A and B.

[*Hint* : Assume unit current at the entrance and exit points.]

(Ans. $\frac{78}{118} = {\cdot}65\ \Omega$)

31. Find the equivalent resistance of the infinite series of the following circuit (Fig. 1-16) between the points a and b.

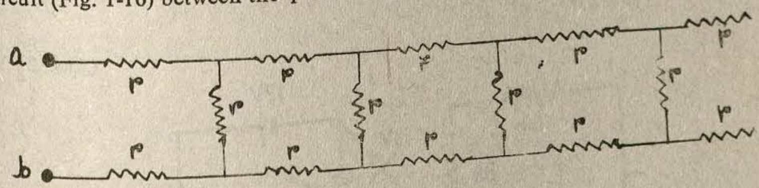


Fig. 1-16

[*Hint* : From an infinite series if we separate out the first unit, the remaining circuit will still be an infinite series and hence will have the equivalent resistance.]

(Ans. $(\sqrt{3}+1)\ r$ ohm)

32. Calculate the equivalent resistance of the circuit shown in Fig. 1-17 between the points *a* and *b*.

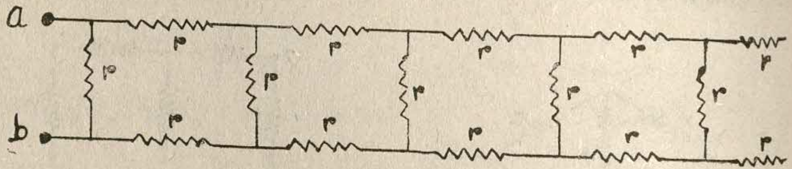


Fig. 1-17

(Ans. $(\sqrt{3}-1)\, r$ ohm)

33. A current of 8 A is to be sent through a resistor of 5 Ω. Calculate the least number of cells required for the purpose when each cell has emf 2 V and internal resistance ·5 Ω.

(Ans. 160)

34. *N* cells, each of emf *e* and internal resistance *r*, are arranged in a ring in series. Two points including *n* cells on one side and *N-n* cells on the other side are connected to a resistor *R*. Calculate the current through *R*.

(Ans. zero)

35. Calculate the equivalent resistance between the terminals of the cell (Fig. 1-18). The resistance of each quadrant is 1 ohm and the intersecting diameters have resistance 2 ohm each.

(Ans. $\frac{15}{7}$ ohm)

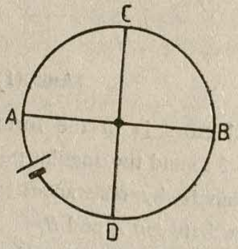


Fig. 1-18

36. Find the potential difference between the points *a*, *b* of the given circuit (Fig. 1-19).

[*Hint* : $V_{ab}=\Sigma\, ir-\Sigma\, e$]

(Ans. $2\frac{2}{3}$ V)

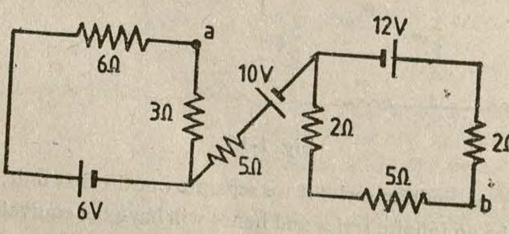


Fig. 1-19

37. A part of a circuit in the steady state with the currents flowing in the branches, the values of resistances are shown in the figure (Fig. 1-20). Calculate the energy stored in the capacitor $C=4\ \mu F$.

(IIT 1986)

(Ans. 8×10^{-4} J)

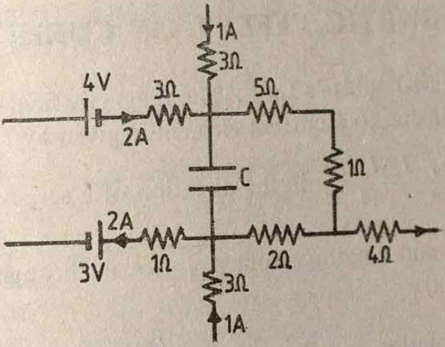


Fig. 1-20

38. Calculate the current through the 5 V cell of the figure (Fig. 1-21).

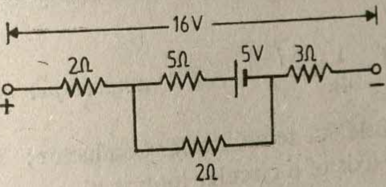


Fig. 1-21

(Ans. $\frac{1}{18}$ A in opposition)

□□□

CHAPTER 2

MAGNETIC EFFECT OF CURRENT

1. Biot-Savart-Laplace's Law : The magnetic field at a point in a vacuum or air due to a current element is given by,

$\triangle B = \frac{\mu_0}{4\pi} \cdot \frac{I \triangle l \sin\theta}{r^2}$ in the direction of a screw perpendicular to $\triangle \vec{l}$ and $\vec{r}$ and rotating from $\triangle \vec{l}$ to $\vec{r}$ through the smaller angle θ $(0 < \theta < \pi)$.

In vector notation, $\triangle \vec{B} = \frac{\mu_0}{4\pi} I \frac{\triangle \vec{l} \times \vec{r}}{r^3}$ *tesla* (T).

The corresponding H-vector (called intensity vector) of the field is given by,

$$\triangle \vec{H} = \frac{1}{4\pi} I \frac{\triangle \vec{l} \times \vec{r}}{r^3} \text{ Am}^{-1} \text{ (ampere per metre).}$$

2. Magnetic field due to a Circular Conductor : The magnetic field at a point on the axis of a circular loop is :

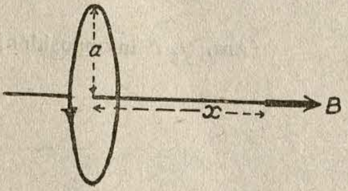


Fig. 2-1

$$B = \frac{\mu_0 N I a^2}{2(a^2 + x^2)^{3/2}} \text{ tesla (T)}$$

where N = number of turns,

a = radius of loop,

x = distance of point from the centre of the loop.

The corresponding H-vector (intensity vector) of the field is :

$$H = \frac{N I a^2}{2(a^2 + x^2)^{3/2}} \text{ Am}^{-1}.$$

The magnitude of the field at the centre is :

$$B = \frac{\mu_0 N I}{2a} \text{ tesla (T) and } H = \frac{NI}{2a} \text{ Am}^{-1}.$$

3. Magnetic Field due to a long Straight conductor : The magnetic field due to a *long straight* conductor is given by,

$$B=\frac{\mu_0 I}{2\pi d}\text{ tesla (T)}$$

where I = current through the conductor, d = perpendicular distance of the point. In general it is given by,

$$B=\frac{\mu_0 I}{4\pi d}(\sin\theta_1+\sin\theta_2)\text{ tesla (T)}$$

where θ_1 and θ_2 are the angles subtended by the two portions of the conductor on the two sides of the plane perpendicular to the conductor and passing through the point so that $(\theta_1+\theta_2)$ is the total angle subtended by the conductor at the given point. The direction of the field is given by the direction of motion of the top of the handle of a screw being rotated to advance in the direction of the current through the conductor. The corresponding H-vector is given by

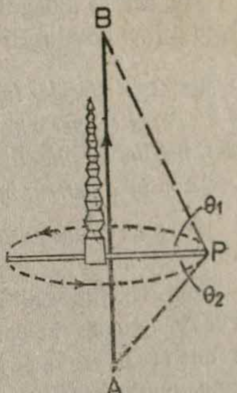


Fig. 2-2

$$H=\frac{I}{2\pi d}\text{ Am}^{-1}$$

or
$$H=\frac{I}{4\pi d}(\sin\theta_1+\sin\theta_2)\text{ Am}^{-1}.$$

EXAMPLES

Ex. 1. *A small current element* 5×10^{-5} Am *is placed at the corner A along AB of an equilateral triangle ABC. Calculate the intensity of the magnetic field at B and C. The triangle is* 20 cm *on each side.*

Sol. $\triangle B$ at $B=\frac{\mu_0}{4\pi}\cdot\frac{(5\times10^{-5})\sin 0^\circ}{\cdot 2^2}=0$

$\therefore\quad \triangle H=0 \qquad \left(\because\quad \triangle H=\frac{\triangle B}{\mu_0}\right)$

$\triangle B$ at $C=\frac{\mu_0}{4\pi}\cdot\frac{(5\times10^{-5})\sin 60^\circ}{\cdot 2^2}$

$=10^{-7}\frac{5\times10^{-5}\times\cdot 866}{\cdot 04}\left(\because\quad \frac{\mu_0}{4\pi}=10^{-7}\right)$

$=1{\cdot}08\times10^{-10}$ T.

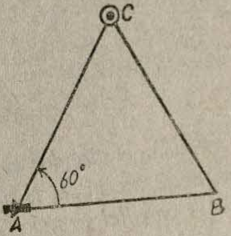


Fig. 2-3

The field is along the normal to the plane of the triangle and away from it (i.e. dot-direction).

Ex. 2. *A circular loop of mean radius* 5 cm *and having* 40 *turns of insulated copper wire is placed in the magnetic meridian. A compass needle is placed at the centre of the loop. What deflection is produced by a current of* ·1 A *through the coil ?*

(*H-vector of the earth's field at the place* = 29 Am^{-1})

Sol. When the coil is placed in the magnetic meridian, the magnetic field due to the coil is at right angles to the magnetic meridian. Let the needle make an angle θ with the magnetic meridian. Then torque on the needle due to the earth's field $= mB_0 \sin\theta$ and torque due to the field of coil $= mB\sin(90° - \theta)$, where B_0 = B-vector of earth's field, B = the same vector of the field due to the coil, m = magnetic moment of the needle.

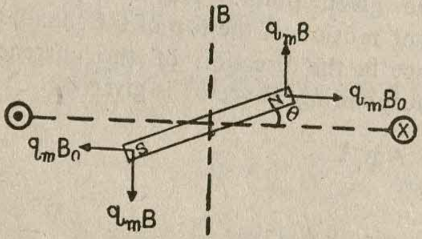


Fig. 2-4

For equilibrium,

$$mB_0 \sin\theta = mB\sin(90° - \theta)$$

or $\quad B = B_0 \tan\theta$

($\because \quad B = \mu_0 H$ and $B_0 = \mu_0 H_0$)

or
$$H = H_0 \tan\theta.$$

But
$$H = \frac{NI}{2a}.$$

$\therefore$
$$\frac{NI}{2a} = H_0 \tan\theta$$

or
$$\tan\theta = \frac{NI}{2aH_0} = \frac{40 \times \cdot 1}{2 \times 5 \times 10^{-2} \times 29} = 1{\cdot}38,$$

$\therefore$
$$\theta = 54°4'.$$

Ans.

Ex. 3. *A circular coil carrying* ·5 A *current, initially set in the magnetic meridian, is turned in proper direction till the needle becomes parallel to the plane of coil. If in this position it is* 30° *away from the magnetic meridian, calculate horizontal component of earth's magnetic field.* (*Radius of coil* = 5 cm *and number of turns* = 50.)

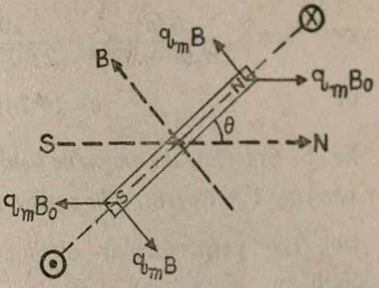


Fig. 2-5

Sol. Obviously the needle makes angle θ with B_0 (earth's field) and 90° with B (field due to the coil).

∴ Torque due to B_0
$=mB_0\sin\theta$.

Torque due to B
$=mB\sin 90°$.

For equilibrium of the needle, $mB_0\sin\theta=mB$

or $$B=B_0\sin\theta.$$

But, $$B=\frac{\mu_0NI}{2a}.$$

$$\therefore \quad \frac{\mu_0NI}{2a}=B_0\sin\theta \text{ or } I=\frac{2aB_0}{\mu_0N}\sin\theta$$

(*Note* : This is the theory of *sine galvanometer*.)

or $$B_0=\frac{\mu_0NI}{2a\sin\theta}=\frac{4\pi\times10^{-7}\times50\times\cdot5}{2\times5\times10^{-2}\times\sin30°} \quad (\because \mu_0=4\pi\times10^{-7})$$

or $$B_0=\frac{4\pi\times10^{-7}\times25}{\cdot1\times\cdot5}=62{\cdot}8\times10^{-5}\text{ T}$$ **Ans.**

and $$H_0=\frac{B_0}{\mu_0}=\frac{62{\cdot}8\times10^{-5}}{4\pi\times10^{-7}}=500\text{ Am}^{-1}.$$ **Ans.**

Ex. 4. *A circular loop is placed in the magnetic meridian. Find what deflection will it produce in a compass needle placed at a distance* 10 cm *from its centre. H-vector of earth's field* = 30 Am^{-1}. *Radius of the loop* = 10 cm *and number of turns* = 10. *Current through the loop* = ·5 A.

Sol. *H*-vector of the field at a distance x is given by

$$H=\frac{NIa^2}{2(a^2+x^2)^{3/2}}=H_0\tan\theta$$

($\because$ H and H_0 are at right angles to each other) or $$\tan\theta = \frac{NIa^2}{2H_0(a^2+x^2)^{3/2}} = \frac{10\times\cdot5\times10^2\times10^{-4}}{2\times30(10^2+10^2)^{3/2}\times10^{-6}} = \cdot295$$

or $$\theta = 16°24'.$$ **Ans.**

Ex. 5. *Calculate the magnetic field at the centre of a square conductor carrying* 1 A *current. The square is* 10 cm *on each side.*

Sol. The perpendicular distance of the centre from each side $= \cdot05$ m.

The perpendicular line bisects each side into two equal halves, each subtending 45° at the centre of the square.

∴ The magnetic field due to one side $= \frac{\mu_0 I}{4\pi d}(\sin\theta_1+\sin\theta_2)$

$$= \frac{10^{-7}}{\cdot05}(\sin45°+\sin45°)$$

$$= 28\times10^{-7}\text{ T}.$$

The total field at the centre of the square is four times the field due to one side.

∴ $B = 4\times28\times10^{-7} = 11\cdot2\ \mu\text{T}\ (1\ \mu\text{T} = 10^{-6}\text{ tesla})$. **Ans.**

Ex. 6. *Calculate the magnetic field at the centre of a semi-circular conductor carrying I ampere current. The radius of the semi-circle is equal to a.*

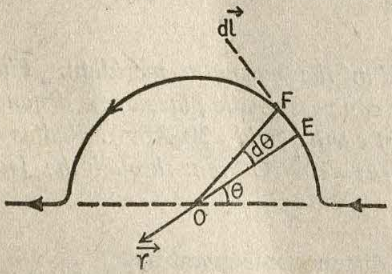


Fig. 2-6

Sol. Consider an element $dl = EF$ of the conductor. Let $d\theta$ be the angle subtended by it at the centre. Then $dl = a\,d\theta$. It can be seen easily that the angle between $d\vec{l}$ and $\vec{r}$ is 90° and the mod of $\vec{r}$ is a.

∴ $$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin90°}{a^2}$$

$$=\frac{\mu_0}{4\pi}\cdot\frac{Iad\theta}{a^2}=\frac{\mu_0 I}{4\pi a}\cdot d\theta.$$

$$\therefore \qquad B=\frac{\mu_0 I}{4\pi a}\int_0^{\pi} d\theta=\frac{\mu_0 I}{4a} \text{ tesla.} \qquad \text{Ans.}$$

EXERCISES

(A)

1. A current element of strength 2×10^{-4} Am is at the corner A of a cube $ABCDEFGH$ of side length 10 cm, the element lying along the edge AB. Calculate B-field and H-field of the element at the diagonally opposite corner.

(Ans. (i) $5{\cdot}44\times10^{-10}$ T; (ii) $4{\cdot}33\times10^{-4}$ Am^{-1})

2. Two current elements of strengths ·2 Am and ·4 Am form a cross at the centre of a circle of radius 10 cm. Calculate the magnetic field at a point on the circle 30° away (anticlockwise) from the first element.

(Ans. $2{\cdot}464\times10^{-6}$ T; along the normal to the circle and towards its plane)

3. What current must be passed through a vertical coil of 50 turns of average radius 8 cm in order to neutralise the horizontal component of earth's field (15 Am^{-1}) at its centre ? (Ans. ·048 A)

4. A circular loop of 100 turns and radius 10 cm is placed with its plane at an angle of 60° with the magnetic meridian. Calculate the angle made with the magnetic meridian by a small magnetic needle placed at the centre of the loop when 1 A current is passed through it. Horizontal component of earth's field is $3{\cdot}6\times10^{-5}$ tesla.

(Ans. 28°28′ for one direction of current and 148°42′ for the other direction)

5. A coil of 50 turns and radius 10 cm is placed with its plane at right angles to the magnetic meridian. It is connected to a cell of steady e. m. f. and a rheostat. The current through the coil is gradually increased till a compass needle at its centre just turns through 180°. Find the strength and direction of current through the coil. (Horizontal intensity of earth's field=30 Am^{-1}.) (Ans. ·12 A)

6. Calculate the current in ampere through a circular loop of 40 turns and mean radius 5 cm placed with its plane in the magnetic meridian that will produce a deflection of 60° of a compass needle placed at its centre. (The value of horizontal induction=$3{\cdot}5\times10^{-5}$ tesla.) (Ans. ·12 A)

7. A circular coil has 63 turns, each of radius 22 cm. It is placed in the magnetic meridian. When a current of ·1 A passed through the coil, the deflection of compass needle placed at its centre is 45°. Calculate the intensity of earth's horizontal field. (Ans. $H=14{\cdot}3$ Am^{-1})

8. A circular coil having 10 turns of mean radius 10 cm is placed in the magnetic meridian. A current of ·5 A is passed through the coil. What is the

deflection of the compass needle placed at the centre of the coil ? If the coil is rotated at what angle the needle will set parallel to the plane of the coil ? (H of earth's field$=30$ Am^{-1})
(Ans. 39°48′; 56°24′)

9. A current of ·1 A passes through a circular coil of 15 cm in diameter and consisting of 50 closely wound turns of fine insulated wire. Calculate the intensity of field (H-vector) (i) at the centre of the coil, (ii) at a point on its axis distant 7·5 cm from the centre.
(Ans. 33·33 Am^{-1}; 11·79 Am^{-1})

10. Calculate the magnetic field at the centre of a regular hexagon, 10 cm on each side when the current through the wire forming the hexagon is 5 A.

(Ans. 34·6 μT)

11. Two long straight parallel conductors each carrying 2 A current in the same direction are 10 cm apart. Calculate the magnetic field at a distance (a) 5 cm, (b) 15 cm from one of the conductors towards the other.

[Ans. (a) zero; (b) $1{\cdot}07\times10^{-5}$ T]

12. Calculate the magnetic field at the orthocentre of an equilateral triangle, 20 cm on each side. The current through the wire forming the triangle is 10 A.
(Ans. 9×10^{-5} T)

13. Calculate the magnetic field at the corner of a right angled triangle ABC, A being 90°, when two long conductors each carrying 2 A passes perpendicular into the plane of the triangle through the corners B and C. The sides of the triangle are $AB=3$ cm, $AC=4$ cm.
(Ans. 16·7 μT)

14. The circular coil of so called tangent galvanometer is set at right angles to the magnetic meridian and a current of 1·2 A is passed through it. The coil is now rotated till the needle set itself at right angles to the magnetic meridian. It is found that the coil has gone through 20° from its initial position till this situation is attained. Calculate horizontal intensity of earth's magnetic field. The coil has only two turns of mean radius 8 cm.

(Ans. $H=14{\cdot}3$ Am^{-1})

15. A long thin wire of a frame of reference is placed along the y-axis (just out side). There exists a uniform magnetic field of 10^{-6} T along x-axis. Calculate the magnetic field at the points (0, 0, 2m), (0, 2m, 0) and (2m, 0, 0) when the wire carries 10 A current. Revise the calculations for a thick wire.
(Ans. 2×10^{-6} T; $\propto$; $\sqrt{2}\times10^{-6}$ T, 2×10^{-6} T, 10^{-6} T, $\sqrt{2}\times10^{-6}$ T)

16. A circular loop of radius 10 cm carries a current of 15 A. At its centre is placed a small loop of radius 1 cm having 50 turns and a current of 1 A. (a) What is B-vector of the magnetic field produced by the large loop at its centre ? (b) What is H vector of the field ? (c) What torque acts on the small loop when its plane is perpendicular to the plane of the large loop ?
(Ans. (a) $9{\cdot}4\times10^{-5}$ T; (b) 74·96 Am^{-1}; (c) $1{\cdot}5\times10^{-6}$ Nm)

17. A thin nonconducting ring of radius a and having a total charge $+q$ distributed uniformly over it rotates with angular speed ω about an axis perpendicular to its plane and passing through its centre. Find the magnetic induction field at its centre and at a distance x.

$$\left(\text{Ans. } \frac{\mu_0 q\omega}{4\pi a};\ \frac{\mu_0 q\omega a^2}{4\pi(a^2+x^2)^{3/2}}\right)$$

18. A thin disc of dielectric material, having a total charge $+q$ distributed uniformly over its surface rotates n times per second about an axis perpendicular to the surface of the disc and passing through its centre. Find the magnetic induction at the centre of the disc. The radius of the disc is equal to r.

$$\left(\text{Ans. } \frac{\mu_0 nq}{r}.\right)$$

19. Two circular loops, each of radius 'a', and carrying equal currents in the same direction, are placed with their planes parallel at a separation b. Find the magnetic induction at the centre of either turn.

[*Hint* : Add the field due to one to the field due to the other.]

$$\left(\text{Ans. } \frac{\mu_0 i}{2a}+\frac{\mu_0 i a^2}{2(a^2+b^2)^{3/2}}\right)$$

20. A long wire is bent into the shape as shown in the fig. 2-7 without cross-contact at P. Determine the magnitude and direction of B at the centre of the circular portion of radius R when the current i flows as indicated.

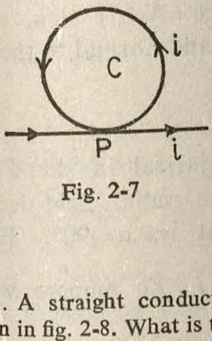


Fig. 2-7

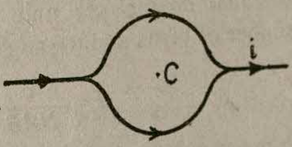


Fig. 2-8

$$\left[\text{Ans. } \frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right),\ \text{out of page}\right]$$

21. A straight conductor is split into two identical semicircular shape as shown in fig. 2-8. What is the magnetic field at the centre of the circular loop so-formed

(Ans. zero)

22. Calculate the force of attraction per unit length between two long straight conductors 10 cm apart and each carrying 5 A current when they are at (i) 0°, (ii) 90° and (iii) 60° inclination to each other.

$$\left[\textit{Hint}:\ \Delta F=\frac{\mu_0 I_1 I_2 \cos\theta}{2\pi d}\right]$$

(Ans. 5×10^{-5} N; 0; $2{\cdot}5\times10^{-5}$ N)

□□□

CHAPTER 3

GALVANOMETERS : AMMETERS AND VOLTMETERS

1. When a current loop (rectangular or circular) is placed in a uniform magnetic field of strength B tesla with the normal to loop at an angle θ with B, it experiences torque given by

$$\tau = IAB \sin\theta \text{ Nm.}$$

Vectorially, $\vec{\tau} = I\vec{A} \times \vec{B}$.

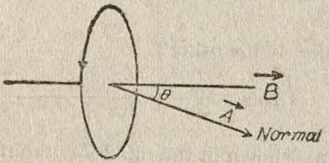


Fig. 3-1

2. Moving coil galvanometer : A moving coil galvanometer works on the above fact that when a loop is placed in a magnetic field it experiences a torque. Measuring this torque by the usual torsional method current is calculated. If c be the torsional couple per unit twist then $c\theta = NABI \sin\alpha$, where N = number of turns and α = angle between B and normal to the coil;

$$\text{or} \qquad I = \frac{c}{NAB} \cdot \frac{\theta}{\sin\alpha}.$$

When the poles of the magnet are cylindrical in shape field between them is radial. The advantage of the radial field is that *plane of loop is always parallel to the field*, that is, $\alpha = 90°$. Hence when poles are cylindrical, $I = \frac{c}{NAB} \cdot \theta$, or $I = K\theta$ ampere where $K = \frac{c}{NAB}$ is a constant of the galvanometer and is called its current reduction factor.

3. The tangent galvanometer : A tangent galvanometer is a combination of a vertical circular coil and a compass box placed exactly at its centre. The coil is placed in the magnetic meridian and the current to be measured is passed. The current through it is given by,

$$I = K \tan\theta \text{ ampere (A)}$$

where K is a constant of the galvanometer called the reduction factor of the galvanometer. $K = \frac{2H_0 a}{N}$ or $\frac{2B_0 a}{\mu_0 N}$.

4. Conversion into ammeter : When a moving coil galvanometer is shunted with a suitable low resistance, it is converted into an ammeter. Suppose i = current sensitivity of the galvanometer i.e. it is the current which produces full scale deflection in the galvanometer.

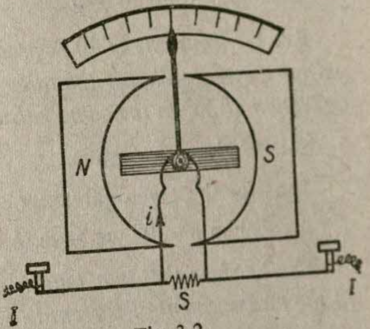


Fig. 3-2

G = resistance of the galvanometer coil. We want to convert it into an ammeter reading up to I ampere. For this we have to select a shunt of such a value that when the main current is I, the current through the galvanometer coil is i and the current through the shunt is $(I-i)$.

Since G and S are in parallel and the same p.d. exists across both,

$$i\,G=(I-i)\,S;$$

$$\therefore \quad S=\left(\frac{i}{I-i}\right)G.$$

This formula gives the shunt required to convert a galvanometer of given sensitivity (i) into an ammeter of assigned range (I).

5. Conversion into voltmeter : When a high resistance is connected in 'series' with a galvanometer coil, it is converted into a 'voltmeter'. Suppose i = current sensitivity of the galvanometer and we want to convert it into a voltmeter reading up to 'V' volt. For this we have to select a high resistance of such value that when p,d. of V volt is applied across the terminals, current through the galvanometer coil is only i ampere.

$$\therefore \quad i=\frac{V}{G+R}, \quad \text{or} \quad R=\left(\frac{V}{i}-G\right)\text{ohm}.$$

This formula gives the resistance required to convert a galvanometer of sensitivity (i) into a voltmeter of assigned range (V). The quality of a voltmeter is indicated by a term 'Sensitivity' which is defined as '*ohm per volt*'.

$$\therefore \quad \text{Sensitivity of a voltmeter}=\frac{R+G}{V}=\frac{1}{i}\,\Omega\,\mathrm{V}^{-1}.$$

EXAMPLES

Tangent galvanometer :

Ex. 1. *A current of* 10 A *produces a deflection of* 45° *in a tangent galvanometer. What is the value of the current which will produce a deflection of* 30° *in the same galvanometer ?*

Sol. We have $I = K\tan\theta$. $\therefore$ $10 = K\tan 45^\circ$ or $K = 10$.

Thus for the given galvanometer we have, $I = 10\tan\theta$.

$\therefore$ When $\theta = 30^\circ$ we have, $I = 10\tan 30^\circ = 5{\cdot}77$ A. **Ans.**

Ex. 2. *Show that in a tangent galvanometer the percentage error in the measurement of current is a minimum when the deflection is 45°.*

Sol. We have for a tangent galvanometer, $I = K\tan\theta$.

Differentiating, $\triangle I = K\sec^2\theta \triangle\theta$.

$$\therefore \text{ The percentage error} = \frac{\triangle I}{I} = \frac{K\sec^2\theta\triangle\theta}{K\tan\theta} \times 100 = \frac{200\triangle\theta}{\sin 2\theta}.$$

Obviously the percentage error is minimum when $\sin 2\theta = 1$

or
$$\theta = 45^\circ.$$
Proved.

Moving coil galvanometers :

Ex. 3. *A suspended vertical circular coil of* 100 *turns and area* 100 cm² *carries a current of* 30 mA *and it is in equilibrium in a uniform magnetic field of intensity* ·4 *tesla when the normal to the plane of coil makes an angle* 30° *with the field. Calculate the torque on the coil due to current.*

Sol. We have, $\tau = NIAB \sin\alpha$

$\tau = 50 \times (30 \times 10^{-3})(100 \times 10^{-4}) \times {\cdot}4 \times \sin 30^\circ = {\cdot}003$ Nm. **Ans.**

Ex. 4. *A Weston galvanometer whose resistance is* 25 Ω *shows full scale deflection for* 500 μA. *Explain how you would convert it into (i) ammeter reading up to* 10 A *and (ii) voltmeter reading up to* 10 V. *What will be the 'sensitivity' of the voltmeter so formed ?*

Sol. To convert it into an ammeter we have to shunt the coil with a shunt given by

$$S = \frac{i}{I - i} \times G \text{ (see article 4 and deduce it here).}$$

Here
$$i = 500\ \mu\text{A} = 500 \times 10^{-6}\ \text{A}$$
$$I = 10\ \text{A and } G = 25\ \Omega.$$

$$\therefore \quad S=\frac{500\times10^{-6}}{10-500\times10^{-6}}\times25=\frac{5\times25\times10^{-4}}{10}$$

(neglecting 500×10^{-6} in comparison to 10)

or $S=\cdot00125\ \Omega$.

To convert it into a voltmeter we have to add a high resistance in series given by

$$R=\left(\frac{V}{i}-G\right) \text{ (see article 3 and deduce it).}$$

Here, $V=10$ volt; $i=500\times10^{-6}$ A.

$$\therefore \quad R=\frac{10}{500\times10^{-6}}-25, \quad \text{or} \quad R=2\times10^{4}-25$$

or $R=20{,}000\ \Omega$ (neglecting 25 in comparison to 2×10^{4}).

Sensitivity of the voltmeter $=\frac{20000}{10}=2000\ \Omega$/volt. **Ans.**

Ex. 5. *A cheap voltmeter having resistance 500 Ω is used to measure the e.m.f. of a cell of internal resistance 4 Ω. What is the % error in the reading of the voltmeter ?*

Sol. Let $e=$ e.m.f. of the cell.

Current in the circuit is given by $i=\frac{e}{R+r}=\frac{e}{500+4}$.

$\therefore$ P.D. across terminals of the voltmeter $=Ri=500\times\frac{e}{500+4}$

$$=\frac{500e}{504}.$$

The voltmeter will read $\left(\frac{500e}{504}\right)$ instead of e.

$$\therefore \quad \%\text{ error in the reading}=\frac{\left(e-\frac{500e}{504}\right)}{\frac{500e}{504}}\times100=\cdot8\%. \qquad \textbf{Ans.}$$

Ex. 6. *In a circuit there is a cell of e.m.f. 1·5 V and internal resistance 2 Ω and a resistor 8 Ω. An ammeter of resistance ·5 Ω is connected in the usual way to measure the current of the circuit. What is the percentage error in the reading of the ammeter ?*

Sol. The actual current through the circuit is,

$$i=\frac{1{\cdot}5}{2+8}={\cdot}15\text{ A.}$$

When the ammeter is connected in series, the current through the circuit is,

$$i'=\frac{1{\cdot}5}{2+8+{\cdot}5}=\frac{1{\cdot}5}{10{\cdot}5}.$$

The ammeter will read $\frac{1{\cdot}5}{10{\cdot}5}$ instead of ·15.

$$\%\text{ error in the reading}=\frac{\left({\cdot}15-\frac{1{\cdot}5}{10{\cdot}5}\right)}{\frac{1{\cdot}5}{10{\cdot}5}}\times 100=5.$$ **Ans.**

Ex. 7. *A voltmeter of* 20 *volt range and* 50000 Ω *is connected by mistake in series with a resistor* 30000 Ω. *If the e.m.f. of the battery to which they are connected is* 6 *volt; what will be the reading of the voltmeter ?*

Sol. A voltmeter will always read the p.d. across it.

The current through the circuit $=\frac{6}{5\times 10^4+3\times 10^4}=\frac{3}{4}\times 10^{-4}$ A.

The p. d. across the voltmeter $=\frac{3}{4}\times 10^{-4}\times 5\times 10^4=\frac{15}{4}$

$=3{\cdot}75$ volt.

Hence the voltmeter will read 3·75 volt. **Ans.**

Ex. 8. *In a suspended type moving coil galvanometer the coil has* 60 *turns each of area* 2×10^{-4} m² *and the strength of the magnetic field is* ·01 *tesla. A current of* 20 mA *produced a deflection of* 10 cm *on a scale* 1 *metre from the instrument. Calculate the torque per unit twist.*

Sol. In a moving coil galvanometer the field is radial and therefore, $\tau=NIAB$. This is the deflecting torque. Controlling torque $=c\theta$ where c is the controlling torque per unit twist. For equilibrium of the coil,

$$NIAB=c\theta.$$

θ is measured by 'lamp and scale' arrangement which works on the principle that *when a mirror rotates through* θ, *reflected ray* *rotates through* 2θ. In the galvanometer a mirror is attached to the suspension wire.

$$\therefore \quad \tan 2\theta=\frac{d}{D}, \quad \text{or} \quad 2\theta=\frac{d}{D}$$

$$\text{or} \quad \theta=\frac{d}{2D}=\frac{10\times 10^{-2}}{2\times 1}=5\times 10^{-2} \text{ radian.}$$

$$\therefore \quad 60\times(20\times 10^{-3})\times(2\times 10^{-4})\times \cdot 01=c\times 5\times 10^{-2}$$

$$\text{or} \quad c=\frac{24\times 10^{-7}}{5\times 10^{-2}}=\mathbf{4{\cdot}8\times 10^{-5}\ Nm.}$$

Ans.

EXERCISES

(A)

Tangent galvanometer :

1. A circular coil of radius 20 cm and 20 turns of wire is mounted vertically with its plane in the magnetic meridian. A small magnetic needle placed at the centre of the coil is deflected through 45° when a current is passed through the coil. What is the value of the current ? (Horizontal intensity of earth's field = $3{\cdot}6\times 10^{-5}$ weber per square metre) **(Ans.** ·573 A)

2. Calculate the current through a tangent galvanometer whose coil has 50 turns and a mean diameter 22 cm when the needle is deflected through 30° at a place where $H=30\ \text{Am}^{-1}$. **(Ans.** ·076 A)

3. A current is sent through two tangent galvanometers in series. The deflection is seen to be the same in both the galvanometers. Compare the radii of the coils of the galvanometers, if the number of turns in the coil of the first galvanometer is 110 and 25 in the second. **(Ans.** 22 : 5)

4. Show that a tangent galvanometer is most sensitive when the deflection is near zero. $\left[\textit{Hint}: \text{Sensitiveness}=\frac{d\theta}{dI}.\right]$

5. The total resistance of a simple circuit is 80 Ω including the resistance of a tangent galvanometer which is 4 Ω. The galvanometer gives deflection of 60°. It is then shunted with 1 Ω. What is the new deflection ? **(Ans.** 19°48′)

6. A tangent galvanometer of resistance 10 ohm shows a deflection of 60° when inserted in a circuit of which the total resistance including that of the galvanometer is 100 ohm. What shunt must be used to reduce the deflection to 30° ? **(Ans.** 4·5 ohm)

Galvanometers : Ammeters : Voltmeters :

7. A circular loop of 50 turns and area 20 cm² is placed in a magnetic field of ·2 T. Calculate the torque required to deflect the loop through 90° in the magnetic field when 1 A current is passing through the loop. **(Ans** ·02 Nm)

8. A galvanometer coil of rectangular shape of length 4 cm and width 2 cm contains 200 turns of insulated copper wire. It is suspended in a magnetic field of ·05 tesla and the suspension produces torque of 10^{-6} Nm per unit twist. What steady current passing through the coil will produce deflection of 5° ?

(**Ans.** $1{\cdot}09\times10^{-5}$ A)

9. A moving coil reflecting type galvanometer has a coil of 50 turns of mean area 1·5 cm^2 per turn and the effective strength of the field (radial) is ·02 T. If a current of 10^{-9} A produces a deflection of 1 cm on a scale placed at 1 m away, calculate the steady torque required to hold the suspension twisted through 1 radian.

(**Ans.** 3×10^{-11} Nm)

10. In a moving coil galvanometer the magnetic field used is 5 tesla. The coil has 100 turns each of mean area 10×10^{-4} m^2. If a current of 10^{-7} A produces a deflection of 1 mm on a distant scale 1 m away, calculate the current reduction factor of the galvanometer and torsional rigidity of the suspension wire.

(**Ans.** 2×10^{-4} A per radian; 10^{-4} Nm per radian)

11. A milliammeter has a resistance of 10 Ω and each divison of its graduation reads one milliampere. How will you convert it into a voltmeter reading 1 volt per division ? What will be the sensitivity of the voltmeter ? (There are 10 divisions on the scale.)

(**Ans.** Place a resistance 990 Ω in series with the ammeter; 100 ohm per volt)

12. A moving coil galvanometer has a resistance of 00 ohm and gives a full scale deflection with a current of 10 milliampere. Explain how you would convert it into (i) ammeter reading up to 1 ampere, (ii) voltmeter reading up to 10 volt. Also calculate the resistance of ammeter and voltmeter so formed.

(**Ans.** 2·02 Ω in parallel, 800 Ω in series; Ammeter = 2 Ω; voltmeter = 1000 Ω)

13. A voltmeter having resistance 8000 ohm is used to measure the e.m.f. of a cell of internal resistance 4 ohm. What is the percentage error in the reading of voltmeter ?

(**Ans.** ·05%)

14. A cheap voltmeter having an internal resistance of 30 ohm is connected to the terminals of a battery whose internal resistance is 5 ohm and true e.m.f. 1·5 volt. What would be the terminal voltage as read by the voltmeter ?

(**Ans.** 1·29 volt)

15. A voltmeter of range of 10 V and resistance 5000 Ω is connected by mistake in series with a battery and a resistor 500 Ω. If the voltmeter reading is 8 V, what is the e. m. f. of the battery ? (Neglect internal resistance of battery.)

(**Ans.** 8·8 V)

16. There is a milliammeter each division of which reads 1 milliampere. It has a resistance of 15 Ω. How would you convert it into a voltmeter so that each division of its graduation would read 1 volt ? (**Ans.** by putting 985 Ω in series)

17. In a circuit consisting of a cell of e. m. f. 2 V and internal resistance 1 Ω and a resistor of 9 Ω, an ammeter of ·02 Ω is connected properly to measure current of the circuit. What is the percentage error in the reading of the ammeter ?

$$\left[\textit{Hint}: C_{\text{real}}=\frac{2}{10}={\cdot}2,\ C_{\text{apparent}}=\frac{2}{10{\cdot}02}\right]$$

(**Ans.** ·2%)

18. A voltmeter of resistance 3000 Ω can read up to 10 volt. How will you adjust it to measure voltages up to 220 volt ? How will the sensitivity of voltmeter be affected ? (**Ans.** Putting 63000 Ω in series; sensitivity will not change)

19. A milliammeter of resistance 20 Ω reads up to 50 mA. Explain how the instrument can be used to read (a) current up to 5 A, (b) voltage up to 50 volt. [**Ans.** (a) 2 Ω in parallel; (b) 980 Ω in series]

20. A battery of e. m. f. 5 V and internal resistance 20 Ω is connected to a combination of two resistances 50 Ω and 40 Ω in series. If the p. d. across 50 Ω resistor is measured by a 1000 Ω resistance voltmeter, what will be the precentage error ? (**Ans.** 2·7%)

21. A 150-volt voltmeter has a resistance of 20,000 Ω. When connected in series with a large resistance *R* across a 110-volt line the meter reads 5 volts. Find the resistance *R*. (**Ans.** $4{\cdot}2\times10^5$ Ω)

(B)

22. A moving coil galvanometer of resistance 20 Ω gives a full scale deflection when a current of 1 mA is passed through it. It is to be converted into an ammeter reading 20 A on full scale. But the shunt of ·005 Ω only is available. What resistance should be connected in series with the galvanometer coil ? (**Ans.** 80 Ω)

23. Two tangent galvanometers of 50 and 100 turns of same wire and radii 10 cm and 5 cm respectively are connected in parallel. Calculate the ratio $\tan\theta_1 : \tan\theta_2$ where θ_1 and θ_2 are the deflections produced in them. (**Ans.** 1 : 4)

24. A 600 Ω resistor and a 400 Ω resistor are connected in series across a 90-volt line. A voltmeter across the 600 Ω resistor reads 45 volt. What is the value of voltmeter resistance ? If the same voltmeter is connected across 400 Ω resistor, what will be its reading ? (**Ans.** 1200 Ω; 30 volt)

25. Three voltmeters *A*, *B* and *C* have the same range but different resistance R_A=1200 Ω, R_B=1000 Ω and R_C=800 Ω. *A* is connected in parallel to *C* and *B* is connected in series with the combination of *A* and *C*. When the combination of *ABC* so formed is connected across a constant potential difference V_{ab}, meter *A* reads 4·8 volts and meter *B* reads 10 volts. What will be the reading of each meter when all are connected in series across the same potential difference V_{ab} ?

(**Ans.** 5·92 volts, 4·94 volts and 3·94 volts)

26. A galvanometer having a coil resistance of 100 ohms gives a full scale deflection when a current of one milliampere is passed through it. What is the value of the resistance which can convert this galvanometer into an ammeter giving a full scale deflection for a current of 10 amperes ?

A resistance of the required value is available but it will get burnt if the energy dissipated in it is greater than one watt. Can it be used for the above described conversion of the galvanometer ? (I. I. T. 1972)

(**Ans.** ·01 Ω; Yes)

(*Hint* : Power dissipated=I^2R watt)

27. When the modified galvanometer of the previous exercise (exercise 26) is connected across the terminals of a battery, it shows a current of 4 ampere. The current drops to 1 ampere when a resistance of 1·5 Ω is connected in series with

the modified galvanometer. Find the e. m. f. and the internal resistance of the battery. (I. I. T. 1972) (**Ans.** ·49 Ω; 2 volt)

28. An ammeter and a voltmeter are connected in series to a battery of emf $E=6$ V. When a certain resistance is connected in parallel with the voltmeter, the reading of the latter decreases $n=2$ times, whereas the reading of the ammeter increases the same number of times. What is the ratio of the voltmeter resistance to the ammeter resistance ? Find the voltmeter reading after the connection.

$$\left(\textbf{Ans.}\ \ n:1;\ \frac{E}{n+1}=2\text{ V}\right)$$

□□□

CHAPTER 4

WHEATSTONE BRIDGE, METRE BRIDGE AND P.O. BOX

1. Wheatstone Bridge : Wheatstone bridge is an arrangement of resistors in which four resistors $AB=P, BC=Q, AD=R$ and $DC=S$ are arranged in a quadrilateral $ABCD$, A being joined to C through a cell and B to D through a galvanometer. The distribution of currents in the different arms of a Wheatstone bridge in compliance with Kirchhoff's 'point rule' is shown in the Fig. 4.1. When there is no current through the galvanometer, the bridge is said to be balanced.

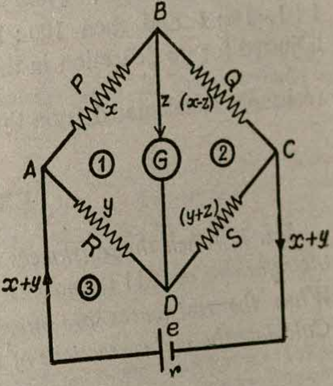


Fig. 4-1

$$\frac{P}{Q}=\frac{R}{S}$$ —*Condition for balance.*

2. Metre Bridge : Metre bridge is an application of Wheatstone bridge. Here P and Q are obtained by dividing a uniform 1 metre long resistance wire by a slider.

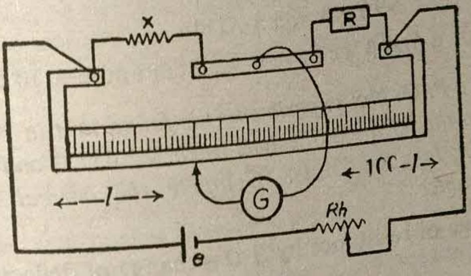


Fig. 4-2

$$\therefore \quad \frac{X}{R}=\frac{l}{100-l}$$

$$X=\frac{l}{100-l}\times R.$$

More accurately, $$X=\frac{l+\alpha}{(100-l+\beta)}\times R$$

where α and β are the resistances of the wire beyond '0' and '100' of the scale including the resistance of the end strips. These are called 'end corrections'.

3. Post Office Box (P O. box) : This is a compact arrangement of Wheatstone bridge. Here keeping $P:Q$ at three values, namely, 1 : 1, 10 : 1 and then 100 : 1, the resistance in the rheostat arm is adjusted for no deflection in the galvanometer and then unknown resistance is calculated from the formula $\frac{P}{Q}=\frac{R}{S}$.

EXAMPLES

Ex. 1. *When the resistances of the left gap and the right gap of a metre bridge are* 111 Ω *and* 1 Ω, *the null point is found at* 99·8 cm. *When the resistances are interchanged in the gaps it is now at* ·3 cm. *Calculate the end corrections of the bridge wire.*

Sol. We have, $$\frac{111}{1}=\frac{99{\cdot}8+\alpha}{100-99{\cdot}8+\beta}=\frac{99{\cdot}8+\alpha}{{\cdot}2+\beta}$$

or $$22{\cdot}2+111\beta=99{\cdot}8+\alpha \qquad \text{.. (i).}$$

Again, $$\frac{1}{111}=\frac{{\cdot}3+\alpha}{100-{\cdot}3+\beta}=\frac{{\cdot}3+\alpha}{99{\cdot}7+\beta}$$

or $$99{\cdot}7+\beta=33{\cdot}3+111\alpha \qquad \text{.. (ii).}$$

Solving for α and β we have $\alpha={\cdot}59$ cm and $\beta={\cdot}70$ cm. **Ans.**

Ex. 2. *In a P. O. box experiment it is found that in* 1000 : 10 *ratio deflection is to the left by* 2 *divisions when* $R=437\ \Omega$ *and to the right by* 1·3 *division for* $R=436\ \Omega$. *Compute the correct value of the unknown resistance.*

Sol. Change of resistance by 1 Ω ≡ change of deflection by 3·3.

∴ Change of deflection by 1 dv. ≡ change of resistance by $\frac{1}{3{\cdot}3}$

„ „ „ „ 2 „ ≡ „ „ „ $\frac{2}{3{\cdot}3}$

$$={\cdot}6\ \Omega.$$

Here for null point, $R=437-{\cdot}6=436{\cdot}4\ \Omega$

$$\therefore \qquad X=\frac{R}{100}=\frac{436{\cdot}4}{100}=4{\cdot}364\ \Omega.$$

EXERCISES

1. When the resistances in left gap and right gap of a metre bridge are 101 Ω and 1 Ω respectively, the null point is found at 99·5 cm. When the resistances are interchanged it is now at ·7 cm. Calculate the end-corrections of the metre bridge. (**Ans.** ·29 cm; ·49 cm)

2. Show that in a metre bridge percentage error in the determination of unknown resistance is minimum when the null point is at the centre of the wire.

3. In a P. O. box experiment it is found that in 100 : 1 ratio deflection in the galvanometer is to the left by ·7 division of the galvanometer scale when resistance in rheostat arm is 978 ohm and to the right by 1·2 division when it is 979 ohm. Calculate the unknown resistance. (**Ans.** 9·7837 ohm)

4. In a simple metre bridge circuit the gaps are bridged by coils *P* and *Q*, *P* having the smaller resistance. A balance is obtained when the jockey key makes contact at a point of the bridge wire 40 cm from one end. On shunting the coil *Q* with a resistance 50 ohm, the balance point is moved through 10 cm. Find the resistance of *P* and *Q*. (**Ans.** 16·67 and 25 ohm)

5. A coil of copper wire is put in the left gap and some resistance in right gap of a simple metre bridge. The coil is immersed in a water bath. When the temperature of the bath is 0°C, the null point occurs at 50·0 cm. When the water in the bath is boiled, the null point shifts to 52 cm. Calculate the temperature coefficient of resistance of copper. (**Ans.** ·000833/°C)

6. The resistances of the four arms of a Wheatstone bridge are $P=10$, $Q=100$, $R=40$ and $S=10\ \Omega$. What resistance in series or parallel with the last one will be required to obtain no deflection in the galvanometer ? (**Ans.** 390 Ω in series)

7. In a P.O. box experiment a student observes that he does not get null point in any of the ratios. But when the ratio is 1000 : 10 he finds that deflection is to the right by 1 division for 596 Ω in the rheostat arm and 0·2 division to the left when it is 597 Ω. Compute the correct value of unknown resistance from the observation of the student. (**Ans.** 5·9683 Ω)

8. In a metre bridge the wire consists of two parts one of length 30 cm and of radius *r* and the other is of radius 2*r*. Where will the null point occur if the resistances in the left and right gaps are 5 Ω and 8 Ω respectively ? The material of the wires is the same. (**Ans.** 18·3 cm)

□□□

CHAPTER 5

POTENTIOMETER

1. The Potentiometer : The potentiometer is an instrument which can be used to measure the e.m.f. of a seat without drawing any current from the seat. It works on the fact that an e.m.f. or a p. d. can be balanced against another e.m.f. or p.d. and produce zero current.

In a potentiometer the e.m.f. of a battery (called driving battery) is distributed over a long uniform wire AB and p.d. between any two points of it (one fixed at the left end and the other sliding) is applied against e.m.f. or p.d. to be measured. The sliding point is then moved over the wire till the two balance each other. A sensitive galvanometer indicates whether current in the circuit is zero or not.

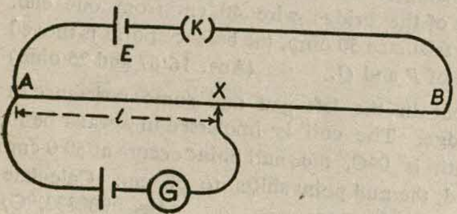


Fig. 5-1

When there is no deflection in the galvanometer we have P. D. between A and X = e.m.f. of the test cell.

Suppose l = distance of the sliding point from the left end when there is no deflection in the galvanometer,

r = resistance per unit length of the wire,

e = e.m.f of the test cell,

i = current through the potentiometer wire.

Then $e = lri$ .. (1). —*Working formula of potentiometer.*

The current (i) through potentiometer wire is given by

$$i = \frac{E}{R+r+R'} \qquad .. \quad (2)$$

where E = e.m.f. of the driving cell or battery,

r = internal resistance of it,

R = resistance of the potentiometer wire,

R' = additional resistance if any.

Note. If the resistance of the driving cell circuit is not disturbed, i may be considered as a constant. However, if the resistance is changed, i is to be calculated everytime using formula (2).

EXAMPLES

Ex. 1. *A battery of e.m.f.* 2 *volt and internal resistance* 1 *ohm is used to send a current through a potentiometer wire of length* 200 cm *and resistance* 4 ohm. *What length of the potentiometer wire will be required to balance a Daniell cell of e. m. f.* 1·08 *volt ?*

Sol. Here

$$i=\frac{2}{4+1}=\cdot 4 \text{ ampere},$$

$$r=\frac{4}{2}=2 \text{ ohm per metre},$$

$$r=lri;$$

$$\therefore \quad 1{\cdot}08=l\times 2\times \cdot 4, \quad \text{or} \quad l=\frac{1{\cdot}08}{\cdot 8}=\frac{10{\cdot}8}{8}=1{\cdot}35 \text{ m.}$$

Ans.

Ex. 2. *In a potentiometer experiment it is found that no current passes through the galvanometer when the terminals of a cell are connected across* 509 cm *of the potentiometer wire. When current is drawn from the cell through a resistor of* 20 Ω, *a balance is found when the cell is connected across* 490 cm *of the wire. Find the internal resistance of the cell.*

Sol. Let e = e.m.f. of the cell, V = p.d. of the cell and R = external resistance. From circuit equation we have, $i=\frac{e}{R+r}$. Applying Ohm's law to the external resistance we have, $i=\frac{V}{R}$.

$$\therefore \quad \frac{V}{R}=\frac{e}{R+r}$$

or

$$r=\frac{e-V}{V}R.$$

Here, $e=509\,\rho i$ where ρ = resistance per unit length of the wire and $V=490\rho i$.

$$\therefore \quad r=\frac{509-490}{490}\times 20=\cdot 776\ \Omega.$$

Ans.

Ex. 3. *A 5-wire potentiometer is connected to a storage cell of steady e.m.f.* 2 V *and* 1 Ω *resistance. A primary cell is balanced against* 3·5 m *of it. What resistance will be required in series with the storage cell to push the null point to the centre of the last wire i.e. at* 4·5 m ? (*The wire has* 3 Ω *resistance per metre of it.*)

Sol. The potentiometer current (i) is given by

$$i=\frac{2}{5\times3+1}=\frac{2}{16}=\frac{1}{8}\text{ A.}$$

$$\therefore \qquad e=3{\cdot}5\times3\times\tfrac{1}{8}.$$

Let X= resistance required.

Now potentiometer current (i) is,

$$i=\frac{2}{15+1+X}=\frac{2}{16+X};$$

$$\therefore \qquad \frac{3{\cdot}5\times3}{8}=4{\cdot}5\times3\times\frac{2}{16+X}$$

or $56+3{\cdot}5X=72$, or $3{\cdot}5X=16$, or $X=4{\cdot}57\ \Omega$. **Ans.**

EXERCISES

(A)

1. A metre bridge wire of resistance 3 Ω is connected to a cell of e.m.f. 2 volt and internal resistance 1 Ω Calculate the p.d. per cm of the wire. What length of this wire will balance a fresh dry cell of e.m.f. 1·5 volt ?

(**Ans.** ·015 V/cm; full length of it)

2. A potentiometer wire 4 m long is connected to the terminals of a battery of steady voltage. A Lechlanche cell gives null point at 1 m. If the length of the potentiometer wire be increased by 1 m, find the position of the null point.

(**Ans.** 1·25 m)

3. A potentiometer, having a wire of 10 m long stretched on it, is connected to an accumulator having a steady voltage. A Lechlanche cell gives a null point at 7 5 m. Jf the length of the potentiometer wire is increased by 1 m, find the new position of the null point.

(**Ans.** 8·25 m)

4. An accumulator giving a steady e.m.f. of 2 volt is connected across a potentiometer wire 10 m long having resistance 107 Ω per metre. A cell gives a null point at 6·732 m. If a resistance of 2·5 Ω is put in series with the wire, find the new position of null point.

(**Ans.** 6·748 m)

5. A 10-wire potentiometer is connected to an accumulator of steady voltage. A length of 7·8 m of it balances the e.m.f. of a cell on 'open circuit'. When the cell delivers current through a conductor of resistance 10 Ω, it is now balanced against 7·0 m of the same potentiometer. Calculate the internal resistance of the cell.

(**Ans.** 1·14 Ω)

6. A secondary cell of e.m.f. 2 volt and internal resistance ·1 Ω is connected to the ends of a uniform wire of length 1 m and resistance 12 ohm. A primary cell of e.m.f. 1·5 volt in series with a galvanometer is connected to two points on the wire. If the galvanometer shows no deflection, find the distance between the points. (Ans. ·757 m)

7. In a potentiometer experiment it is found that no current passes through the galvanometer when the terminals of the cell are connected across ·52 m of the potentiometer wire. If the cell is shunted by a resistance of 5 ohm, a balance is obtained when the cell is connected across ·4 m of the wire. Find the internal resistance of the cell. (Ans. 1·5 ohm)

(B)

8. The resistance of a potentiometer wire 8 m long is 8 ohm. A high resistance box and a 2-volt accumulator are connected in series with it. What should be the value of the resistance in the box, if it is desired to have a potential drop of 1 micro volt per mm ? (Ans 1992 Ω)

9. The terminals of a cell are connected to resistance R and the fall of potential across R is balanced against the fall of potential on a potentiometer wire. When R is 20 Ω and 10 Ω respectively, the corresponding length on the potentiometer wire are 1·5 m and 1·2 m. Calculate the internal resistance of the cell. (Ans. 6·67 Ω)

10 An accumulator of e.m.f. 2 volt and negligible internal resistance is connected across a uniform wire of length 10 m and resistance 30 Ω. The appropriate terminals of a cell of e.m.f. 1·5 volt and internal resistance 1 Ω is connected to one end of the wire, and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire. What length of the wire will be required to produce zero deflection of the galvanometer ? How will the balancing change (a) when a coil of resistance 5 Ω is placed in series with the accumulator, (b) the cell of 1·5 volt is shunted with 5 Ω resistor ?

(Ans. 7·5 m; 8·75 m; 6·25 m)

11. A certain thermocouple (treat it as a seat of e.m f.) which has a total resistance of 10 Ω, has one junction in melting ice and the other junction in steam. The e.m.f. between its ends as measured with a potentiometer is 4 millivolt. What would be its reading when it is connected to a millivoltmeter which has a resistance of 50 ohm ? (Ans. 3·33 millivolt)

12. A 10-wire potentiometer has resistance 10 Ω and is connected to an accumulator of 2 volt and negligible internal resistance. There are two resistance boxes R_1 and R_2 in series with the accumulator and one can have any integral values of resistance from resistance boxes. A standard *Cd*-cell of 1·018 V having a sensitive galvanometer in series with it is connected across R_1. How would you proceed with the above arrangement to obtain a potential drop of 1 μV per mm of the potentiometer wire ? Calculate values of R_1 and R_2 required. What length of this potentiometer will balance the thermo e.m.f. of iron-copper couple at 300°C which develops 17 μV/°C ?

(Ans. Put $R_1 = 1018$ (one thousand times the e.m.f. of *Cd*-cell); adjust R_2 till no deflection in the galvanometer; $R_2 = 972$ Ω, 5·1 m)

13. A five wire potentiometer is connected to an accumulator of e.m.f. 2·2 V and 1 Ω internal resistance. The potentiometer wire has resistance 1 Ω per metre. What is the maximum voltage that you can measure with this particular arrangement of potentiometer ? What length of this potentiometer will balance the e.m.f. of a Daniell cell (e.m.f.$=1{\cdot}18$ V) ? What resistance in series with accumulator will be required to balance this cell exactly at the centre of the last wire ?

(Ans. 1·833 V; 3·218 m; 2·39 Ω)

14 A potentiometer wire of length 1000 cm has a resistance of 10 ohms. It is connected in series with a resistance and a cell of e m f. 2 volts and of negligible internal resistance. A source of e.m.f. 10 millivolts is balanced against a length of 40 cm of the potentiometer wire. What is the value of the external resistance ?

(I.I.T. 1976)

(Ans. 70 Ω)

15. In a ten wire potentiometer the first five wires are of radius r and the next five wires are of radius $2r$. The wire is connected to a battery of steady voltage 2 V and negligible internal resistance. What lengths of this potentiometric arrangement will balance that e.m.f. of (a) a Daniell cell (emf$=1{\cdot}0$ volt), (b) a Lechlanche cell (emf$=1{\cdot}5$ V) and (c) an unknown cell of emf 1·8 volt ?

(Ans. (a) 312·5 cm, (b) 468·75 cm, (c) 750 cm)

□□□

CHAPTER 6

ELECTRICAL WORK : POWER : ENERGY

[Heating and Chemical Effect of Current]

1. **Electrical work :** When Q coulomb of electricity flows through a potential difference of V volt the electrical work done is

$$W = VQ \text{ joule (J).} \qquad \cdot\cdot \qquad (1a)$$

Since $I = Q/t$ (current = rate of flow of charge),

$\therefore \quad Q = It$

and $\quad \therefore \quad W = VIt$ joule. $\quad \cdot\cdot \quad$ (1b)

2. **Electrical power :** The rate at which electrical work is done in a circuit is called power of that circuit.

$$\therefore \quad P = \frac{VIt}{t} \text{ watt}$$

or $\quad P = VI$ watt (W) or I^2R or $\frac{V^2}{R}$ $\quad (\because \; V = RI)$.

3. **Electrical energy :** The energy consumed in an electrical circuit in a certain time is the work done in the circuit.

$\therefore$ Energy consumed in a circuit $= VIt$ joule $= Pt$ joule.

Board of Trade Unit : The unit for which electrical energy charged at a certain rate is called Board of Trade Unit (B.O.T. unit) and it is kilowatt-hour (K.W.H.).

$$3600 \text{ joule} = 1 \text{ watt hour}$$

and, $\quad 3600 \times 1000$ joule $= 1$ K.W.H.

$$\therefore \quad \text{Energy consumed} = \frac{V \text{ (in volt)} \times I \text{ (in ampere)} \times t \text{ (in hour)}}{1000} \text{ B. O. T. units}$$

or $$\frac{P \text{ (in watt)} \times t \text{ (in hour)}}{1000} \text{ B. O. T. units.}$$

4. **Heat produced by electric current :** If the whole of the electrical energy given up by charge carriers as they flow down a potential

drop is fully converted into heat energy, it is said to be a pure resistance. The heat produced is given by

$$W = JH.$$

$\therefore$ H (heat produced by current) $= \dfrac{V.It}{J}$ calorie.

We have, $V = RI$ .. Ohm's law.

$$\therefore \quad H = \frac{I^2Rt}{J} \text{ calorie}$$

$$\text{or} \quad H = \frac{V^2t}{JR} \text{ calorie.}$$

5. Chemical effect of Current :

(a) Faraday's laws of electrolysis : (1) The mass of ions liberated (and hence deposited also) is proportional to the amount of electricity that passes through it.

(2) The masses of different ions liberated by the same quantity of electricity are proportional to their chemical equivalents.

(b) Electro-chemical equivalent (E.C.E.) : If W = mass of ions liberated in kg by Q coulombs of electricity, then

$$W \propto Q, \text{ or } W = ZQ, \text{ or } W = ZIt$$

($\because$ Charge = current × time), where Z is a constant of the ions and is called electrochemical equivalent.

If $Q = 1$ C, then $W = Z$ kg. Thus E.C.E. is the mass of ions in kg liberated by 1 coulomb of electricity.

Unit, kilogramme per coulomb (kg C^{-1}).

(c) Faraday : To liberate 1 gm equivalent of any substance, a fixed amount of electricity is required. That fixed amount of electricity is called *a faraday*. A faraday is nearly 96,540 C per gm equivalent or 96540000 C per kg equivalent.

(d) Relation of chemical equivalent and electrochemical equivalent : The ratio of chemical equivalent to electrochemical equivalent of a substance is a universal constant and that constant is a Faraday.

i.e. $$\frac{E_1}{Z_1} = \frac{E_2}{Z_2} = \frac{E_3}{Z_3} = \dots = 96500000 \text{ C per kg equivalent.}$$

EXAMPLES

Heating effect :

Ex. 1. *In a hostel there are one hundred* 40-*watt lamps, ten* 60-*watt fans and a* 1000-*watt electric oven. If all the appliances are used for* 5 *hours daily, find the electric bill in a month of* 30 *days if the cost of* 1 B. O. T. *unit is* 30 *paise.*

Sol. Energy consumed $= \frac{(100\times 40+10\times 60+1000)\times 30\times 5}{1000}$ B. O. T. units

$$=\frac{5600\times 150}{1000}=56\times 15 \text{ units.}$$

$\therefore$ Bill $=$ Rs. $\frac{56\times 15\times 30}{100}=$ Rs. 252. **Ans.**

Ex. 2. *A* 220 *V*-500 *watt aluminium electric kettle weighing* 1·2 *kg is used to boil water from a* 180 *volt supply. How long will it take to bring* 1 *kg of water from* 30°C to 100°C ? (*Relative heat capacity of aluminium* = ·3.)

Sol. A '220 V-500 watt' means that kettle will consume 500 watt only when voltage supply is exactly 220 V. When voltage will change power consumed will also change. *The thing that will not change appreciably is its resistance.* We have,

$$P=\frac{V^2}{R}, \text{ or } R=\frac{V^2}{P}=\frac{220^2}{500}=96\cdot 8\ \Omega.$$

When we put it on 180 volt supply,

$$P=\frac{180^2}{96\cdot 8} \text{ joule per second.}$$

Let t be the time required.

Then, heat produced $=\frac{180^2}{96\cdot 8}\times t$ joule.

Heat absorbed by aluminium and water

$=1\cdot 2\times\cdot 3\times 4200\times(100-30)+1\times 4200(100-30)$

$=1\cdot 36\times 4200\times 70$

($\because$ Heat capacity of water $=4200$ J kg^{-1} K^{-1})

$$\therefore \quad 1\cdot 36\times 4200\times 70=\frac{180^2}{96\cdot 8}\times t$$

or $$t=\frac{1\cdot 36\times 4200\times 70\times 96\cdot 8}{180^2}=1194\text{ s}=19\text{ min }54\text{ s.}\quad \textbf{Ans.}$$

Ex. 3. *Compare the amounts of heat developed in three wires having lengths in the ratio* 1 : 5 : 8 *and radii* 1 : 2 : 3 *when joined in parallel.*

Sol. We have, $H=\frac{V^2}{R}t$ and $R=\rho.\frac{l}{\pi r^2}$.

$$\therefore \quad H=\frac{V^2}{\rho.l/\pi r^2}.\,t=\frac{V^2\pi r^2}{\rho l}t.$$

Here V, π, t and ρ are same for all.

$\therefore\ H\propto r^2/l.\quad \therefore\quad H_1:H_2:H_3=1^2/1:2^2/5:3^2/8$

$=1:\frac{4}{5}:\frac{9}{8}:=40:32:45.$ **Ans.**

Ex. 4. *A* 9000 Ω *voltmeter is connected across a resistor R and an ammeter of* ·015 Ω in *series with it. The voltmeter reads* 117 *volt and ammeter reads* ·13 *A. Calculate the value of R and power input to R.*

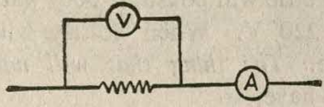


Fig. 6-1

Sol. Suppose R' = effective resistance of resistor and voltmeter.

Then $\frac{1}{R'}=\frac{1}{R}+\frac{1}{9000}=\frac{9000+R}{9000R}$

or $R'=\frac{9000R}{9000+R}$.

$$\therefore \quad \text{P.D. across voltmeter}=\frac{9000R}{9000+R}\times\cdot 13$$

$=117$ (given).

$$\therefore \quad \frac{9000R}{9000+R}=\frac{117}{\cdot 13}=900,\text{ or } R=1000\ \Omega,$$

Power input $=\frac{V^2}{R}=\frac{117^2}{1000}=13\cdot 7$ watt. **Ans.**

Chemical effect :

Ex. 5. *A metal plate weighing* 750 *gm is to be electroplated with* ·05% *of its weight of silver. If the current of* 0·8 *ampere is used, find the time needed for depositing the required weight of silver.* (*E. C. E. of silver is* $11\cdot 18\times 10^{-7}$ *kg* C^{-1}.)

Sol. We have, $W=ZIt.$

Here $W=\cdot75\times\dfrac{\cdot05}{100}=375\times10^{-6}$ kg;

$\therefore$ $375\times10^{-6}=11\cdot18\times10^{-7}\times\cdot8\times t$

or $t=407\cdot6$ s $=6$ m 47 s. **Ans.**

Ex. 6. *The electrochemical equivalent of silver is* $11\cdot18\times10^{-7}$ *kg* C^{-1}. *If the atomic weight of silver is* 108 *and Avogadro's number* $6\cdot06\times10^{23}$, *calculate the charge on silver ion in silver nitrate solution.*

Sol. Let us consider liberation of 1 gm equivalent of silver. One gm equivalent of silver is 108 gm. One gm mole of silver is also 108 gm because silver is monovalent. Let Q be the charge required to liberate 108 gm of silver.

Then $108=11\cdot18\times10^{-4}\,Q$ $\quad(\because\ Z=11\cdot18\times10^{-7}\text{ kg C}^{-1}$
$=11\cdot18\times10^{-4}\text{ gm C}^{-1})$

or $Q=9\cdot66\times10^{4}$ C.

This amount of charge is carried by $6\cdot06\times10^{23}$ ions of silver.

$\therefore$ Charge on a silver ion $=\dfrac{9\cdot66\times10^{4}}{6\cdot06\times10^{23}}=1\cdot594\times10^{-19}$ C. **Ans.**

Ex. 7. *If* 96,500 *C of electricity liberates* 1 *gm equivalent of any substance, what time will it take for a current of* ·15 *ampere to liberate* 20 *mgm of copper from a solution of copper sulphate ? (Atomic weight of copper* = 64.)

Sol. In copper sulphate salt copper is *divalent*.

$\therefore$ Chemical equivalent of copper $=\dfrac{64}{2}=32.$

$$\therefore \quad \frac{Q}{20\times10^{-3}}=\frac{96500}{32} \quad\text{or}\quad Q=60\cdot3\text{ C.}$$

Now $Q=It,\ \therefore\ t=\dfrac{60\cdot3}{\cdot15}=402$ s $=6$ m 42 s. **Ans.**

EXERCISES

(A)

Heating effect :

1. In a house there are ten 60-watt lamps, three 40 watt fluorescent tubes and three 40 watt fans. When all these are switched on what is the current drawn from the mains if the supply voltage is 220 V ? Calculate the power drawn and the cost of using them for 6 hours daily in a month of 30 days. (1 B. O. T. unit costs 30 paise.) (Ans. 3·82 A, 840 W and Rs. 45 36)

2. A 500 watt—220 V electric kettle is used to prepare 4 cups of tea How long will it take ? Assume that a cup contains 250 c.c. of water. Specific heat capacity of water $=4200$ J kg^{-1} K^{-1} and initial temperature of water $=20°C$. Also calculate the cost of preparing tea if 1 B. O. T. costs 35 paise.

(Ans. 11 min. 12 sec; 3·3 paise)

3. Show that heat developed in n wires each of resistance r joined in parallel in between two points at constant potential difference is n^2 times the heat developed in the same wires when they are joined in series between same two points.

4. Show that the power supply by a battery to an external resistance is maximum when it is equal to the internal resistance of the battery. Further show that this maximum power is $\frac{e^2}{4r}$ where e is the e. m. f. of the battery and r is internal resistance.

5. The average temperature coefficient of tungsten is $5{\cdot}1\times10^{-3}/°C$. The filament of an electric lamp has a cold resistance of 9·7 Ω at 20°C. When glowing fully well it has resistance of 121 Ω. Calculate its temperature. If it is a 100 watt lamp, what current does it consume ?

(Ans. 2499°C; 3·2 A when cold and ·91 A when hot)

6. Compare the heat developed in the four arms of a balanced Wheatstone bridge. The resistances of the arms are 10 Ω, 100 Ω, 40 Ω and 400 Ω. Also indicate in which arm minimum heat will be developed and in which the maximum.

(Ans. 4 : 40 : 1 : 10; maximum in 100 Ω and minimum in 40 Ω)

7. A wire of resistance 5 Ω is connected to a battery of e. m. f. 2 V and internal resistance 1Ω. In 2 minutes (a) how much energy is transferred from chemical to electrical form, (b) how much appear in the wire as Joule heat ? (c) Account for the difference between (a) and (b).

(Ans. 80 joule; 66·67 joule; the rest is lost as heat in internal resistance of the battery)

8. An electric kettle taking 3·5 amperes at 210 volts brings 1200 c.c. of water from 25°C to 95°C in 12 minutes. Find the percentage of the supplied energy which goes towards heating water. (Ans. 66·67%)

9. In a hostel 160 bulbs, having a resistance of 1200 ohm each are lighted on 220-volt supply everyday from 6 P. M. to 11 P.M. If electrical energy costs 25 paise per kilowatt hour, find the cost of lighting the hostel for a month of 30 days. (Ans. Rs. 242)

10. A 220 volt electric kettle takes 990 watts. Calculate (i) the resistance of the heating element, (ii the time required to melt and boil away 1 kg of ice assuming a heat loss of 40% by conduction and radiation.

(Ans 1·414 hour; 49 Ω)

11. Two batteries of e. m. f. 'e' and internal resistance r are connected in parallel across a resistor R. For what value of R is the power delivered to the resistor a maximum ? What is the maximum power ?

$\left(\text{Ans. } R=r/2;\ \text{max. power}=\frac{e^2}{2r}\right)$

12. A 660 watt electric heater is designed to operate from 120 volt lines. What is its resistance ? What current does it draw ? What is the rate of deve-

lopment of heat in calorie per second ? If the line voltage drops to 110 volt, what power does the heater take in watts ? (J=4·2 joule per calorie.)

(Ans. 22 Ω; 5·5 A, 157 cal/sec; 555 watts)

(B)

13. A 2 m long wire of resistance 4 ohms and radius 0·64 mm is coated with plastic insulation of thickness 0·06 mm. When a current of 5 amperes flows through the wire, find the temperature difference across the insulation in the steady state. Thermal conductivity of insulator=·15 $\text{Wm}^{-1}\ \text{K}^{-1}$.

(I. I. T. 1974) (Ans. 4·7°C)

[*Hint* : Q (rate of flow of heat across a thin co-axial cylindrical surface)

$$=\frac{\lambda(2\pi x l)\,d\theta}{dx}.$$

$$\therefore \quad d\theta=\frac{Q}{2\pi\lambda l}\cdot\frac{dx}{x}; \quad \therefore \quad \int_{\theta_1}^{\theta_2} d\theta=\frac{Q}{2\pi\lambda l}\int_{r_1}^{r_2}\frac{dx}{x}$$

or $$(\theta_2-\theta_1)=\frac{Q}{2\pi\lambda l}\cdot\log_e\frac{r_2}{r_1}.\Bigg]$$

14. A fuse made of lead wire has an area of cross-section 0·2 mm². On short circuiting, the current through the fuse wire reaches 32 A. How long after the short circuiting will the fuse begin to melt ? For lead, specific heat capacity $=134\ \text{J kg}^{-1}\ \text{K}^{-1}$; melting point=327°C; relative density=11·34; intial temperature=20°C; electrical resistivity$=11\times10^{-8}\ \Omega\text{m}$.

(I.I.T. 1976)

(Ans. ·165 second)

15. An electric tea-kettle has two windings. When one of them is switched on the kettle begins to boil in t_1 minutes, and when the other is switched on it begins to boil in t_2 minutes. In what time will the kettle begin to boil if both windings are switched on simultaneously in series and parallel ?

$$\left[\text{Ans. } (t_1+t_2);\ \frac{t_1t_2}{t_1+t_2}\right]$$

16. A storage battery with an e.m f. of 10 V and an internal resistance 1 Ω is connected across an external resistance R and liberates in it 9 watt power. Find the potential difference across the terminals of the battery and value or values of R.

(Ans. 9 V or 1 V; 9 Ω or 1/9 Ω)

Chemical effect :

17. A copper voltameter is connected in series with a battery and 1 Ω coil. The current is passed for an hour and the increase in weight of the cathode is 2·952 gm. The reading of the voltmeter connected across 1-ohm coil is 2·5 volt. Calculate e.c.e. of copper.

(Ans. 33×10^{-8} kg/coulomb)

18. A metal plate having a total surface area of 300 cm² is to be nickel plated. If a current of 1·5 A is used for 3 hours, find the thickness of nickel deposited. (Density of nickel$=8800\ \text{kgm}^{-3}$; E. C. E. of nickel$=30{\cdot}4\times10^{-8}\ \text{kg C}^{-1}$.)

(Ans. $1{\cdot}87\times10^{-5}$ m)

19. A copper voltameter is connected in series with a tangent galvanometer of 10 turns and 5 cm in radius. Calculate the weight of copper deposited in 30

minutes if the deflection of the galvanometer is 60°. (Horizontal component of earth's field$=28$ Am^{-1} and E.C.E. of copper$=32{\cdot}8\times10^{-8}$ kg C^{-1}.)

(**Ans.** $\cdot2863\times10^{-3}$ kg)

20. In a copper voltameter 20 mgm of copper is deposited on the cathode of the voltameter by ·15 A current in 6 minutes 42 seconds. Calculate (i) amount of electricity required to liberate one gm equivalent of any substance, (ii) charge carried by an electron. (Atomic weight of copper$=64$; Avogadro's constant $=6{\cdot}02\times10^{23}$ mol^{-1} and copper is divalent.)

[**Ans.** (i) 96500 C; (ii) 1.6×10^{-19}C]

21. A steady current is passed for 25 minutes through a silver voltameter and ammeter in series and ·559 gm of silver is deposited. The ammeter reads ·3 A. Find the error, if any, in the ammeter reading, given that the E.C.E of silver $=11{\cdot}18\times10^{-7}$ kgC^{-1}.

(**Ans.** The true current is ·333 A and therefore the error is ·033 A)

22. A current of 5A is passed for 5 hours through three voltameters arranged in series, containing solutions of copper sulphate, silver nitrate and sulphuric acid. Calculate the masses of silver, copper and hydrogen liberated. (E.C.E of silver$=11{\cdot}18\times10^{-7}$ kg C^{-1}, atomic weight of silver$=107{\cdot}88$, atomic weight of copper$=63$ 57 and atomic weight of hydrogen$=1{\cdot}008$.)

(**Ans.** $100{\cdot}62\times10^{-3}$; $29{\cdot}65\times10^{-3}$ and $\cdot94\times10^{-3}$ kg respectively)

23. A current passes through copper voltameter and water voltameter in series. How much hydrogen at 27°C and pressure of 100 cm of mercury will be liberated during the time 17·5 gm of copper will be deposited ? (Atomic weight of copper$=63$ and density of hydrogen at S.T.P.$=\cdot09$ kgm^{-3}.)

(**Ans.** $5{\cdot}15\times10^{-3}$ m^3)

24. When a detonating gas explodes 34·5 kcal are liberated per gm of reacted hydrogen. Use this observation to find the minimum e.m.f. of a battery at which the electrolysis of water is possible. (A faraday$=96{,}500$ C.) (**Ans.** 1·5 V)

* * *

CHAPTER 7

ELECTROMAGNETIC INDUCTION

1. Magnetic flux : Magnetic flux through an area $\triangle S$ is defined as the dot product of the area vector and magnetic B-vector.

That is, magnetic flux through $\triangle S = \vec{B} \cdot \triangle\vec{S}$
or $B \triangle S \cos\theta$ weber (wb)

and magnetic flux through an extended area

$= \sum \vec{B} . \triangle\vec{S}$ where $\sum$ extends over all the elementary areas.

2. Electromagnetic induction : Faraday discovered incidentally that whenever there is a charge of magnetic flux through a closed coil, a momentary current is produced in the coil so long as the change of magnetic flux takes place. This phenomenon is called electromagnetic induction.

3. Faraday's law of electromagnetic induction : In a compact form Faraday's law of electromagnetic induction states that

$$e = -N \frac{d\varphi_B}{dt} \qquad .. \quad (1)$$

where N = number of turns, φ_B = instantaneous flux through the coil at time t, e = induced e.m.f.

4. Induced e.m.f. in moving straight conductor : When a straight conductor of length l is moved at right angles to a uniform magnetic field B with velocity v, the induced e.m.f. between its ends is given by

$$e = Blv \text{ volt (V).} \qquad .. \quad (2)$$

The direction of e is given by Fleming's right hand rule.

5. Induced e.m.f in a rotating coil : If a coil of area A is rotated with uniform velocity in a uniform magnetic field B about an axis perpendicular to the magnetic field, the induced e.m.f. in the coil is given by

$$e = NAB\,\omega \sin\omega t \text{ volt where } \omega = \text{angular velocity} \qquad .. \quad (3)$$

or $\quad e = e_0 \sin \omega t \qquad .. \quad (3a)$

where $\quad e_0 = NAB\omega$ is the maximum e.m.f. induced.

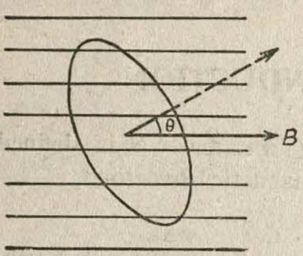


Fig. 7-1

6. Flux through a coil : Let the normal to a coil of area S make an angle θ with the magnetic field B. Then φ_B (flux through coil)

$$=\Sigma\vec{B}.\triangle\vec{S}=\Sigma B\triangle S\cos\theta$$

$$=B\cos\theta\Sigma\triangle S=B\cos\theta.S$$

where S = total area of the coil.

$$\therefore\quad \varphi_B=BS\cos\theta \text{ weber (wb)}. \quad ..\ (4)$$

7. Self induction : When current through a coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current is produced in the coil itself. This phenomenon is called self induction.

According to the law of electromagnetic induction e (self induced e.m.f.) $=-N\frac{d\varphi_B}{dt}$. But φ_B (instantaneous flux) $\propto i$ (instantaneous current through coil).

$$\therefore\qquad \frac{d\varphi_B}{dt}\propto\frac{di}{dt} \text{ and } e\propto\frac{di}{dt}$$

or $e=-L\frac{di}{dt}$, where L is a constant of the coil and is called coefficient of self induction. Its unit is henry (H). Also see Ex. 5.

8. (i) L of a circular coil is : $L=\frac{1}{2}\pi\mu_0\mu_r N^2 a$ henry (H) where a = radius of the coil and N = number of turns.

(ii) L of a solenoid is : $L=\frac{\mu_0\mu_r N^2 A}{l}$ henry (H) where l = length of the solenoid and A = area of cross-section of the solenoid.

9. Mutual induction : When two coils are placed side by side or one above the other, on changing current in one (primary coil), current is induced in the other (secondary coil). This phenomenon is called Mutual induction. The e.m.f. in the secondary coil is given by

$$e_s=-N_s\frac{d\varphi_B}{dt}. \text{ But } \varphi_B\propto i_p.$$

$$\therefore\qquad e_s\propto-\frac{di_p}{dt}, \text{ or } e_s=-M\frac{di_p}{dt}$$

when M is a constant for the coils and is called coefficient of mutual induction. It depends on the distance between the coils, their number of turns, area of cross-section and the medium in their cores. Its unit is also henry (H).

10. Mutual inductance between two similar parallel but co-axial coils :

$$M=\frac{\mu_0\mu_r\pi N^2a^4}{2(a^2+x^2)^{3/2}}\text{ H.}$$

When they are placed in touch :

$$M=\tfrac{1}{2}\mu_0\mu_r\pi N^2a\text{ H.}$$

Mutual inductance between two co-axial solenoids :

$$M=\frac{\mu_0\mu_r N_1N_2S}{l}\text{ H}$$

where N_1 = number of turns of the primary coil,
N_2 = number of turns of the secondary coil,
S = area of cross-section of the secondary coil,
l = length of the primary coil.

EXAMPLES

Ex. 1. *The axles of the carriage of a train travelling at 72 km per hour are* 1·6 m *long. Find the difference in potential at their ends if total intensity of the earth's field is* ·5 × 10⁻⁴ *tesla and angle of dip is* 60°. (Similar I.I.T. 1981)

Sol. As the train moves its axle moves at *right angles* to the vertical component of the earth's field.

$$B_v=B\sin\delta=\cdot5\times10^{-4}\sin60^\circ=\frac{\cdot5\sqrt{3}}{2}\times10^{-4}\text{ tesla.}$$

Now $e=Blv$ volt.

$$\therefore\quad e=\frac{\cdot5\sqrt{3}\times10^{-4}}{2}\times1\cdot6\times\frac{72000}{3600}$$

$$=\cdot5\times1\cdot732\times\cdot8\times20\times10^{-4}=1\cdot385\times10^{-3}\text{ V.}$$ **Ans.**

Ex. 2. *A copper disc of radius* 10 cm *rotates* 20 *times per second with its axis parallel to a uniform magnetic field of* ·05 *tesla. Calculate the induced e.m.f. between the centre and the edge of the disc.*

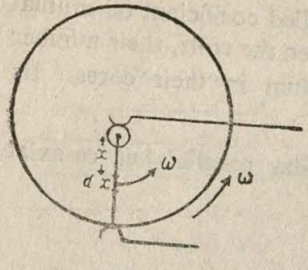


Fig. 7-2

Sol. Consider an element dx at a distance x from the centre. This element is moving with speed $v=\omega x$.

$\therefore$ Induced e.m.f. across dx
$=B(dx)v=Bdx\omega x=B\omega xdx;$

$\therefore$ e.m.f. between the centre and the edge of the disc $=\int_0^r B\omega xdx=\frac{B\omega r^2}{2}.$

Here $B=\cdot05$ T; $\omega=2\pi n=2\pi\times20=40\pi$ and $r=10$ cm $=\cdot1$ m.

$$e=\frac{\cdot05\times40\pi\times\cdot1^2}{2}=\cdot0314 \text{ volt.}$$ **Ans.**

Ex. 3. *Calculate the maximum e.m.f. induced in a coil of* 100 *turns and* ·01 m² *in area rotating at the rate of* 50 *revolutions per second about an axis perpendicular to a uniform magnetic field of* ·05 *tesla. If resistance of the coil is* 30 Ω, *what is the maximum power generated by it ?*

Sol. We have, $e_0=NAB\omega=100\times\cdot01\times\cdot05\times2\pi50$

($\because$ $\omega=2\pi f$ where $f=$ frequency of revolution)

or $e_0=15\cdot7$ volt.

Current produced $=\frac{15\cdot7}{30}=\cdot524$ ampere.

Power generated $=15\cdot7\times\cdot524=8\cdot23$ watt. **Ans.**

Ex. 4. *Calculate the self inductance of a solenoid (iron-cored) of length* 30 cm *comprising of* 100 *turns and radius* 5 cm (μ_r *of iron* $=500$).

Sol. We have, $L=\frac{\mu_0\mu_r N^2 A}{l}$ henry.

$$\therefore \quad L=\frac{4\pi\times10^{-7}\times500\times100^2\times\pi\times\cdot05^2}{\cdot3} \text{ henry}$$

$$=\frac{4\pi^2\times125}{3}\times10^{-4}=16\cdot08\times10^{-4}=1\cdot61 \text{ m H.}$$ **Ans.**

Ex. 5. *Show that coefficient of self induction is also magnetic flux linked with unit current.*

Sol. We have, $e=-N\frac{d\varphi_B}{dt}$, by law of electromagnetic induction.

Also $e=-L\dfrac{di}{dt}$, formula through which L is introduced.

∴ $$N\frac{d\varphi_B}{dt}=L\frac{di}{dt}$$

or $\int N d\varphi_B=\int L\,di$, or $N\varphi_B=Li+$a constant.

When $i=0$, $\varphi_B=0$; ∴ constant$=0$.

∴ $N\varphi_B=Li$, or $L=\dfrac{N\varphi_B}{i}$.

Put $i=1$, then $L=N\varphi_B=\varphi_B$, the total flux linked with the coil.

Thus *coefficient of self induction is the flux linked with the coil per unit current.* **Proved.**

EXERCISES

(A)

1. A circular loop of radius 10 cm and 500 turns is turned upside down on a horizontal table in ·5 s. Calculate the mean e.m.f generated. (Earth's vertical field$=\cdot 43\times10^{-4}$ tesla) (Ans. 2·69 mV)

2. A copper disc of radius 20 cm makes 1200 revolutions per minute about its axis which is parallel to a uniform magnetic field of ·01 tesla. Find the potential difference between the centre and the edge of the disc. (Ans. 25·14 mV)

3. Find the difference of potential between the ends of a horizontal rod 1 m long pointing east and west, two second after it has fallen from rest. (Horizontal intensity of earth's field$=2\times10^{-5}$ tesla) (Ans. $3\cdot92\times10^{-4}$ volt)

4. A rivulet 10 m long is flowing northward along an insulated bed with a velocity of $\cdot 3$ ms^{-1}. Calculate the potential difference between the water at the sides of the rivulet. ($B_0=34\times10^{-6}$ tesla and dip$=60°$) (Ans. ·177 mV)

5. Calculate the inductance of a coil of 100 turns of wire wound on an iron ring of radius 10 cm and 10 cm^2 in cross-section, the relative permeability of iron being 700. (Ans. ·014 henry)

6. An all metal aeroplane flies horizontally at 600 km per hour at a place where vertical intensity is 4×10^{-5} tesla. If the wing span is 10 m, what will be the resulting P.D. between the tips of wing? (Ans. ·067 V)

7. A field of ·2 tesla acts at right angles to a coil of area 100 cm^2 with 50 turns. The coil is removed from the field in ·1 s. Find the e.m.f. induced. (Ans. 1 volt)

8. A millivoltmeter is connected between the rails of a track. Calculate the voltmeter reading when a train passes at 60 km per hour. The vertical component of the earth's field is 2×10^{-5} tesla and the distance between the rails is 1·5 m. (Ans. ·5 milli volt)

9. Calculate the coefficient of self induction of solenoid having 500 turns and a length 1 m. The area of cross-section is 7 cm^2 and permeability of the core is 1000. (Ans. ·22 H)

10. Calculate the coefficient of self induction of a coil of 500 turns when a current of 1·25 ampere produces a magnetic flux of one microweber.

[*Hint* : See Ex. 5] (Ans. ·4 mH)

11. A rectangular conductor of area ·2 m^2 is placed in a uniform magnetic field having B-vector strength 2 T with its normal at an angle 30°. Calculate the magnetic flux linked with the conductor. (Ans. ·35 wb)

12. The self inductance of a closely wound coil of 100 turns is 5 mH. What is the flux through the coil when the current in it is 10 mA ?

[*Hint* : See Ex. 5] (Ans. 5×10^{-5} wb)

(B)

13. A solenoid has a core of 2×10^{-3} m^2 cross-section, half air and half iron ($\mu_r=500$). The length of the solenoid is 2 m. If the number of turns is 1000, calculate its coefficient of self induction. (Ans. ·315 H)

14. A solenoid 50 cm long and 8 cm in diameter is wound with 500 turns. Another coil of 20 turns of insulated wire is closely wound over it at its mid-region. Calculate the coefficient of mutual induction.

[*Hint* : Use the formula $M=\frac{\mu_0\mu_r N_1 N_2 S}{l}$ henry] (Ans. ·126 mH)

15. A metal wire of mass m slides without friction on two horizontal rails l distance apart. The track is in a vertical uniform magnetic field of induction B. A battery of constant e.m.f. ε is connected to the rails. What is the terminal speed of the slider ? $\left(\text{Ans. } \frac{\varepsilon}{Bl}\right)$

16. A stiff wire bent into a semicircle of radius R is rotated with angular speed ω in a uniform filed of induction B about its diameter perpendicular to the field. Calculate the maximum e.m.f. developed. (Ans. $\frac{1}{2}\pi BR^2\omega$)

17. A copper bar of mass m slides under gravity on two smooth parallel rails 'l' distance apart and set at an angle α to the horizontal. At the top the rails are joined by a resistor R. Calculate the steady velocity of the bar when there is a uniform magnetic field B perpendicular to the plane of the rails. $\left(\text{Ans. } \frac{mgR\sin\alpha}{B^2l^2}\right)$

18. A copper bar of mass m rest at right angles to two parallel horizontal rails 'l' distance apart. The rails are connected by a resistor R at one end and kept open at the other ends. There is a uniform upward magnetic field of induction B. The bar is pulled away from the closed end by a constant force F. Calculate the terminal velocity of the bar when μ is the coefficient of friction etween rails and the bar. $\left(\text{Ans. } \frac{(F-\mu mg)R}{B^2l^2}\right)$

□□□

CHAPTER 8

A. C. DYNAMO : D. C. DYNAMO : D. C. MOTOR : A. C. CIRCUITS : TRANSFORMER

1. A. C. Dynamo : Principle : When a coil is rotated at a constant speed about an axis perpendicular to a magnetic field, there is continuous change of magnetic flux. Following this change of flux, a continuous e. m. f. is produced in the coil.

When a coil of area A and having N turns is rotated uniformly with angular velocity ω in a magnetic field B about an axis perpendicular to the field the e. m. f. induced is given by

$$e = NAB\omega \sin\omega t$$

or $$e = e_0 \sin\omega t$$

where $e_0 = NAB\omega$ is the maximum induced e. m. f. Thus we see that the e. m. f. induced in a rotating coil changes harmonically. The corresponding current will also change harmonically but not necessarily in the same phase. So we can write for current, $i = i_0 \sin(\omega t + \alpha)$; i_0 is the maximum value of current and α is the phase difference between the current and the e. m. f.

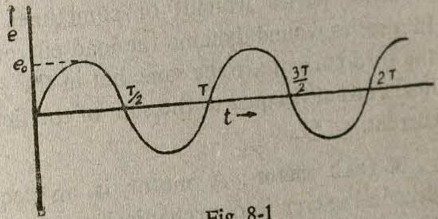


Fig. 8-1

Due to rotation of the coil in the magnetic field, it experiences a magnetic torque which tends to stop the motion of the coil. Hence an external agent must apply a torque to just overcome this torque and maintain rotation of the coil in the magnetic field. This is how a dynamo converts energy of the external agent into electrical energy. The instantaneous torque is given by

$$\tau = iNAB\sin\omega t$$

where i is the instantaneous current through the armature coil.

2. D.C. Dynamo : A D.C. dynamo consists of several coils wound on a laminated iron cylinder called 'armature'. The induced e.m.fs in the coils are rectified by half rings and are collected by common collectors. The e.m.f. across the terminals of the dynamo is fairly constant. Due to rotation of the armature coils in the magnetic field, the armature experiences a torque which tends to oppose the motion of the armature. Hence an external agent must apply a torque to overcome this magnetic torque and the torque due to friction at the collectors. If E is the steady emf generated and I_A is the armature current, then EI_A is the electrical power developed. If τ is the torque and ω is the constant speed of the armature then mechanical power expended in running the armature is $\tau\omega$.

In an ideal dynamo, $\tau\omega = EI_A$

and $E = kI_F f$ where k is a constant, I_F = field current and f = frequency of revolution of the armature.

The field magnet is generally electromagnet excited by the dynamo itself. The exciting coil of the electromagnet is sometimes connected in series (series wound) or sometimes in parallel (shunt wound). In a series wound dynamo the load current, the armature current and the field current are the same but in the shunt wound dynamo the load current is equal to the sum of the field current and the armature current.

3. D.C. motor : A motor is an electrical appliance to convert electrical energy into mechanical energy. The construction of a DC motor is the same as the DC dynamo. When a DC voltage is applied across its terminals, the armature experiences a magnetic torque due to the fact that a current-carrying loop placed in a magnetic field experiences a torque. A motor is used to drive some machinery. When it is coupled to some machinery, it is said to be loaded. The machinery applies a mechanical torque tending to stop it. So the motor does some work. Thus in an ideal motor

$$\tau_{\text{applied}} = \tau_{\text{magnetic}}.$$

Due to the motion of armature coils in the magnetic field an emf is induced which tends to oppose the armature. This is called the back emf. The back emf depends on the speed of the armature and the flux density of the magnetic field. If E is the applied emf, I_A is the armature current and e is the back emf, then

$$I_A = \frac{E - e}{R_A}$$

where R_A is the armature resistance of the motor.

4. Average value *or* **Mean value** : The average or mean value of instantaneous currents over a half cycle of change i.e. from 0 to $T/2$ or $T/2$ to T is termed as average or mean value of the current.

$$\bar{i} = \frac{\int_0^{T/2} i\,dt}{T/2} = \frac{2}{T}\int_0^{T/2} i_0 \sin\omega t\,dt = \frac{2i_0}{T}\cdot\frac{T}{\pi}.$$

$$\therefore \quad \bar{i} = \frac{2}{\pi} i_0.$$

Similarly, $$\bar{e} = \frac{2}{\pi} e_0.$$

5. Root mean square *or* **Virtual** *or* **Effective value** : The square root of the mean of the squares of instantaneous currents is called root mean square (r. m. s.) value or virtual or effective value of current.

$$\overline{i^2} = \frac{\int_0^{T} i^2 dt}{T} = \frac{i_0^2}{T}\int_0^{T} \sin^2\omega t\,dt = \frac{i_0^2}{T}\cdot\frac{T}{2}.$$

$$i_{r\cdot m\cdot s\cdot} = \sqrt{\overline{i^2}} = \frac{i_0}{\sqrt{2}}.$$

Similarly, $$e_{r\cdot m\cdot s\cdot} = \frac{e_0}{\sqrt{2}}.$$

6. A. C. Circuits : (i) *Circuit with resistance only* :

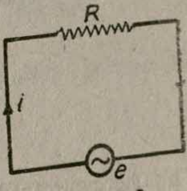


Fig. 8-2

If e = instantaneous e.m.f.

i = instantaneous current

then, $i = \frac{e}{R}$, or $e = iR$.

Let us assume $i = i_0 \sin\omega t$.

$\therefore \quad e = i_0 R \sin\omega t = e_0 \sin\omega t$

(where $e_0 = i_0 R$).

Thus current and e. m. f. change in phase.

(ii) *Circuit containing inductance only* :

Here an e. m. f. is induced in the coil.

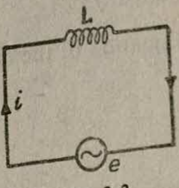


Fig. 8-3

$$e_L = -L\frac{di}{dt}.$$

$$\therefore \quad e - L\frac{di}{dt} = 0.i = 0$$

or $$e = L\frac{di}{dt} = \omega L i_0 \cos\omega t$$

$$e = e_0 \sin(\omega t + \pi/2)$$

where $$e_0 = i_0 \omega L, \text{ or } i_0 = \frac{e_0}{\omega L}.$$

Thus current lags behind e. m. f. by $\pi/2$ or by a quarter cycle. We note here that though the coil has no pure resistance, it reacts to the flow of alternating current by developing an e. m. f. This amounts to opposition to all current. This is why the quantity 'ωL' is called **inductive reactance** and is expressed by $X_L = \omega L$ ohm.

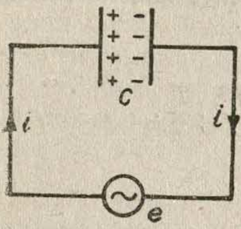


Fig. 8-4

(iii) *Circuit containing capacitor only* : Suppose e, i and q are instantaneous e. m. f., current and charge on the capacitor; p. d. across capacitor $= q/C$.

$\therefore$ $e - q/C = 0$ (by circuit equation)

or $e = q/C$.

Assuming $e = e_0 \sin\omega t$,

we have $$q = eC = e_0 C \sin\omega t.$$

$$\therefore \quad i = \frac{dq}{dt} = \frac{\cos\omega t}{e_0 \omega C} = i_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where } e_0 = \frac{i_0}{\omega C}$$

or $$i_0 = \frac{e_0}{\left(\frac{1}{\omega C}\right)}.$$

Thus here current leads the e.m.f. by $\pi/2$ and $X_C = \frac{1}{\omega C}$ ohm.

(iv) *Circuit containing resistance* (R) *and inductance* (L) (i.e. an L-R circuit) :

Equation of the circuit is

$$e - L\frac{di}{dt} = Ri, \text{ or } e = Ri + L\frac{di}{dt}.$$

Assuming $i = i_0 \sin\omega t$ and hence taking the value of i and $\frac{di}{dt}$ and

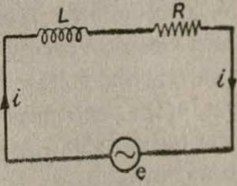


Fig. 8-5

putting them in equation of circuit we have $e = e_0 \sin(\omega t + \alpha)$

where $e_0 = i_0\sqrt{R^2 + \omega^2 L^2}$

and $\tan\alpha = \dfrac{\omega L}{R}$.

Thus in L-R circuit the current lags behind the e. m. f. by $\alpha = \tan^{-1} \omega L/R$ and the effective opposition to the flow of current is $\sqrt{R^2 + \omega^2 L^2}$. This quantity is called Impedance. It is denoted by Z. Thus **impedance** (Z) of L-R circuit is given by :

$$Z = \sqrt{R^2 + \omega^2 L^2} \text{ ohm } (\Omega).$$

(v) *Circuit containing resistance R, inductance L and capacitance C.* (i.e. L-C-R circuit) :

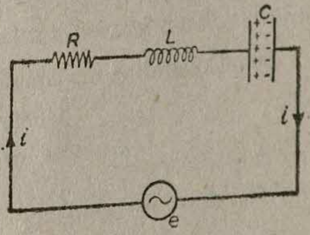


Fig. 8-6

Equation of the circuit is :

$$e - L\frac{di}{dt} - \frac{q}{C} = Ri$$

or $e = L\dfrac{di}{dt} + Ri + \dfrac{q}{C}$.

Assuming $i = i_0 \sin\omega t$ and hence taking the value of i, $\dfrac{di}{dt}$ and $q = \int i dt$ and putting them in circuit equation we have $e = e_0 \sin(\omega t + \alpha)$

where $e_0 = i_0 \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

and $\tan\alpha = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$.

$\therefore$ Z (Impedance of L-C-R circuit) $= \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}\ \Omega$

and X (Reactance of L-C-R circuit) $= \left(\omega L - \dfrac{1}{\omega C}\right) \Omega$.

6. Transformer : Transformer is an electrical appliance to transfer electrical energy from one circuit to another circuit from one current and voltage level to another current and voltage level. It works on the phenomenon of mutual induction. In an ideal transformer (using suffix 1 for the primary coil and 2 for the secondary coil) we have

$$\frac{E_2}{E_1}=\frac{N_2}{N_1}=n, \text{ called transformation ratio.}$$

Also $E_1I_1=E_2I_2$ i.e., Power in the primary coil = Power in the secondary coil.

EXAMPLES

A. C. and A. C. Circuits :

Ex. 1. *The equation of an alternating e. m. f. is* $e=50\sin 100\pi t$. *Calculate* (i) *peak value of the e. m. f.*, (ii) *the mean value of the e. m. f.*, (iii) *r. m. s. value of the e. m. f.* (iv) *What is the interval at which it attains maximum value ?*

Sol. Comparing the equation with general formula $e=e_0\sin\omega t$,

we have $e_0=50$ volt. **Ans.**

$\omega=100\pi$, or $2\pi f=100\pi$, or $f=50$.

We know $\bar{e}=\frac{2}{\pi}.e_0$. $\therefore$ $\bar{e}=\frac{2}{\pi}.50=\frac{100}{\pi}=31{\cdot}8$ V. **Ans.**

We have, $e_{r \cdot m \cdot s \cdot}=\frac{e_0}{\sqrt{2}}$; $\therefore$ $e_{r \cdot m \cdot s \cdot}=\frac{50}{\sqrt{2}}=\frac{50\sqrt{2}}{2}=35{\cdot}35$ V. **Ans.**

The interval between two successive maxima is $T/2$.

$\therefore$ Required interval $=\frac{T}{2}=\frac{1}{2}\left(\frac{1}{f}\right)$ $\left(\because\ T=\frac{1}{f}\right)$

$$=\frac{1}{2f}=\frac{1}{2\times 50}={\cdot}01 \text{ second.}$$ **Ans.**

Ex. 2. *A coil of* 20 Ω *resistance and* ·1 *henry inductance is connected to* 220 *volt* 50 *cycle mains. What is the impedance of the coil ? What current does it draw ? Does the current lead or lag and by how much ?*

Sol. We have, Z (impedance of L-R loop)

$$=\sqrt{R^2+\omega^2L^2}=\sqrt{20^2+4\pi^2.50^2\times\cdot 1^2}=\sqrt{400+987}$$

$$=\sqrt{1387}=37\cdot 24 \text{ ohm.}$$ **Ans.**

$$\text{Current}=\frac{220}{37\cdot 24}=5\cdot 9 \text{ A.}$$ **Ans.**

Current lags by angle $\alpha=\tan^{-1}\dfrac{\omega L}{R}=\tan^{-1}\dfrac{2\pi.50\times\cdot 1}{20}$

$$=\tan^{-1}1\cdot 57=57°30'.$$ **Ans.**

Ex. 3. *Calculate the resistance required which must be connected in series with an inductance of ·2H in order that the phase difference between the current and the e. m. f. may be 45° when the frequency is 50 Hz.*

Sol. We have, in an L-R circuit the phase difference between the current and the e. m. f. is given by $\tan\varphi=\dfrac{\omega L}{R}$.

Here $\varphi=45°$,

$\therefore$ $\tan 45°=\dfrac{\omega L}{R}$, or $R=\omega L$

or $R=2\pi fL=2\pi\times 50\times 2=62\cdot 84\ \Omega$. **Ans.**

Ex. 4. *A 40 watt-110 V fluorescent tube is to be operated on 220 volt-50 Hz a. c. supply. Calculate the choke required for the purpose.*

Sol. We have, $P=VC$; $\therefore$ $C=\dfrac{P}{V}=\dfrac{40}{110}=\dfrac{4}{11}$ A.

Also, $P=\dfrac{V^2}{R}$; $\therefore$ $R=\dfrac{V^2}{P}=\dfrac{110^2}{40}=302\cdot 5\ \Omega$.

When put on 220 volt supply, suppose X is the reactance of the choke required. *A choke is a purely inductive coil.*

$\therefore$ $Z=\sqrt{302\cdot 5^2+X^2}$.

Now, $\dfrac{4}{11}=\dfrac{220}{Z}$, or $\dfrac{16}{121}=\dfrac{220^2}{302\cdot 5^2+X^2}$

or $X^2+302\cdot 5^2=\dfrac{220^2\times 121}{16}$, or $X^2+91506=366025$

or $X^2=274519$, or $X=\sqrt{274519}=524\ \Omega$.

Now, $X=\omega L,\quad \therefore\quad L=\frac{X}{\omega}=\frac{524}{2\pi\times 50}=1{\cdot}67$ henry (H).

A choke of 1·67 H is required for the purpose. **Ans.**

DC Dynamo and Motor :

Ex. 5. *A DC dynamo armature is driven at* 1200 *rpm (revolutions per minute). With no-load, the torque to drive the armature is only* 5 *Nm; with an out-put current of* 20 *A, the torque required is* 45 *Nm. Estimate the generated emf.*

Sol. When there is no load, the armature current is zero. Hence there is no magnetic torque experienced by it. Now voltage is developed but *not any electrical power.* Under such circumstances the external agent has to overcome the torque due to friction only.

$$\therefore\qquad \tau_{friction}=5\text{ Nm}.$$

When there is load the external agent has to overcome the magnetic torque and the torque due to friction.

$$\tau_{magnetic}+\tau_{friction}=45\quad\text{or}\quad \tau_{mag}=40\text{ Nm}.$$

In a simple dynamo, $\tau_{magnetic}\times\omega=EI.$

$$\therefore\qquad 40\times\frac{2\pi\times 1200}{60}=E\times 20\quad\text{or}\quad E=251{\cdot}3\text{ V}.$$ **Ans.**

Ex. 6. *A series wound generator is rated as* 5 *kW*, 100*V*, 1000 *rpm. The resistance of the armature coil and the field coil is* ·05 Ω *If the speed is increased to* 1500 *rpm and the full-load power generated by it is* 8 *kW, find the terminal voltage and the full-load current.*

Sol. The above rating means that at full load its terminal voltage is 100 V at 1000 rpm and the power delivered by it is 5 kW. Using '*fl*' for full-load we have here $V_{fl}=100$ V.

$$\therefore\quad I_{fl}=\frac{5000}{100}=50\text{ A}.$$

By the circuit equation (dynamo treated as a cell)

$$E=V_{fl}+I_A R_A.$$

$$\therefore\quad E=100+50\times{\cdot}05=125\text{ V}.$$

When the speed is increased, $I'_{fl}=\frac{8000}{V'_{fl}}$ where V'_{fl} is the new terminal voltage.

$\therefore$ E' (the new e.m.f. generated)

$$=V'_{fl}+I'_{fl}\times\cdot5=V'_{fl}+\frac{4000}{V'_{fl}}. \quad \cdots \text{(i)}$$

The greater the e.m.f. generated the greater the flux linkage and the angular speed. But the flux is proportional to the field current (I_F).

$\therefore$ $E\propto I_F\times\omega$. That is, $\frac{E}{E'}=\frac{I_F\times\omega}{I'_F\times\omega'}$.

Since it is a series wound dynamo, $I_A=I_F=I_L$.

$$\therefore \quad E'=\frac{I'_L\times\omega'}{\omega}\times E=\frac{8000}{V'_{fl}}\times\frac{1500}{1000}\times125=\frac{240}{V'_{fl}}\times125. \quad \cdots \text{(ii)}$$

Thus from (i) and (ii), $V'_{fl}+\frac{4000}{V'_{fl}}=\frac{240}{V'_{fl}}\times125$

or $V'^2_{fl}=2600$ or $V'_{fl}=161$ volt **Ans.**

and $I'_{fl}=\frac{8000}{161}=49\cdot7$ A. **Ans.**

Ex. 7. *An electric dc motor whose armature is* $\cdot25\ \Omega$ *takes a current of* 10 *A from* 100 *V mains when running at* 1300 *rpm. Calculate* (a) *the power loss in the armature,* (b) *the power of the motor and* (c) *the available horse power (H.P.) when the load is increased so that the speed falls to* 1200 *rpm assuming that the field current remains constant.*

Sol. (a) Power loss in the armature $=I_A^2R_A=10^2\times\cdot25=25$ W. **Ans.**

(b) Power of the motor $=100\times10-25=975$ W. **Ans.**

(c) At 1300 rpm, $10=\frac{100-e}{\cdot25}$ where e is the back e.m.f. at 1300 rpm

or $e=97\cdot5$ V.

The back e.m.f. is proportional to the field current and the speed. Here field current remains constant (given) and hence the back e.m.f. is proportional to the speed only.

$\therefore$ The back emf at 1200 rpm $=\frac{1200}{1300}\times97\cdot5=90$ V.

$\therefore$ Armature current at 1200 rpm $=\frac{100-90}{\cdot25}=40$ A.

Power loss in the armature $=40^2\times\cdot25=400$ W.

$\therefore$ Power available $=100\times40-400=3600\text{ W}=\frac{3600}{746}=4\cdot8$ H.P. **Ans.**

Ex. 8. *A step-down transformer is designed to step down the power line voltage of 2200 V, 50 Hz down to 220 V for distribution to the houses. Calculate the primary current when the transformer delivers power to a load of only 2·4 Ω, assuming it to be an ideal one. What is its transformation ratio ?*

Sol. We have in an ideal transformer, $\frac{E_2}{E_1}=\frac{N_2}{N_1}$

and $$E_1I_1=E_2I_2.$$

$$\therefore \quad I_1=\frac{E_2}{E_1}\times I_2=\frac{220}{2200}\times\frac{220}{2\cdot5}=10\times 88=880 \text{ A.}$$ **Ans.**

$$n \text{ (transformation ratio)}=\frac{N_2}{N_1}=\frac{E_2}{E_1}=\frac{2200}{220}=10.$$ **Ans.**

EXERCISES

A. C. and A. C. Circuits :

1. The equation of an alternating current is $i=2 \sin 200\pi t$. Calculate (i) the peak value of current, (ii) the mean value of current. (iii) virtual current.
[**Ans.** (i) 2 A; (ii) 1·273 A; (iii) 1·414 A]

2. An electric lamp which runs at 100 V D.C. and 10 A current is connected to 200 V, 50 cycle A. C. mains. Calculate the inductance of the choke required.
[*Hint* : Consider choke as a pure inductor.] (**Ans.** ·055 H)

3. A coil of resistance 100 Ω and inductance ·2 H is connected across 50 Hz, 220 V supply. Calculate (i) reactance of coil, (ii) impedance, (iii) fraction of the total effective resistance (i.e. impedance) which is purely resistive and reactive. Why is the sum of the two percentage not zero ? [**Ans.** (i) 62·84 Ω; (ii) 118·1 Ω; (iii) 84·7%, 53·2%; because of the phase difference]

4. A 60 volt 10 watt lamp is to be run on 100 volts 60 cycle mains. Calculate the inductance of the choke coil that would serve the purpose. How much pure resistance would serve the same purpose ? (**Ans.** 1·273 H; 240 Ω)

5. The impedance of a choke-coil is 14 Ω and its reactance is 13 Ω. What is the resistance of the choke coil ? (**Ans.** 5·2 Ω)

6. An alternating e.m.f. of 100 volts and 50 cycles is applied to a circuit of inductance ·02 henry and resistance 4 Ω. Find the peak value of the alternating current. (**Ans.** 19 A)

7. An A. C. flourescent lamp needs 1 A current at 100 volts. Calculate the inductance of the choke required to run the lamp at 50 cycle 220 volt supply. (**Ans.** ·624 H)

8. A 60 volt 10 watt lamp is to be run on A.C. supply of 100 volt 50 cycle in series with a suitable capacitor. Find the capacitance of the capacitor that would be needed for the purpose. (**Ans.** 6·63 μF)

9. An alternating e.m.f. of 200 V, 50 cycles is applied to a circuit having a resistance 10 Ω, an inductance ·1 henry and a capacitance 1 farad, all in series. Calculate the value of the current and also its phase.

(Ans. 6·07 A; current lags by 72·3°)

10. Show that for certain value of C an L-C-R series circuit is nonreactive and calculate this value of C for the case $L=10$ mH, $R=100$ Ω and frequency $=1000$ per sec. (Ans. $\cdot 3\times10^{-4}$ F)

11. Show that in an L-C-R series circuit current is maximum and is in phase with the e.m f. when the frequency of the applied e.m.f is $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$.

12. A 600 Ω resistor is in series with a ·5 H inductor and a ·2 μF capacitor. Compute the impedance of the curcuit at 400 Hz. (Ans. 947 Ω)

13. The current from a half wave rectifier is given by $i=i_0 \sin\frac{2\pi t}{T}$ in the intervals $t=0$ to $t=T/2$, T to $3T/2$ etc. and during the intervals $t=T/2$ to T etc., the current is zero. Find (a) the average current during the interval from 0 to T. (b) the r. m. s. current during the same interval. [Ans. (a) $\frac{i_0}{\pi}$; (b) $\frac{i_0}{2}$]

DC dynamo and motor :

14. A dc generator is rated as 250 V, 100 A, 2000 rpm, armature resistance (R_A) is ·2 Ω.

(a) Calculate the no-load terminal voltage.

(b) Predict the full-load terminal voltage if the field current is decreased by 10%, assuming the armature current remaining constant.

[Ans. (a) 270 V, (b) 223 V]

15. A dc dynamo is 10 kW, 200 V and 50 A at 1000 rpm. If the armature resistance (R_A) is ·4 Ω, calculate the no-load terminal voltage at 1000 rpm and the full-load voltage at 800 rpm. Assume the field current to be constant.

(Ans. 220 V, 156 V)

16. A 20 kW, 250 V, 1200 rpm dc generator is running without load. The armature resistance of the generator is ·15 Ω. If the speed is increased to 1500 rpm and the field current reduced by 10%, calculate the terminal voltage on no-load. (Ans. 295 V)

17. A dc motor consists of 100 turns of 200 cm² cross-section. The flux density of the field is fixed at ·4 wb m^{-2} and the applied e.m.f. is 110 V. Find the back e.m.f at 1200 rpm. If the resistance of the motor circuit is 1 Ω, find the power delivered. (Ans. 100·5 V, 955 W)

[*Hint* : e (back e.m.f.)$=NAB\,\omega$.]

18. A dc series motor is rated as 220 V, 50 A at 1000 rpm at full-load. The resistance of the armature is ·4 Ω and that of the field coil in series with the armature coil is ·2 Ω. Calculate the torque at the rated speed. (Ans. 91 Nm)

[*Hint* : e (back e.m.f.)$=NAB\omega$ and $\tau=I_A ABN$.]

19. A dc motor is rated 10 kVA, 500 V, 30 A, 1000 rpm. The resistance of the armature is ·6 Ω. If the no-load armature current is 4 A, find the speed at no-load. What is the change in the back e.m.f. ? (Ans. 1032 rpm, 15·6 V)

Transformer :

20. The in-put of a power supply transformer (300 turns on primary) is to be 120 V at 50 Hz. There are two secondaries of it. One secondary provides 120 mA at 500 V and another 2 A at 6 V.

(a) Estimate the number of turns required on the secondaries. (b) Estimate the full-load primary current. [**Ans.** (a) 1250, 15; (b) ·6 A]

21. A step-down transformer is designed to step down the power line voltage of 2200 V for distribution to a certain locality at 220 V. Calculate the primary current when the full-load impedance of the locality is 4 Ω. Assume the transformer to be ideal. Also calculate the transformation ratio of the transformer.

(**Ans.** 5·5 A, ·1)

22. A transformer is to be designed to deliver 500 kW to a locality at 220 V. If power supply is at 2200 V, what is the transformation ratio of the transformer and what is primary current at full-load ? (**Ans.** 1/10, 227 A)

□□□

CHAPTER 1

INTRODUCTION TO MODERN PHYSICS

1. Motion of a charged particale in an electric field : When a charged particle enters into an electric field between two plates it experiences a force F given by, $\vec{F}=q\vec{E}$ where q = charge carried by the particle and E = intensity of the electric field.

That is, it experiences a force qE in the direction of the electric field *whatever be its initial velocity* and *whatever be the inclination of direction of its initial motion with the electric field.*

Force = mass × acceleration.

$$\therefore \qquad qE=m\times f, \qquad \text{or } f=\frac{qE}{m}$$

where m is the mass of particle. Hence to solve problems on motion of non-relativistic* charged particle in an electric field, recall all the equation of kinematics viz., (i) $v=u+ft$, (ii) $s=ut+\frac{1}{2}ft^2$, (iii) $v^2-u^2=2fs$.

2. Electron volt (eV) : When a charged particle moves in the electric field between two plates, it gains in kinetic energy. If V = potential difference between the plates then energy acquired by the charged particle = Vq.

$$\therefore \qquad Vq=\tfrac{1}{2}mv^2$$

$$\text{or} \qquad v=\sqrt{\frac{2Vq}{m}}.$$

When an electron moves down a p.d. of 1 volt, energy acquired by it is called **electron volt.**

Putting $V=1$ volt and $q=1{\cdot}6\times10^{-19}$ C we have,

$$\text{electron volt}=1\times1{\cdot}6\times10^{-19}\text{ J}=1{\cdot}6\times10^{-19}\text{ J}.$$

$$\text{Thus,} \qquad 1\text{ eV}=1{\cdot}6\times10^{-19}\text{ J}.$$

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*Particles moving with speeds which are small in comparison to the speed of light in a vacuum (c).

MeV (Million electron volt) $= 10^6$ eV $= 1{\cdot}6 \times 10^{-13}$ J

BeV (Billion electron volt) $= 10^9$ eV $= 1{\cdot}6 \times 10^{-10}$ J.

3. Motion of a charged particle in a magnetic field : When a charged particle enters into a magnetic field, it experiences a sideways force F given by $\vec{F} = q\,\vec{v} \times \vec{B}$ where v = velocity of charged particle and B is the magnetic field.

That is, it experiences a force $= qvB \sin\theta$ in a direction of a screw perpendicular to v and B and rotating from v to B through the smaller angle.

Since a charged particle experiences force at right angles to it its velocity, its motion in magnetic field is always **circular.** If r = radius of the track of the charged particle then we must have

$$Bqv = \frac{mv^2}{r} \quad \text{or} \quad \omega = \frac{Bq}{m} \quad (\because \quad v = \omega r)$$

or

$$T \text{ (time of revolution)} = \frac{2\pi m}{Bq} \quad \left(\because \quad \omega = \frac{2\pi}{T}\right).$$

Note. If the particle is projected at inclination θ with the field, then v in the above formula is the resolved part of the velocity perpendicular to the field. The motion of the particle is then along a 'helical path'. If v_0 is the velocity of the particle at inclination θ, the 'pitch' of the path $= v_0 \cos\theta \times T$.

EXAMPLES

Ex. 1. *Calculate the velocity with which electrons in a cathode ray tube strike the anode when* 30,000 V *is operating between the cathode and anode of the tube.* (*mass of electron* $= 9 \times 10^{-31}$ kg, *charge of electron* $= 1{\cdot}6 \times 10^{-19}$ C).

Sol. We have, $Vq = \frac{1}{2}mv^2$

or

$$v = \sqrt{\frac{2Vq}{m}} = \sqrt{\frac{2 \times 30000 \times 1{\cdot}6 \times 10^{-19}}{9 \times 10^{-31}}}$$

or

$$v = \sqrt{\frac{2 \times 3 \times 1{\cdot}6}{9}} \times 10^8 = 1{\cdot}033 \times 10^8 \text{ m/s.}$$ **Ans.**

Ex. 2. *An electron entering a magnetic field of* ·01 *tesla with a velocity of* 10^7 ms^{-1} *describes a circle of radius* ·6 cm. *Calculate* e/m *(specific charge) of the electron.*

Sol. We have, $Bev = \frac{mv^2}{r}$ or $\frac{e}{m} = \frac{v}{Br}$.

$$\therefore \quad \frac{e}{m} = \frac{10^7}{\cdot 01 \times \cdot 6 \times 10^{-2}}$$

$$= 1{\cdot}67 \times 10^{11} \text{ coulomb per kilogramme (Ckg}^{-1}\text{).} \qquad \textbf{Ans.}$$

Ex. 3. *An electron is accelerated in between two plates maintained at* 1000 *volt. What is the energy in eV acquired by the electrons ?* $(e = 1{\cdot}6 \times 10^{-19}$ C)

Sol. Energy acquired in the form of kinetic energy

$$= \text{Loss in electrical potential energy}$$

$$= Vq \text{ joule}$$

$$= 1000 \times 1{\cdot}6 \times 10^{-19} \text{ joule} \qquad (\because \ 1{\cdot}6 \times 10^{-19} \text{ J} = 1 \text{ eV})$$

$$= \frac{10000 \times 1{\cdot}6 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} \text{eV} = 1000 \text{ eV}. \qquad \textbf{Ans.}$$

EXERCISES

1. An electron falls through a potential difference of 1000 volt and gives up $1{\cdot}6 \times 10^{-16}$ joule of energy. What is the charge on the electron ? (Ans. $1{\cdot}6 \times 10^{-19}$ C)

2. The forces of cathode ray particles due to electric and magnetic fields which are at right angles to each other balance each other. If the magnetic field has an intensity ·1 tesla, what is the intensity of electric field ? The velocity of cathode ray particle is 10^6 ms^{-1}. What p.d. is required to produce this velocity ? (Ans. 10^5 Vm^{-1}; 2·8 volt)

3. An electron falls through a potential differenc of 50,000 V. What is the energy acquired by it ? (Ans. 5×10^4 eV)

4. An electron of energy 150 eV describes a circle in a magnetic field of ·1 tesla. Calculate the radius of the circle.. (Ans. $4{\cdot}1 \times 10^{-4}$ m)

5. An electron is moving with a velocity of 10^7 ms^{-1}. Find its energy in electron volts. (mass of electron$=9{\cdot}1 \times 10^{-31}$ kg and charge of electron $=1{\cdot}6 \times 10^{-19}$ C) (Ans. 284·4 eV)

6. A horizontal stream of charged particles is accelerated to velocity 3×10^7 ms^{-1} immediately before allowing them into an electric field between two horizontal plates separated by 2 cm and maintained at a p.d. of 100 V. The stream is deflected by 5 mm. Calculate e/m of the charged particles of the stream. (Length of plates$=10$ cm.)

[*Hint* : Electric field between plates=rate of change of potential

$$= \frac{\text{P.D. between plates}}{\text{Distance between plates}}.\Big]$$

(Ans. $1{\cdot}8 \times 10^{11}$ Ckg^{-1})

7. Electrons move at right angles to a magnetic field of ·015 tesla with velocity 6×10^7 ms^{-1}. Find the radius of the orbit.

(Ans. $2{\cdot}27\times10^{-2}$ m)

8. In a cathode ray tube electric and magnetic fields are simultaneously applied at right angles to each other and the intensity of magnetic field is so adjusted that the beam remains undeflected. Show that the velocity of cathode ray particles is given by $v=\frac{E}{B}$.

Calculate this velocity when $B=\cdot01$ tesla and $E=50{,}000$ volt per metre.

(Ans. 5×10^6 m/s)

9. A non-relativistic electron enters into a right-handed co-ordinate space with velocity 10^4 ms^{-1} at an angle 60° with the x-axis in the xy-plane. There is a uniform magnetic field of 50 mT along the y-axis. Find the pitch of the helical trajectory. What is the axis of the helix ? (Ans. 6·0 μm; y-axis)

10. A non-relativistic proton enters at right angles to a uniform magnetic field of 30 mT. What is the frequency of revolution of the particle if the field is sufficiently deep ?

(Ans. $4{\cdot}6\times10^5$)

11. A non-relativistic electron enters into a uniform magnetic field of 50 mT with speed 10^6 m/s at right angles to it and emerges from the field after ·1 nanosecond. What is the deviation produced by the field ? Show that it does not depend on the initial speed of the electron. (Ans. 51°)

□□□

CHAPTER 2

ATOMIC STRUCTURE

1. Atom : An atom is a miniature solar system with a central heavy mass called the nucleus and electrons revolving round the nucleus in different orbits.

The nucleus broadly consists of two types of particles—protons and neutrons. Protrons are positively charged particles and have mass nearly equal to mass of a hydrogen atom. Neutrons are electrically neutral and have mass nearly equal to mass of protons. Electrons are negatively charged particles having mass $\frac{1}{1840}$ of a proton.

The number of protons in the nucleus is the same as the number of electrons in the orbits. Each proton carries the same amount of positive charge as that carried by an electron. This is why atom as a whole appears electrically neutral. *The number of protons in the nucleus is called Atomic number* (Z) *and the total number of protons and neutrons* (*commonly named nucleons*) *is called Mass number* (A). Mass number of an atom is the nearest integer to the atomic weight of atom. Symbolically in atomic physics an atom is written as M_A where M is the usual chemical symbol of the atom.

Surrounding the nucleus electrons revolve in different orbits but all the orbits are not allowed. Electrons can revolve only in those orbits called stationary orbits or quantised orbits which satisfy the quantum condition, $I\omega = n\frac{h}{2\pi}$ where I = moment of inertia of electron, ω = angular velocity, h is a universal constant called Planck's constant and $n = 1, 2, 3, \ldots$ called principal quantum number. The energy of electrons in orbits is graded being lowest in the first and gradually increasing with increase of order of orbits. Thus electrons in different orbits will be in different energy states (or levels or shells). These energy states (or level or shells) are termed as $K, L, M, N, \ldots$ shells characterised by quantum number $n = 1, 2, 3, 4 \ldots$

Each shell has again several subshells (or levels) characterised by another quantum number called orbital quantum number l which can assume values $0, 1, 2, 3, \ldots\ldots n-1$ called $s, p, d, f, g \ldots\ldots$ states (or subshells or levels). In each subshell again there are several energy states characterised by a third quantum number called magnetic quantum number (m) which can assume values from $+l$ to $-l$. As electrons have two spins, each of these states will have two spin states. Thus for example in p subshell m can assume three values 1, 0, -1, i.e. there are three states and each of these three states have two spin states. Therefore in all there are six states in p subshell. These levels are termed as $p_1, p_2, p_3, \ldots, p_6$ states. How many electrons are there in a shell ? This is governed by a famous principle known as Pauli's exclusion principle. According to this principle *no two electrons can have all their quantum numbers identical.* Thus according to this principle the maximum number of electrons in an orbit is $2n^2$ and maximum number of electrons in a subshell is $2(2l+1)$. Filling of the shells and subshells takes place according to another principle—'*A system is stable when it is in the lowest energy level*'. The various energy states are graded in this way :

$$1s_1 < 1s_2 < 2s_1 < 2s_2 < 2p_1 < 2p_2 \ldots\ldots\ldots < 2p_6 < 3s_1 < 3s_2 < 3p_1 < 3p_2 \ldots\ldots < 3p_6 < 4s_1 < 4s_2 < 3d_1 < 3d_2 < 3d_3 \ldots\ldots < 3d_{10} < 4p_1 < 4p_2 \ldots\ldots < 4p_6 < 5s_1 < 5s_2 < 4d_1 < 4d_2 \ldots\ldots < 4d_{10}.$$

2. Energy of an Atom : From the consideration of dynamics of circular motion of an electron in an orbit of radius r_n,

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} = m\omega^2 r_n. \qquad \text{.. (i)}$$

From quantum condition we have

$$mr_n^2\omega = n.\frac{h}{2\pi}. \qquad \text{.. (ii)}$$

Eliminating ω between (i) and (ii) we have

$$r_n = \frac{\epsilon_0 n^2 h^2}{Ze^2\pi m} = \left(\frac{\epsilon_0 h^2}{\pi m e^2}\right) \times \frac{n^2}{Z} = r_0 \times \frac{n^2}{Z},$$

where $$r_0 = \frac{\epsilon_0 h^2}{\pi m e^4} = 5{\cdot}3 \times 10^{-11}.$$

The potential energy of the electron

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze(-e)}{r_n} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n}.$$

The kinetic energy of the electron $= \frac{1}{2}m(\omega r_n)^2$.

Adding the kinetic energy and the potential energy we have

$$E_n = -\frac{Z^2e^4m}{8\epsilon_0{}^2h^2n^2} = -\left(\frac{e^4m}{8\epsilon_0{}^2h^2}\right)\cdot\frac{Z^2}{n^2} = -E_0\times\frac{Z^2}{n^2},$$

where $E_0 = \frac{e^4m}{8\epsilon_0{}^2h^2} = 2{\cdot}18\times10^{-18}$ joule or 13·6 eV and is called one 'rydberg' unit of energy.

This expression gives the energy of the electron of an atom in the *n*th orbit or the energy of the atom in the *n*th state.

When the atom reverts from an *n*th excited state to *s*th lower state, the difference of energy is emitted as light wave, the frequency of which is given by

$$E_n - E_s = h\nu$$

or

$$\nu = \frac{Z^2e^4m}{8\epsilon_0{}^2h^3}\left(\frac{1}{s^2}-\frac{1}{n^2}\right).$$

$\bar{\nu}$ (wave number) = Number of waves per metre $= \frac{1}{\lambda} = \frac{\nu}{c}$.

$$\therefore \qquad \bar{\nu} = \frac{Z^2e^4m}{8\epsilon_0{}^2h^3c}\left(\frac{1}{s^2}-\frac{1}{n^2}\right) = RZ^2\left(\frac{1}{s^2}-\frac{1}{n^2}\right);$$

$R = \frac{e^4m}{8\epsilon_0{}^2h^3c} = 1{\cdot}097\times10^7\ \text{m}^{-1}$, a universal constant called Rydberg constant. Thus $\bar{\nu} = 1{\cdot}097\times10^7Z^2\left(\frac{1}{s^2}-\frac{1}{n^2}\right)$.

3. Excitation Energy of a State and Excitation Potential : The energy required to excite an atom from *the ground state* ($n=1$) to the given state is called the excitation energy of the state. The accelerating potential needed to accelerate a free electron to provide the excitation energy to the atom on collision with it is called the excitation potential. If V_{ex} is the excitation potential of a state and E_{ex} is the excitation energy of the state, then $E_{ex} = V_{ex}\times e$ or $V_{ex} = \frac{E_{ex}}{e}$.

Remember, *excitation energy in electron volt is numerically equal to the excitation potential in volts.*

4. Binding Energy of a State : The energy required to remove the electron to *infinity from the present state* is called the binding energy of the state.

5. Ionisation Potential and Energy : The energy required to *remove the outermost electron* (most loosely bound) *to infinity* is called the ionisation (principal one) energy of the atom and the accelerating potential needed to accelerate an external free electron to provide this energy on collision with the atom is called the ionisation potential of the atom. *The ionisation energy in electron volt is numerically equal to the ionisation potential in volts.*

EXAMPLES

Ex. 1. *Calculate the radius of the first orbit of hydrogen atom. Given that charge of electron* $=1{\cdot}6\times10^{-19}$ C, *mass of electron* $=9{\cdot}1\times10^{-31}$ kg, *Planck's constant* $=6{\cdot}6\times10^{-34}$ Js.

Sol. From consideration we have

coulomb's force = centripetal force.

$$\frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{r^2}=m\omega^2 r. \qquad \cdots (1)$$

From quantum condition we have

$$mr^2\omega=1.\frac{h}{2\pi} \qquad \cdots (2)$$

($\because$ I = moment of inertia of electron $=mr^2$).

Squaring (2) and dividing by (1),

$$4\pi^2.\frac{4\pi\epsilon_0 r^2}{e^2}=\frac{m^2r^4\omega^2}{m\omega^2 r}, \text{ or } r=\frac{\epsilon_0 h^2}{\pi m e^2}.$$

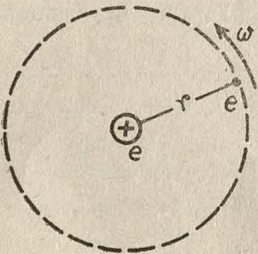

Fig. 2-1

$$\therefore\ r=\frac{8{\cdot}85\times10^{-12}\times(6{\cdot}6\times10^{-34})^2}{3{\cdot}142\times9{\cdot}1\times10^{-31}(1{\cdot}6\times10^{-19})^2}=5{\cdot}3\times10^{-11}\text{ m.} \qquad \text{Ans.}$$

Ex. 2. *Find the atomic structure of* ${}_{20}Ca^{40}$.

Sol. $Z=20$ = Number of electrons and also number of protons; $A=40$, number of nucleons.

$\therefore$ Number of neutrons $=A-Z=40-20=20$.

Arrangement of electrons—

$1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \ldots 2p_6 3s_1 3s_2 3p_1 3p_2 \ldots 3p_6 4s_1 4s_2 = 20$ electrons.

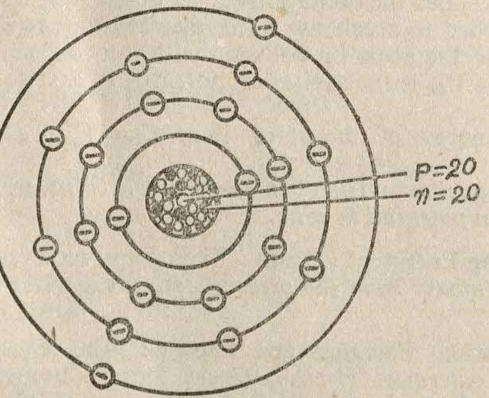

Fig. 2-2

Ex. 3. *Show that the kinetic energy of electrons in the first Bohr orbit (ground state) is numerically half of its potential energy.*

Sol. Considering dynamics of circular motion of electron we have

$$\frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{r^2}=m\omega^2 r \quad .. \text{ (i). (See Fig. 2-1 of example 1)}$$

From quantum condition,

$$mr^2\omega=1.\frac{h}{2\pi} \qquad (n=1, \text{ for first orbit}).$$

Now, kinetic energy of electron $=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2r^2$ $\qquad (\because \; v=\omega r)$
and potential energy of electron

$$=\frac{e}{4\pi\epsilon_0 r}\times(-e)=-\frac{e^2}{4\pi\epsilon_0 r}.$$

Proceeding with kinetic energy expression,

$$\text{k.e.}=\frac{1}{2}\,m\omega^2r^2=\frac{1}{2}mr^2.\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{mr^3}\right)$$

[taking ω^2 from (i) and putting it here)

$$=\frac{1}{2}\cdot\frac{e^2}{4\pi\epsilon_0 r}.$$ **Proved.**

EXERCISES

(A)

1. Find the structure of the following atoms :
(i) ${}_8O^{16}$, (ii) ${}_{17}Cl^{35}$, (iii) ${}_{13}Al^{27}$.

[**Ans.** (i) Number of electrons or protons=8; No. of neutrons=8.
$1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 2p_3 2p_4$.
(ii) Number of electrons or protons=17; Number of neutrons=18.
$1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \cdots\cdots\cdots\cdots 2p_6 3s_1 3s_2 3p_1 3p_2 \cdots\cdots\cdots 3p_5$.
(iii) Number of electrons or protons=13; Number of neutrons=14.
$1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \cdots\cdots\cdots\cdots 2p_6 3s_1 3s_2 3p_1$.]

2. Calculate the frequency of revolution of an electron in the first orbit of an aluminium atom. Given that mass of electron $=9\times10^{-31}$ kg; $e=1{\cdot}6\times10^{-19}$ C; radius of the orbit$={\cdot}2\times10^{-11}$ m. (**Ans.** $3{\cdot}25\times13^{18}$ revolutions/sec.)

3. Show that the energy of hydrogen atom in the ground state is

$$E=-\frac{1}{8\pi\epsilon_0}\cdot\frac{e^2}{r}.$$

[*Hint* : Energy in the ground state=Kinetic energy of electron in the first orbit+electrostatic potential energy of the electron.]

4. Calculate the energy of hydrogen atom in the ground state given that the first Bohr orbit of hydrogen is 5×10^{-11} m and electronic charge is $1 \cdot 6 \times 10^{-19}$ C. (Ans. $-2 \cdot 304 \times 10^{-18}$ joule)

5. Calculate the speed of the electron in the first Bohr's orbit given that $h = 6 \cdot 6 \times 10^{-34}$ Js; $m = 9 \cdot 11 \times 10^{-31}$ kg and $e = 1 \cdot 603 \times 10^{-19}$ C. (Ans. $2 \cdot 2 \times 10^{6}$ ms^{-1})

6. Find the atomic structure of $_{12}Mg^{24}$. What is its valency ?

(Ans. number of electrons or protons $= 12$; number of neutrons $= 12$;
electron arrangement : $1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \ldots\ldots\ldots\ldots 2p_6 3s_1 3s_2$;
valency $= 2$)

7. Calculate the radius of the first and second orbit of sodium atom $(Z = 11)$.
$h = 6 \cdot 6 \times 10^{-34}$ Js, $e = 1 \cdot 6 \times 10^{-19}$ C and $m = 9 \cdot 1 \times 10^{-31}$ kg.
(Ans. $4 \cdot 82 \times 10^{-12}$ m, $1 \cdot 928 \times 10^{-11}$ m)

8. An α-particle of velocity $1 \cdot 6 \times 10^{7}$ ms^{-1} approaches gold nucleii $(Z = 79)$. Calculate the distance of 'closest approach'. Mass of an α-particle $= 6 \cdot 6 \times 10^{-27}$ kg. (Ans. $4 \cdot 3 \times 10^{-14}$ m)

9. A single electron orbits around a stationary nucleus of charge $+Ze$, where Z is a constant and e is the electronic charge. It requires 47·2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit Find (i) the value of Z, (ii) the energy required to excite the electron from the third to the fourth Bohr orbit and (iii) the wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity. (I. I T. 1981)
[Ans (i) 5, (ii) 16·6 eV, (iii) 36·3 A°]

10. Calculate for a hydrogen atom the ionisation potential, the first excitation potential and the wavelength of the resonance line $(n' = 2 \rightarrow n = 1)$.
(Ans. 13·6 V, 10·2 V, 1215 A°)

11. An electron in an excited hydrogen atom acquired an energy of 12·1 eV. To what energy level did it jump ? How many spectral lines may be emitted in the course of the transition to lower energy levels ? Calculate the corresponding wavelengths. (Ans. $n = 3$; 1025 A°; 6563 A°; 1215 A°)

12. Calculate the radius of the first Bohr orbit of a He^{+} ion and the binding energy of its electron in the ground state. (Ans. $2 \cdot 65 \times 10^{-11}$ m; 54·5 eV)

13. A hydrogen atom in a state having binding energy 0·85 eV makes a transition to a state with an excitation energy of 10·2 eV. Find the energy of the photon emitted. (Ans. 2·55 eV)

14. Calculate the wavelengths of the first member of Lyman, Balmer and Paschen series. (Ans. 1215 A°, 6563 A°, 18752 A°)

15. A doubly ionised lithium atom is hydrogen-like with atomic number 3. Find the wavelength of radiation required to excite the electron in Li^{++} from the first to the third orbit (ionisation energy of the hydrogen atom equals to 13·6 eV). How many spectral lines are observed in the emission spectrum of the excited system ? (I. I. T. 1985)
(Ans. 1022 A°; 3)

CHAPTER 3

PHOTO-ELECTRICITY, X-RAYS AND RADIOACTIVITY

1. Particle and wave aspect of light: According to French scientist De Broglie a particle of momentum p is associated with a wave of wavelength λ given by

$$p=\frac{h}{\lambda} \quad \text{or} \quad p=\frac{h\nu}{c} \qquad .. \quad (1)$$

where ν is the corresponding frequency and c = velocity of wave.

According to this principle light may be supposed to consist of particles called photons of momentum $p=\frac{h\nu}{c}$ where ν = frequency of light and c = velocity of light and each photon is a packet of energy,

$$E=h\nu. \qquad .. \quad (2)$$

2. X-rays: When fast moving electrons strike a hard target like tungsten, platinum etc., electromagnetic waves of extremely short wavelengths are emitted. These are called X-rays.

If V is the voltage by which electrons are accelerated then Ve is the energy acquired by each electron. When it is stopped by the target, most of its energy is converted into heat energy and very little into X-rays. However, assuming complete conversion of its energy into X-rays we can calculate maximum frequency (and hence minimum wavelength) of X-rays produced. $Ve=h\nu_{max}$

$$\text{or} \qquad \nu_{max}=\frac{Ve}{h}\ \text{s}^{-1} \text{ and } \lambda_{min}=\frac{hc}{Ve}\ \text{m}. \qquad .. \quad (3)$$

3. Photoelectric emission: When light of frequency greater than a certain minimum called the threshold frequency (ν_0) falls on certain metals like *Na*, *K*, *Ce* etc., in a vacuum, electrons are ejected from them. This phenomenon of emission of electrons by light is called photoelectric emission and the electrons so ejected are called **Photoelectrons.**

4. Work function (φ): The energy required to expel an *electron from inside of a metal to outside of it* is called its work function.

If light of frequency ν falls on a metal then we may think as if particles of energy $h\nu$ are striking the electrons and imparting their energy to the electrons. If this energy is equal to the work function of the metal, electrons will just come out of the metal. If $h\nu$ is greater than the work function φ, then the balance $(h\nu-\varphi)$ appears as kinetic energy of the electron.

$$\therefore \qquad h\nu-\varphi=\tfrac{1}{2}mv^2.$$

Let $$\varphi=h\nu_0$$

then $$h\nu-h\nu_0=\tfrac{1}{2}mv^2$$

or $$\tfrac{1}{2}mv^2=h(\nu-\nu_0). \qquad .. \quad (4)$$

This is known as Einstein's photoelectric equation.

5 Radioactivity : The phenomenon of spontaneous disintegration of heavy elements with emission of certain rays like α, β and γ-rays is known as radioactivity.

If N_0 is the number of atoms present now and N is the number of atoms present t seconds after, then

$$N=N_0e^{-\lambda t} \qquad .. \quad (5)$$

where λ is a constant called **disintegration constant.**

Half life period : The time (T) in which the number of atoms present at any instant is reduced to half is called the half life period.

$$\therefore \qquad \frac{N_0}{2}=N_0e^{-\lambda T}, \quad \text{or} \quad \frac{1}{2}=e^{-\lambda T}.$$

Taking log of both sides, we have

$$-\log_e 2=-\lambda T, \quad \text{or} \quad T=\frac{\log_e 2}{\lambda}=\frac{2{\cdot}3\times\log_{10}2}{T} \qquad .. \quad (6)$$

or $$T=\frac{2{\cdot}3\times{\cdot}3010}{\lambda}=\frac{{\cdot}6931}{\lambda}. \qquad .. \quad (6a)$$

Average life : In a radioactive substance atoms are continuously breaking. Those atoms which break earlier have very short life, those breaking later will enjoy longer life.

Average life is defined as the mean of the lives of all atoms averaged over all atoms.

$$\text{Average life}=\frac{\text{Sum of lives of all atoms}}{\text{Number of atoms}}=\frac{1}{\lambda}.$$

Measure of activity of a radioactive substance : The rate at which disintegration takes place is taken as measure of activity. Its unit is curie (Ci). 1 curie $=3{\cdot}7\times10^{10}$ disintegrations per second. Another unit is rutherford corresponding to 10^6 disintegrations per second.

A third unit is 'bequerel' = 1 disintegration per second.

$\therefore$ 37 rutherford = 1 mCi (milli curie).

Activity of any substance

$$= -\frac{dN}{dt} = \frac{\lambda N}{3{\cdot}7 \times 10^{10}} \text{ curie (Ci) or } \frac{\lambda N}{10^6} \text{ rutherford.} \quad \ldots \quad (7)$$

EXAMPLES

Ex. 1. *Calculate the de Broglie wavelength of neutron whose energy is* 1 *electron volt.*

Mass of neutron $= 1{\cdot}676 \times 10^{-27}$ *kg*

Planck's constant $= 6{\cdot}622 \times 10^{-34}$ *joule second.*

Sol. We have, $E = \frac{1}{2} mv^2 = \frac{1}{2m}(mv)^2 = \frac{1}{2m} p^2$;

$$\therefore \qquad p = \sqrt{2mE}\text{. But } p = \frac{h}{\lambda}\cdot$$

$$\therefore \qquad \frac{h}{\lambda} = \sqrt{2mE}, \quad \text{or } \lambda = \frac{h}{\sqrt{2mE}}$$

$$\text{or} \qquad \lambda = \frac{6{\cdot}622 \times 10^{-34}}{\sqrt{2 \times 1{\cdot}676 \times 10^{-27} \times 1{\cdot}6 \times 10^{-19}}}$$

($\because$ $E = 1$ eV $= 1{\cdot}6 \times 10^{-19}$ joule)

or $\lambda = {\cdot}2857 \times 10^{-10}$ m

[$\because$ 10^{-10} m = 1 angstrom (A°)]

or $\lambda = {\cdot}2857$ A°. **Ans.**

Ex. 2. *Calculate energy in electron volt of a photon of wavelength* 6000 *A.U.*

$e = 1{\cdot}6 \times 10^{-19}$ C

$h = 6{\cdot}63 \times 10^{-34}$ Js

$c = 3 \times 10^8$ ms^{-1}.

Sol. We have, $E = h\nu$ where E = energy, h = Planck's constant and ν = frequency of light wave.

We have also, $\nu\lambda = c$; $\therefore$ $E = h\nu/\lambda$.

Here, $\lambda = 6000$ A U. $= 6000 \times 10^{-10}$ m

($\because$ 1 A.U $= 10^{-10}$ m)

$$\therefore \qquad E = \frac{6{\cdot}63 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}} = 3{\cdot}31 \times 10^{-19} \text{ joule}$$

$$=\frac{3{\cdot}31\times10^{-19}}{1{\cdot}6\times10^{-19}} \qquad (\because\ 1\text{ eV}=1{\cdot}6\times10^{-19}\text{ J})$$

or $$E=2{\cdot}07\text{ eV}.$$ **Ans.**

Ex. 3. *A photoelectric surface has work function* 2 *eV. What is the maximum velocity of the photoelectrons ejected by a light of wavelength* 3000 *A.U.? Also calculate stopping potential of the electrons.*

($h=6{\cdot}6\times10^{-34}\,Js$; $e=1{\cdot}6\times10^{-19}C$; *mass of electron* $=9\times10^{-31}\,kg$.)

Sol. Energy of incident photon $=h\nu=\dfrac{hc}{\lambda}$ $\qquad(\because\ \nu\lambda=c)$

$$=\frac{6{\cdot}6\times10^{-34}\times3\times10^{8}}{3000\times10^{-10}}$$

$$=6{\cdot}6\times10^{-19}\text{ J}.$$

Work function $=2\text{ eV}=2\times1{\cdot}6\times10^{-19}=3{\cdot}2\times10^{-19}$ J

$(1\text{ eV}=1{\cdot}6\times10^{-19}\text{ J})$.

The difference appears as kinetic energy of electrons.

$\therefore$ Kinetic energy of electrons $=6{\cdot}6\times10^{-19}-3{\cdot}2\times10^{-19}$

$=3{\cdot}4\times10^{-19}\text{ J}=\frac{1}{2}mv^2$

or $$v=\sqrt{\frac{2\times3{\cdot}4\times10^{-19}}{9\times10^{-31}}}=8{\cdot}7\times10^{5}\text{ ms}^{-1}.$$ **Ans.**

Let $V=$ stopping potential.

Then, $$\text{K.E.}=eV.$$

or $$V=\frac{\text{K.E.}}{e}=\frac{3{\cdot}4\times10^{-19}}{1{\cdot}6\times10^{-19}}=2{\cdot}12\text{ volt}.$$ **Ans.**

Ex. 4. *One gramme of a radioactive substance* ($A=39$) *takes* 50 *seconds to lose* 1 *centigram. Find* (i) *disintegration constant,* (ii) *half life,* (iii) *average life,* (iv) *activity in the beginning.* (*Avogadro's number* $=6\times10^{23}\,mol^{-1}$.)

Sol. We have, $N=N_0e^{-\lambda t}$. Mass is proportional to number of atoms.

$\therefore$ $W=W_0e^{-\lambda t}$.

Here $W_0=1$ gm, $W=1$ gm $-$ 1 cgm

$=1\text{ gm}-{\cdot}01\text{ gm}={\cdot}99\text{ gm}.$

$\therefore$ $${\cdot}99=1\,e^{-\lambda 50};\quad\text{or}\quad e^{\lambda 50}=\frac{100}{99}$$

or $$\log_e e^{\lambda 50} = \log_e \frac{100}{99}$$

or $$\lambda 50 = 2{\cdot}3 \log_{10} \frac{100}{99} = 2{\cdot}3(2 - 1{\cdot}9956)$$

or $$\lambda = \frac{2{\cdot}3 \times {\cdot}0044}{50} = {\cdot}000206 \text{ s}^{-1}$$

$$\text{Half life} = \frac{{\cdot}6932}{\lambda} = \frac{{\cdot}6932}{{\cdot}000206} = 3397 \text{ s} = 56 \text{ min } 37 \text{ s}.$$

$$\text{Average life} = \frac{1}{\lambda} = \frac{1}{{\cdot}000206} = 4{\cdot}85 \times 10^3 \text{ s}.$$

$$\text{Activity} = N\lambda = \frac{6 \times 10^{23}}{39} \times \frac{{\cdot}000206}{3{\cdot}7 \times 10^{10}} = 8{\cdot}5 \times 10^{7} \text{ curie (Ci)}.$$

EXERCISES

1. Calculate the de Broglie wavelength of an electron which falls through a P.D. of 100 volt. ($h = 6{\cdot}62 \times 10^{-34}$ Js, $e = 1{\cdot}6 \times 10^{-19}$ C and $m = 9{\cdot}11 \times 10^{-31}$ kg)
(Ans. $1{\cdot}227 \times 10^{-10}$ m)

2. Calculate the wavelength associated with an electron of energy 1000 eV. (Ans. ·388 A°)
($h = 6{\cdot}62 \times 10^{-34}$ Js; 1 eV $= 1{\cdot}6 \times 10^{-19}$ joule)

3. Calculate the energy of a photon of wavelength 5500 A.U.
($h = 6{\cdot}62 \times 10^{-34}$ Js).
(Ans. 2·256 eV)

[*Hint* : $E = h\nu$ and $\nu\lambda = c$; $\therefore \; E = \dfrac{hc}{\lambda}$]

4. What voltage should be applied to a cathode ray tube to produce electrons of wavelength ·4 A° ? What would be the speed of these electrons ?
(Charge of electron $= 1{\cdot}6 \times 10^{-19}$C; mass of electron $= 9 \times 10^{-31}$ kg and Planck's constant $= 6{\cdot}62 \times 10^{-34}$ Js)
(Ans. 951·3 V; $1{\cdot}84 \times 10^{7}$ m/s)

5. An X-ray tube operates at 50 kilovolt. Find the shortest wavelength of X-rays produced.
($h = 6{\cdot}62 \times 10^{-34}$ Js; Velocity of light $= 3 \times 10^{8}$ ms^{-1})
(Ans. ·248 A°)

[*Hint* : $\lambda_{min} = \dfrac{hc}{Ve}$]

6. A X-ray tube operates at 30,000 volts and the current through it is 2 mA. Calculate (a) power of the tube, (b) the number of electrons striking the target per second, (c) velocity of electrons when they hit the target, and (d) the lower wavelength limit of the X-rays emitted.
[Ans. (a) 60 watt; (b) $1{\cdot}25 \times 10^{16}$; (c) 10^{8} ms^{-1}; (d) $4{\cdot}1 \times 10^{-11}$ m]

7. An X-ray tube operates at 30 kilo volt and emits a continuous X-ray spectrum with shortest wavelength limit ·414 A.U. Calculate Planck's constant. (**Ans.** $h=6{\cdot}624\times10^{-34}$ Js)

8. An X-ray tube operates at 50 kilo volt. Assuming that only 10% of energy of electrons is converted into X-ray radiations calculate the wavelength of X-rays produced. (**Ans.** 2·48 A.U.)

Photoelectricity :

9. Calculate the work function of sodium metal given that the threshold wavelength is 6800 A.U. ($h=6{\cdot}625\times10^{-34}$ Js)

[*Hint* : $\varphi=h\nu_0$ and $\nu_0\lambda_0=c$; $\therefore\ \varphi=\dfrac{hc}{\lambda_0}$] (**Ans.** 1·827 eV)

10. The photoelectric threshold wavelength for tungsten is 2300 A.U. Find the energy of the photoelectrons emitted when light of wavelength 1800 A° falls on tungsten. (Planck's constant$=6{\cdot}6\times10^{-34}$ Js) (**Ans.** $2{\cdot}39\times10^{-19}$ J)

11. A ray of ultra-violet light of wavelength 3000 A° falls on the surface of a material whose work function is 2·28 eV. Calculate the velocity of the fastest electrons. ($h=6{\cdot}62\times10^{-34}$ Js, $m=9{\cdot}1\times10^{-31}$ kg)

[*Hint* : Use $\frac{1}{2}\,mv^2=h\nu-\varphi$] (**Ans.** $8{\cdot}1\times10^{5}$ ms^{-1})

12. Calculate the threshold frequency of a metal whose work function is 1·8 eV. ($h=6{\cdot}0\times10^{-34}$ Js) (**Ans.** $4{\cdot}8\times10^{14}$ hertz)

13. A photon of wavelength 3000 A° falls on the cathode of a photoelectric tube. Calculate the stopping potential of electrons emitted if the threshold wavelength of the material of cathode is 6000A°.

($h=6\ 0\times10^{-34}$ Js, $e=1{\cdot}6\times10^{-19}$ C and $c=3\times10^{8}$ ms^{-1})

[*Hint* : Stopping potential is the negative potential required to be applied on the anode to prevent even the fastest electrons to reach anode i.e. in other words potential required to reduce photoelectric current to zero.

Use $$\frac{1}{2}\,mv^2=h(\nu-\nu_0)=hc\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)=Ve$$

$$\therefore\qquad V=\frac{hc}{e}\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)\Bigg]$$ (**Ans.** 1·875 volts)

14. A photon of wavelength 3300 Å falls on the cathode of a photoelectric tube and electrons of energy 3×10^{-19} J are ejected. If the wavelength of the incident light is changed to 5000 A° the energy of ejected electrons decreases to $\cdot972\times10^{-19}$ joule. Calculate Planck's constant and the threshold wavelength of the material of cathode. (**Ans.** $6\ 6\times10^{-34}$ Js; 6640 Å)

Radioactivity :

15. One gram of radium is reduced by 2·1 mgm in 5 years. Calculate half-life period of radium. (**Ans.** 1649 years)

16. If 1 gm of radium emits $3{\cdot}7\times10^{10}$ particles per second, find the half-life period and average life of radium. (Atomic weight of radium$=226$; Avogadro's number$=6\ 025\times10^{23}$ mol^{-1}) (**Ans.** 1584 years; 2285 years)

17. The half life of the radioactive radon is 3·8 days. Calculate the time at the end of which 1/20th of the radon sample will remain undecayed. $(\log_{10}e = \cdot 4343)$ (I.I.T. 1981)

(Ans. 16·4 days)

18. A α-particle of 5 MeV is scattered through 180° by a fixed uranium nucleus. Calculate the distance of closest approach. (I.I.T. 1981)

(Ans. $1{\cdot}35 \times 10^{-13}$ m)

19. The activity of a certain preparation decreases 2·5 times after 7 days. Find its half life. **(Ans.** 5·3 days)

20. At the initial moment the activity of a certain radionucleide totalled 650 particles per minute. What will be the activity of the preparation after half its half-life period ? **(Ans.** 460 per minute)

21. A U^{238} preparation of mass 1 gm emits $1{\cdot}24 \times 10^4$ α-particles per second. Find the half-life of this nucleide and activity of the preparation.

(Ans. $3{\cdot}35 \times 10^{-7}$ Ci; $4{\cdot}5 \times 10^9$ years)

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CHAPTER 4

MASS ENERGY EQUIVALENCE : NUCLEAR FISSION AND FUSION

1. Mass Energy equivalence : Albert Einstein showed in his famous theory of relativity that mass of a system may change (an idea contradictory to what we are told in the beginning) and that mass and energy are equivalent to each other. A mass 'm' is equivalent to energy E given by

$$E = mc^2 \text{ joule (J).} \qquad .. \quad (1)$$

The well known principle of conservation of energy is to be modified and read as "the total energy of a system (mass also being considered as energy) remains constant in any process".

2. Atomic mass unit (a. m. u.) : In atomic physics the masses involved are very small and so we adopt a different unit of mass. This unit is $\frac{1}{12}$ of ${}_6C^{12}$ atom. This is called atomic mass unit

$$1 \text{ a. m. u.} = 1{\cdot}66 \times 10^{-27} \text{ kg.}$$

The masses of all atoms are measured in atomic mass units. On this scale mass of a proton is 1·00758, that of neutron is 1·00893 and that of electron is ·00055.

Applying Einstein's formula $E = mc^2$, we find that 1 a. m. u. corresponds to 931 Mev.

3. Binding energy : The mass of nucleus of an atom is always less than the sum of masses of the constituent nucleons (protons and neutrons). This means that certain mass disappears in the process of formation of the atom which is converted into energy. This energy is called Binding energy of the atom and the difference in mass is called **Mass defect of the atom.**

Thus, Binding energy of an atom $= \delta mc^2$ joule .. (2)
where δm is the **mass defect of the atom.**

4. Nuclear fission : In 1939 Otto Hahn bombarded uranium-238 with fast moving neutrons and found that it splits into two nearly equal parts with release of huge energy. This phenomenon of splitting of heavy nucleus into two nearly equal parts with release of considerable energy is known as Nuclear fission.

5. Nuclear fusion : When two light elements are brought together at very high temperature they fuse into a single heavier element with release of huge energy. This process is known as Nuclear fusion e.g. $_1H^2 + _1H^2 = _2He^3 + _0n^1 + 4$ Mev.

6. Conservation Laws : In any nuclear reactions three conservation rules must always be satisfied—(i) conservation of mass number, (ii) conservation of atomic number and (iii) conservation of atomic mass units.

EXAMPLES

Ex. 1. *Calculate the binding energy of helium nucleus* ($_2He^4$) *and express the quantity in Mev and joule.*

Mass of helium nucleus $= 4{\cdot}0028$ *a. m. u.*

Mass of proton $= 1{\cdot}00758$ *a. m. u.*

Mass of neutron $= 1{\cdot}00897$ *a. m. u.*

Sol. In $_2He^4$ nucleus there are two protons and 2 neutrons.

Mass of 2 neutrons and 2 protons $= 2 \times 1{\cdot}00897 + 2 \times 1{\cdot}00758$

$= 4{\cdot}03310$ a. m. u.

$\therefore$ Mass of helium nucleus $= 4{\cdot}00280$

$\therefore$ Mass defect of helium nucleus $= {\cdot}0303$.

The energy that is required to split a nucleus is called binding energy and this is exactly equal to energy required to form the nucleus.

1 a. m. u. = 931 MeV.

$\therefore$ Binding energy of helium nucleus $= 931 \times {\cdot}0303 = 28{\cdot}2$ MeV.

$1 \text{ eV} = 1{\cdot}6 \times 10^{-19}$ joule

$1 \text{ MeV} = 1{\cdot}6 \times 10^{-13}$ joule.

$\therefore$ Binding energy in joule $= 28{\cdot}2 \times 1{\cdot}6 \times 10^{-13}$

$= 4{\cdot}5 \times 10^{-12}$ joule. **Ans.**

Ex. 2. *Complete the nuclear reaction and calculate the energy released.*

$$_3Li^7 + _1H^1\ (proton) \rightarrow _2He^4 + _2He^4 + Q$$

given that mass of lithium atom $= 7{\cdot}01822$ *a. m. u.*

mass of proton $= 1{\cdot}00812$ *a. m. u.*

mass of α-particle $= 4{\cdot}00390$ *a. m. u.*

Sol. Mass on r. h. s. $= 7{\cdot}01822 + 1{\cdot}00812 = 8{\cdot}02634$ a. m. u.

mass on l. h. s. $= (2 \times 4{\cdot}00390 + Q)$ a. m. u. $= (8{\cdot}00780 + Q)$;

$$\therefore \qquad 8{\cdot}02634 = 8{\cdot}00780 + Q$$

or $$Q = {\cdot}01854 \text{ a. m. u.} = {\cdot}01854 \times 931 \text{ Mev}$$

(∵ 1 a. m. u. = 931 Mev)

$$= 1726 \text{ Mev.}$$ **Ans.**

Ex. 3. *Calculate how much coal is equivalent to* 1 *kg of* U^{235}. *Given that* 200 *Mev is released per fission process and* 8000 *kcalorie is released as heat of combustion of* 1 *kg of coal. Avogadro's number* $= 6{\cdot}1 \times 10^{26}$ *kg* mol^{-1}.

Sol. Let M kg of coal be equivalent to 1 kg of U^{235}.

235 kg of uranium contain $6{\cdot}1 \times 10^{26}$ atoms;

∴ 1 kg " " " $\dfrac{6{\cdot}1 \times 10^{26}}{235}$ atoms.

∴ Energy released by fission of 1 kg of uranium

$$= \frac{6{\cdot}1 \times 10^{26}}{235} \times 200 \text{ Mev.}$$

Now, energy released by M kg of coal

$$= 8000 \times 10^3 \times M \text{ cal.}$$

$$= 8 \times 10^6 M \times 4{\cdot}2 \text{ J} \qquad (\because \ 1 \text{ calorie} = 4{\cdot}2 \text{ joule})$$

$$= \frac{8 \times 10^6 \times 4{\cdot}2M}{1{\cdot}6 \times 10^{-13}} \text{ Mev}$$

$$(\because \ 1 \text{ Mev} = 1{\cdot}6 \times 10^{-13} \text{ J})$$

$$\therefore \qquad \frac{6{\cdot}1 \times 10^{26}}{235} \times 200 = \frac{8 \times 10^6 \times 4{\cdot}2M}{1{\cdot}6 \times 10^{-13}}$$

or $$M = \frac{6{\cdot}1 \times 10^{26} \times 200 \times 1{\cdot}6 \times 10^{-13}}{235 \times 8 \times 10^6 \times 4{\cdot}2} = \frac{61 \times 4}{235 \times 42} \times 10^8$$

$$= 2{\cdot}473 \times 10^6 \text{ kg.}$$ **Ans.**

EXERCISES

1. Calculate the binding energy of the lithium atom ($_3Li^7$) from the following data :

mass of proton = 1·00759 a.m.u.
mass of neutron = 1·00898 a. m. u.
mass of electron = ·00055 a. m. u.
mass of lithium atom = 7·01818 a. m. u. (**Ans.** 39·3 Mev)

2. Find the minimum energy that a γ-ray must have to give rise to an electron-positron pair.

mass of electron=·00055 a. m. u.

" " positron=·00055 a. m. u. (Ans. 1·024 mev)

3. Calculate the energy released by fission of 1 gm of $_{92}U^{235}$ assuming that an energy of 200 Mev is released by fission of each atom of U^{235}. (Avogadro's number=6·023×10²⁶ kg mol⁻¹.) (Ans. 5·1×10²³ Mev)

4. A reactor develops nuclear energy at the rate of 3×10⁴ kilowatt (kW). How many atoms of U^{235} undergo fission per sec ? How much U-235 is burnt in 1000 hours of operation ?

(Assume that 200 Mev is released per fission and Avogadro's number 6·023 ×10²⁶ per kg mole) (Ans. 9·375×10¹⁷; 1·317 kg)

5. Calculate the amount of energy set free by the annihilation of an electron and a positron. Given that mass of the electron=·00055 a. m. u. and positron =·00055 a. m. u. (Ans. 1·0241 Mev)

6. Complete the following nuclear reactions :

(i) $_7N^{14}+He^4\rightarrow{}_8O+{}_1H^1$

(ii) $Mg^{24}+{}_2He^4\rightarrow{}_{13}Al^{27}+$

(iii) $_3Li+n^1\rightarrow He^4+{}_1H^3$.

[Ans. (i) $_7N^{14}+{}_2He^4\rightarrow{}_8O^{17}+{}_1H^1$, (ii) $_{12}Mg^{24}+{}_2He^4\rightarrow{}_{13}Al^{27}+{}_1H^1$, (iii) $_3Li^6+{}_0n^1\rightarrow{}_2He^4+{}_1H^3$]

7. Calculate the energy released in kilowatt hour when 100 gm of $_3Li^7$ are converted into $_2He^4$ by proton bombardment. Mass of $_3Li^7$=7·0183 a. m. u.; mass of $_2He^4$=4 0040 a. m. u.; mass of proton=1·0081 a. m. u. Write down the nuclear reaction. [Ans. $_3Li^7+{}_1H^1$ (proton)$\rightarrow 2{}_2He^4+Q$ (energy released), 6·554×10⁶ kilowatt hour]

8. Calculate the energy released in the nuclear fusion of isotopes of hydrogen

(i) $_1H^2+{}_1H^2\rightarrow{}_2He^3+{}_0n^1$

(ii) $_1H^2+{}_1H^3\rightarrow{}_2He^4+{}_0n^1$.

Given that mass of neutron= 1·00867 a. m. u.

mass of $_1H^2$=2·01410 a. m. u.
mass of $_1H^3$=3·01605 a. m. u.
mass of $_2H^3$=3·01603 a. m. u.
mass of $_2He^4$=4·00260 a. m. u.

[Ans. (i) 3·26 Mev, (ii) 17·6 Mev]

* * *

REVISION EXAMPLES I

1. A capacitor of capacitance C is first charged with Q units of charge and then it is connected to another uncharged capacitor of capacitance c and subsequently disconnected. The latter is completely discharged by short-circuiting its plates and then again connected to the first capacitor. The process is repeated n times. Show that the first capacitor can never be fully discharged by the above process. Why ? What is the charge left after n discharges ?

$$\left[\text{Ans. } Q_n=\left(\frac{C}{C+c}\right)^n\times Q\right]$$

2. A capacitor of capacitance C_1 is charged to a potential difference of V_0. It is then connected across the combination of two capacitors C_2 and C_3 in series. Calculate the charge on C_1, C_2 and C_3.

$$\left[\text{Ans. } q_1=\frac{(C_2+C_3)C_1^2V_0}{C_1C_2+C_2C_3+C_3C_1};\ q_2=q_3=\frac{C_1C_2C_3}{C_1C_2+C_2C_3+C_3C_1}\times V_0\right]$$

3. The minimum deviation of a certain glass prism ($\mu=1{\cdot}5$) is equal to its refracting angle. Find the latter. (**Ans**. 83°)

4. A parallel plate capacitor of capacitance 10 uF is connected to a battery of e.m.f. 30 V. A dielectric plate ($\epsilon_r=5$) is introduced between plates so as to occupy the entire space between them without disconnecting the battery. Calculate (a) the extra energy drawn from the battery and (b) the extra energy stored in the capacitor.

(*Hint* : Extra energy drawn from the battery$=e.\Delta q$ joule and extra energy of the capacitor$=E_f-E_i$.)

(**Ans**. 36×10^{-3} joule, 18×10^{-3} joule)

5. Two point charges each of ·1 pC are placed 10 cm apart. An electron is projected at right angles to the line joining the two charges from the mid-point with velocity 10^6 ms^{-1}. How far along this line will it go ?

(*Hint* : Consider conservation of energy.) (**Ans**. 22 cm)

6. A tube bent twice at right angles at two point 'l' distance apart is held vertically and is filled with a liquid up to a certain height in the vertical arms. Calculate the difference in levels if, (a) the tube has an acceleration 'a' towards the right and (b) if the tube is mounted on a horizontal turn-table rotating with an angular velocity 'ω' with one of the vertical arms as the axis of rotation. Assume diameter of the tube small in comparison to 'l'.

(*Hint* : Apply D'Alembert's principle and consider equilibrium of the horizontal portion of the tube.) (**Ans**. $a\,l/g$; $\omega^2l^2/2g$)

7. A brass rod ($a=18\times10^{-6}$ K^{-1}) has projections right angles at the ends. Another rod of some other material just fits in the space between the projections at 0°C. What is the stress produced in the rod when the system is heated to 100°C ? Consider two cases (a) when the coefficient of expansion and Young's modulus are 29×10^{-6} K^{-1} and 18×10^{9} Nm^{-2}, (b) 12×10^{-6} K^{-1} and $2{\cdot}06\times10^{11}$ Nm^{-2} respectively.

[**Ans**. 198×10^5 Nm^{-2} (compressive), 1236×10^5 Nm^{-2} (tensile)]

8. Two small discs of masses m_1 and m_2 interconnected by a weightless spring rest on a smooth horizontal plane. The discs are set in motion with initial velocities v_1 and v_2 whose directions are at right angles to each other and lie in the same horizontal plane. Find the total energy of this system in the frame of the centre of mass.

$$\left[\text{Ans. } E=\tfrac{1}{2}\,\frac{m_1 m_2}{(m_1+m_2)}\,(v_1^2+v_2^2)\right]$$

[*Hint* : $(m_1+m_2)\,\vec{v}_{cm}=m_1\,\vec{v}_1+m_2\,\vec{v}_2$;

$$E=\tfrac{1}{2}m_1\left|\vec{v}_{cm}-\vec{v}_1\right|^2+\tfrac{1}{2}m_2\left|\vec{v}_{cm}-\vec{v}_2\right|^2.]$$

9. A rod of mass m_2 rests on a wedge of mass m_1 (Fig. A). Guides allo the rod to move only in the vertical direction and the wedge in the horizontal direction. Find the acceleration of both bodies and the reaction of the wedge. Neglect friction and take the wedge angle with the horizontal as α.

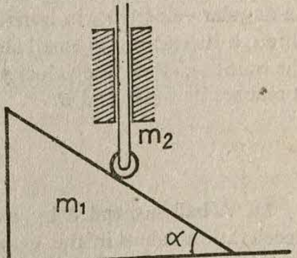


Fig. A

$$\left(\text{Ans. } a_1=\frac{m_2 g}{m_2\tan\alpha+m_1\cot\alpha},\quad a_2=\frac{m_2 g\sin^2\alpha}{m_2\sin^2\alpha+m_1\cos^2\alpha};\quad N=\frac{m_1 m_2 g\cos\alpha}{m_2\sin^2\alpha+m_1\cos^2\alpha}\right)$$

10. A block is held in contact with the vertical wall of a carriage by pressing it against the wall to the left with the thumb. Now the thumb is removed and the carriage is moved to the right with acceleration 'a'. Calculate the acceleration 'a' so that the block may not fall if the μ is the coefficient of friction beween the block and the wall of the carriage.

[*Hint* : Apply D'Alembert's principle and make a free-body diagram of the block.] (Ans. $a=g/\mu$)

11. A block of mass m_3 is connected by an inextensible light string to another block of mass m_2 lying on the top horizontal surface of a carriage of mass m_1. Calculate the force by which the carriage is to be pulled so that the two blocks may remain stationary.

[*Hint* : Apply D'Alembert's principle and make free body diagrams of the three bodies.] [Ans. $F=(m_1+m_2+m_3)\,m_3 g/m_2$]

12. A circular conductor of resistance 4 Ω is tapped at two points dividing the conductor in the ratio 1 : 3. The tappings are connected to a cell. Calculate the magnetic field at the centre of the coil. (Ans. Zero)

13. Two rods of lengths l and l', Young's modulus Y and Y' an coefficient of expansion α and α' respectively are held in between two massive vertical walls. The temperature of the rods is increased by T. Calculate the thrust exerted by the rods on each other assuming that there is no bending of the rods and there is no change in the area of cross-section of the rods. The rods are of equal area of cross-section 'A'.

[*Hint* : $l+l'=l_1+l_1'$ where l_1 and l_1' are the final lengths. Final length of a rod = initial length + thermal expansion − elastic compression]

$$\left[\textbf{Ans. } F=\frac{ATYY'(l\alpha+l'\alpha')}{lY'+l'Y}\right]$$

14. A tapering rod of length l is stretched by F. Calculate the increase in length of the rod if it tapers from an area of cross-section α_1 to α_2. Young's modulus of the rod is Y.

$$\left(\textbf{Ans. } \frac{Fl}{Y\sqrt{\alpha_1\alpha_2}}\right)$$

15. A smooth uniform rod AB of mass M and length l rotates freely with an angular velocity ω_0 in horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m starts sliding along the rod from the point A. Find the velocity v' of the sleeve relative to the rod at the moment it reaches its other end B.

$$\left(\textbf{Ans. } v'=\frac{\omega_0 l}{\sqrt{1+\frac{3m}{M}}}\right)$$

16. A ball suspended by a thread swings in a vertical plane so that its acceleration values in the extreme positions and the lowest positions are equal. Find the thread deflection at the extreme positions. (**Ans.** 53°)

[*Hint* : Acceleration at any point $=\sqrt{a_r^2+a_t^2}$ where a_r = radial acceleration and a_t = tangential acceleration.]

17. A small body of mass m tied to an inextensible thread moves over a smooth horizontal table. The other end of the thread is being drawn through a hole O in the table with a constant velocity (Fig. B). Find the tension of the string as a function of the distance r between the body and the hole if at $r=r_0$ the angular velocity of the body is equal to ω_0.

$$\left(\textbf{Ans. } T=\frac{m\omega_0^2 r_0^4}{r^3}\right)$$

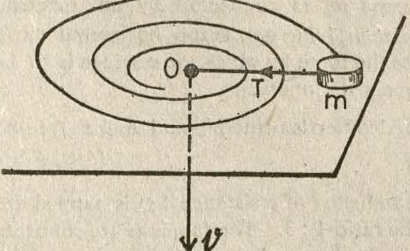


Fig. B

[*Hint* : Since the external force has no moment about O, angular momentum of the body about the point O is conserved.]

18. Three cells of emfs e_1, e_2, e_3 and internal resistances r_1, r_2, r_3 respectively are connected in parallel. Calculate the P.D. of the cells.

$$\left(\text{Ans. } \frac{r_1r_2e_3+r_2r_3e_1+r_3r_1e_2}{r_1r_2+r_2r_3+r_3r_1}\right)$$

19. A horizontal tube of cross-section A has a bend of θ° with the vertical. A liquid of density ρ flows through it with speed v. Calculate the force needed to hold the tube in position. [Ans. $\sqrt{2}A\rho v\,(\sin\theta/2-\cos\theta/2)$]

20. A light rod is suspended horizontally by three identical spring balances (x, x, x), each of weight 4 kg. Below it a second light rod is suspended horizontally by two identical spring balances (y, y) and finally a load of 4 kg is hung from this lower rod. Find the readings of x and y.

[*Hint* : Draw free-body diagrams of the two rods.] (Ans. 1·6 kg, 2 kg)

21. Calculate the resistance of a tapering wire of length l and end sections α_1 and α_2.

[*Hint* : Consider a section of the wire at a distance x from the apex of the wire and calculate the resistance of an elementary length and integrate.)

$$\left(\text{Ans. } R=\rho\frac{l}{\sqrt{\alpha_1\alpha_2}}\right)$$

22. A 100-watt sodium vapour lamp radiates uniformly in all directions. (a) At what distance from the lamp will the average density of photons be 10^7m^{-3} ? (b) What is the average density of photons 2 m from the lamp ? Assume the light to be monochromatic, with $\lambda=5890$ A.

[*Hint* : intensity of the light$=Dc$, where D is the density of energy and c is the speed of the light wave. Energy of a photon$=h\nu$.]

(Ans. 314 m, $2{\cdot}48\times10^{11}$ m^{-3})

23. Show that when a monatomic ideal gas absorbs $\triangle Q$ heat at a constant pressure, $\triangle Q : \triangle U : \triangle W=5:3:2$.

[*Hint* : $\triangle Q=C_p\triangle T$; $\triangle U=C_v\triangle T$; $\triangle W=p\triangle V$)

24. A pole extends 2 m above the bottom of a swimming pool and ·5 m above the water. Sunlight is incident at 45°. What is the length of the shadow of the pole on the bottom of the pool ? ($\mu=4/3$) (Ans. 1·75 m)

25. The earth has a magnetic dipole moment of $6{\cdot}4\times10^{21}$ Am^2. What current would have to be set up in a single turn of wire going around the earth at its magnetic equator if we wished to set up such a dipole ? (Radius of the earth$=6400$ km) (Ans. 5×10^7 m)

26. In a biprism experiment the two halves of the biprism are covered by two glass plates of the same thickness but of different refractive indices 1·4 and 1·7. The point on the screen where the central maximum fell before the glass plates were inserted is now occupied by the fifth bright fringe. Find the thicknes of the glass plates. ($\lambda=4800$ Å) (Ans. 8×10^{-6} m)

27. Show that the capacitance of two metal spheres of the same radius, when far apart, is one-half the capacitance on isolated sphere.

[*Hint* : Consider the two spheres as the two conductors of a capacitor.]

28. An iron wire (diameter 1 mm, length 10 cm) is placed in an evacuated chamber. Calculate the equilibrium temperature of the wire if it carries a current of 10 A. (Stefan's constant (σ) $=5{\cdot}67\times10^{-8}$ Wm^{-2} K^{-4} and resistivity of iron$=10\times10^{-8}$ Ωm^{-1}). Take the temperature of the chamber walls to be 27°C.

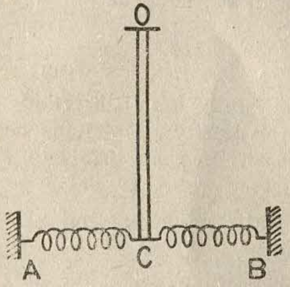
[*Hint* : R (radiancy i.e. energy radiated from unit area of a black body surface in unit time)$=\sigma\,(T^4-T_0^4)$, where T_0 is the temperature of the surroundings.] (Ans. 258°C)

29. In one branch AB of a complicated circuit there is a resistor of 2 Ω and a box concealing a battery of 8 Ω resistance. If the power absorption in the branch be 50 watt and the current through the branch is 1 ampere from A to B, (a) what is the potential difference, (b) what is the e m.f. of the battery, and (c) what is its polarity ? (**Ans.** 50 V, 40 V, + to A and − to B)

30. Find the frequency of small oscillations of a thin uniform vertical rod of mass m and length l hinged at the point O (Fig. C). The force constant of either spring is k. The masses of the springs are negligible.

[*Hint* : τ (torque about O)$=-2k\,lx-mg\,\frac{x}{2}$.]

[Ans. $f=\frac{1}{2\pi}\sqrt{\frac{3g}{2l}\left(1+\frac{4kl}{mg}\right)}$]



(Fig. C)

31. A simple bar magnet of magnetic moment 'm' and mass 'w' is suspended by a string from its south pole. If a uniform magnetic field 'B' directed parallel to the ceiling from which it is suspended is established, show the resulting orientation of the string and the magnet.

(**Ans.** The string will remain vertical as before but the magnet will make an angle $\tan^{-1}\frac{wgl}{mB}$ where $2l$ is the length of the magnet)

32. A particle of charge $+Q$ is assumed to have a fixed position at a point P. A second particle of mass 'm' and charge '$-q$' moves at a constant speed in a circle of radius 'r_1' centred at P. Find the work done by an external agent on the second particle in order to increase the radius of the circle of motion, centred at P, to r_2.

[*Hint* : Apply 'Work-Energy' theorem, i.e., Work done=change in kinetic energy.] [**Ans.** $\frac{Qq}{8\pi\epsilon_0}\,(1/r_1-1/r_2)$]

33. An electron is accelerated through a potential difference of 1000 volts and directed into a region between two parallel plates separated by ·02 m with a voltage difference of 100 volts between them. If the electron enters moving perpendicular to the electric field between the plates, what magnetic field is necessary perpendicular to both the electron path and the electric field so that electron travels in a straight line ?

[*Hint* : $F_m = Bev$ and $F_e = eE = e \times V/d$.) **(Ans.** $2{\cdot}7 \times 10^{-4}$ T)

34. Two identical conducting spheres of radius ·15 m are separated by 10 m from centre to centre. What is the charge on each sphere if the potential of one is +1500 volts and if the other is −1500 volts ?

(Ans. $+2{\cdot}5 \times 10^{-8}$ C and $-2{\cdot}5 \times 10^{-8}$ C)

35. Dimensionally S_t (distance described in t^{th} second)$=u+\frac{1}{2}\,a(2t-1)$ seems to be incorrect. In fact this is a correct relation. Can you explain the flaw ?

36. A plank of mass m_1 with a bar of mass m_2 placed on it lies on a horizontal plane surface. A horizontal force growing with time as $F=at$ (a is constant) is applied to the bar. Find how the accelerations f_1 and f_2 of the plank and the bar respectively depend on time t, if coefficient of friction between the bar and the plank is μ. Draw the approximate plots of these dependences.

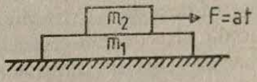


Fig. D

$$\text{(Ans. } f_1 = f_2 = \frac{at}{m_1+m_2} \text{ when } t < t_0 = \frac{\mu m_2 (m_1+m_2) g}{m_1 a}$$

$$\text{and when } t > t_0,\ f_1 = \frac{\mu m_2 g}{m_1} \text{ and } f_2 = \frac{at - \mu m_2 g}{m_2}\Big)$$

37. Each item (a) coulomb-ohm-metre/weber, (b) volt-second, (c) coulomb-ampere/farad, (d) (henry × farad)$^{\frac{1}{2}}$ is equal to one of the following items in the following list : metre, ampere, second, kilogramme, newton, joule watt, coulomb, volt, ohm, weber, henry, farad. Give the equalities.

(Ans. metre, weber, watt, second)

38. Show, by the method of dimensions, that the relation $E=Bc$ between the magnetic field B and the electric field E in an electromagnetic wave is true.

39. A particle moves in a plane under the action of a force which is always perpendicular to the particles velocity and depends on the distance from a certain point on the plane as $1/r^n$, where n is a constant. At what value or values of n will the motion of the particle along the circle be steady ?

(Ans. For all values of n from −1 to +1)

[*Hint* : For equilibrium of a closed system its total energy must be negative.]

□□□

REVISION EXAMPLES II

1. A particle of mass m moves in an elliptical orbit under a central force $=(k/r^2)$, where r is the distance of the particle from the centre of the force. If the distance of the particle from the centre of the force is 'a' at one extreme position of the particle and its linear speed is $\sqrt{k/2ma}$, find from energy considerations and dynamics of angular momentum of the particle the distance of the particle at the other extreme position.

(*Hint.*: Potential energy of the particle$=-k/r$. Apply conservation of angular momentum and energy. Remember that moment of momentum is also angular momentum.)

(Ans. $a/3$)

2. A circular coil is placed in the magnetic meridian and a current is passed through it so as to produce a magnetic field to the west. The compass needle placed at the centre of the coil is deflected through 20°. A short bar magnet of magnetic moment 1·2 Am² is placed to the east of the coil with its midpoint at a distance 10 cm and its axis at a bearing of 30° north of west. Calculate the deflection of the needle. $B_{earth}=3{\cdot}6\times10^{-5}$ tesla.

[*Hint* : Find restoring torque and deflecting torque and apply 'tangent law'.]

(Ans. 83°48′)

3. A particle of mass m is attached to a point O by a string of length l. It is held at a point P distant l from O above the level through O at an angle 60° with the upward vertical. If the particle is released from rest at P, prove that when the particle is again at a distance l from O below the level through O the string exerts an impulsive force of impulse $m\sqrt{\tfrac{1}{2}gl}$, and that immediately afterwards the tension becomes $2mg$.

[*Hint* : Consider conservation of angular momentum of the particle about O to find the speed immediately afterwards, remembering that moment of momentum is also angular momentum.]

4. A motor cyclist moves in a S-bend at a constant speed of 60 kph. The radius of curvature of the first bend is 250 m and that of the second bend is 400 m. Calculate the change in acceleration of the cyclist.

[*Hint* : Remember that acceleration is a vector quantity.]

(Ans. $1{\cdot}17$ ms^{-2})

5. Two bodies A of mass m and B of mass $2m$ connected by a light inextensible string are placed on a smooth cylindrical surface in a plane perpendicular to the axis of the cylinder. Initially the radius at A is horizontal and that at B is inclined to the upward vertical by $\tan^{-1}\frac{8}{4}$. Show that B leaves the surface when each radius describes θ given by $28\cos\theta-11\sin\theta=16$.

6. A point performs harmonic oscillations along a straight line with a period T and amplitude a. Find the mean velocity of the point averaged over the time interval during which it travels a distance $a\sqrt{2}$, starting from (i) the extreme position, (ii) the equilibrium position.

$$\left(\text{Ans. (i) } \bar{v}=\frac{2a}{T}\text{ ; (ii) } \bar{v}=\frac{6a}{T}\right)$$

7. A circuit shown in Fig. E has resistances $R_1=20\ \Omega$ and $R_3=30\ \Omega$. At what value of the resistance S connected in parallel to R_2 will the thermal power absorbed by S be independent of small variations in S itself ? The voltage between the points A and B is a constant.

(Ans. $S=\frac{R_1R_2}{R_1+R_2}=12\ \Omega$)

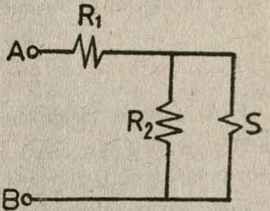


Fig. E

8. Two bars of masses m_1 and m_2 connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between the bars and the surface is equal to k. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar ?

$\left[\text{Ans. } F=k\left(m_1+\frac{m_2}{2}\right)g\right]$

[*Hint* : The force needed is minimum when m_1 is pulled slowly not to create any velocity. Apply 'work-energy' theorem. Remember that the change in kinetic energy of the system is zero.]

9. A motor car is travelling along a level road with a constant speed V, the resistance to the motion being equivalent to a constant back pull 'a' kgf. The car then comes to a hill where the resistance (including gravity) is 'b' kgf, and after the velocity has again become constant, the engine works at the same constant power as on the level. If, while the velocity is varying, the tractive-pull alters uniformly with the distance from its first constant value to the next constant value, show that the distance travelled along the hill before the velocity becomes. constant is $\frac{M(a+b)V^2}{gb^2}$.

[*Hint* : Apply work-energy theorem. The work done by the engine $=\frac{1}{2}(a+b)\,g.x$, taking the average of the initial and final constant value of the force as the constant force while the velocity is varying. This is permissible because the force changes uniformly with distance from its initial constant value to its final constant value.]

10. A spring of unstretched length l and 'modulus' mg is fixed at one end and attached to a body of mass m at the other end. It is stretched by l and then released. Show that the maximum velocity of the particle is $(1-\mu)\sqrt{gl}$, where μ is the coefficient of friction between the body and the table.

[*Hint* : The force called into play in a spring is proportional to the change in length and inversely proportional to its natural length, i.e. $=\frac{\lambda\triangle l}{l}$. This '$\lambda$' is called modulus of the spring. If x is the instantaneous displacement of the body from the point where it is released, then equation of motion of the body is

$$\frac{mg}{l}(l-x)-\mu\, mg=m\frac{dv}{dt}.\Big]$$

11. A particle inside and at the lowest point O of a fixed smooth spherical cavity is projected horizontally with speed $\sqrt{4ga}$. Find the height of the point above O at which the particle leaves the surface of the cavity.

[*Hint* : Apply dynamics of circular motion and conservation of energy.] (Ans. $5a/3$)

12. A vessel contains helium, which expands at a constant pressure when 15 kJ of heat is supplied to it. What will be the variation of the internal energy of the gas ? What is the work performed in the expansion ? Assume the gas to be ideal ($\gamma=\frac{5}{3}$). (Ans. 9 kJ; 6 kJ)

[*Hint* : $\triangle Q=\triangle U+\triangle W$. For an ideal gas, $\triangle U=C_v\triangle T$, $\triangle Q=C_p\triangle T$.

$$\therefore\quad \frac{\triangle U}{\triangle Q}=\frac{C_v}{C_p}=\frac{1}{\gamma}.\Big]$$

13. Show that rate of dissipation of joule heat per unit volume of a conductor is jE where j is the current density and E is the electric field. Assume that j and E are in the same direction.

14. A ball is suspended by a thread of length l at the point O of an inclined wall making a small angle α with the vertical. The ball is deviated through a small angle β $(\beta>\alpha)$ and released. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

$$\left(\text{Ans. } 2\sqrt{\frac{l}{g}}\left[\frac{\pi}{2}+\sin^{-1}\frac{\alpha}{\beta}\right]\right)$$

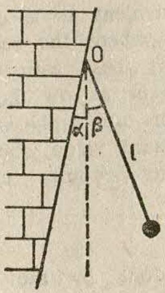


Fig. F

15. A body is thrown up a rough inclined plane with kinetic energy E. Show that the work done against frictional force when the body comes to rest is $\frac{\mu E\cos\alpha}{\sin\alpha+\mu\cos\alpha}$ where μ is the coefficient of friction and α is the inclination of the plane to the horizontal.

[*Hint* : Apply work-energy theorem.]

16. A particle which moves in a straight line and is acted on by a variable force which works at a constant rate, changes its velocity from u to v in passing over a distance x. Prove that the time taken is $\frac{3(v^2-u^2)x}{2(v^3-u^3)}$.

[*Hint* : Work done$=P\int dt=Pt=\frac{1}{2}m(v^2-u^2)$, by work-energy theorem.

Also $P=F.\ v=m.\ dv/dt.\ v=\dfrac{mv^2dv}{dx}$ or $Pdx=mv^2dv$.

$$\therefore \quad \int Pdx=\int_u^v mv^2dv.]$$

17. A particle executes simple harmonic motion of period 7·5 s. The velocity at a point P 1 m away from the centre of motion is 2 ms^{-1}. Find the amplitude of the oscillation. If the particle is passing through P and moving away from the centre of oscillation, find the time elapses before the particle next passes through P. (Ans. 2·6 m, 2·8 s)

[*Hint* : Take the help of the generating circle.]

18. If P_m is the minimum force to move a body up a rough inclined plane, show that the least force parallel to the plane required to move the body up the same plane is $P_m\sqrt{1+\mu^2}$.

[*Hint* : First show that $P_m=mg\sin(\alpha+\lambda)$, where λ is angle of friction.]

19. A bead of mass m is threaded on a smooth circular wire which is held in a vertical plane. The bead is released from one end of a horizontal diameter of the wire. Find the reaction of the wire when the bead is vertically below the centre of the wire. (Ans. $3mg$)

20. A heavy particle hangs from a fixed point O by an inextensible string of length l. When the particle is vertically below O it is projected horizontally with speed $\sqrt{\dfrac{7gl.}{2}}$ Calculate the angle through which the string has turned before it becomes slack. (Ans. 120°)

21. A particle is projected horizontally with a speed v on the smooth inner surface of a hollow right circular cone placed with its axis vertical and vertex downwards. If the particle describes a horizontal circle, show that the height of the circle above the vertex is v^2/g.

22. A circular cone of semi-vertical angle α is fixed with its axis vertical and its vertex upward. An inextensible string of length l is attached at one end to the vertex and at the other end a particle of mass m resting on the smooth external surface of the cone. The particle then revolves with uniform angular velocity 'ω' in a horizontal circle in contact of the cone. Show that $\omega^2<\dfrac{g}{l\cos\alpha}$ and find the tension in the string. [Ans. $m(g\cos\alpha+l\omega^2\sin^2\alpha)$]

23. A particle attached by a light string (which is inelastic and of length l) to a fixed point is describing a horizontal circle of radius nl with uniform speed. The particle is now suddenly stopped and then let go. If in the subsequent motion its speed when the string becomes vertical is equal to half its speed in the original motion, prove that $7n=4\sqrt{3}$.

24. At the moment $t=0$ the force $F=at$ is applied to a small body of mass m resting a smooth plane at a constant angle α with the horizontal. Find (a) the velocity of the mass at the moment of its breaking off the plane, (b) the distance traversed by the mass up to this moment.

$$\left(\text{Ans. } v=\frac{mg^2 \cos\alpha}{2a \sin^2\alpha} \,;\; s=\frac{m^2g^3 \cos\alpha}{6a^2\sin^3\alpha}\right)$$

25. Prove that the 'shortest time from rest to rest' in which a chain which can just bear a steady load of P kgf can lift a weight W kg through a vertical distance h is $\sqrt{\dfrac{2hP}{g(P-W)}}$.

[*Hint* : Move it with maximum acceleration upwards and then let the string be completely loose.]

26. Three vertical walls are of height h, H, h, and are equal distances a apart. A ball is thrown in a vertical plane perpendicular to the walls so that it just clears all three tops. Show that the ball will strike the ground at a distance $a\sqrt{\dfrac{H}{H-h}}$ from the middle wall.

[*Hint* : $H=\dfrac{v^2\sin^2\theta}{2g}$ or $v \sin\theta=\sqrt{2gH}$, $h=v \sin\theta\, t-\frac{1}{2}gt^2$. Eliminate $\sin\theta$ and proceed.]

27. A particle is projected from a given point so as to pass over a wall of height h which is such that the top of the wall is at a distance r from the point of projection. Show that the least speed of projection is $\sqrt{g(h+r)}$.

[*Hint* : $h=x\tan\theta-\frac{1}{2}gx^2/v^2 \cos^2\theta$, where x is the horizontal distance of the foot of the wall. Hence find v and apply the condition for minimum value of v.

28. Projectiles are hurled at a horizontal distance R from the edge of a cliff of height h in such a way as to land a horizontal distance x from the bottom of the cliff. If you want x to be as small as possible, how would you adjust θ and v, assuming v can be varied from zero to some maximum v_m and θ can be varied continuosly ? Only one collision with the ground is allowed.

$$\left(\text{Ans. } \theta=\tfrac{1}{2}\sin^{-1}\frac{Rg}{{v_m}^2}\right)$$

29. A particle rests on the top of a hemisphere of radius R. Find the smallest velocity that must be imparted to the particle if it is to leave the hemisphere without sliding down it. (Ans. $\sqrt{gR}$)

30. A flat car of mass m_0 starts moving to the right due to the action of a constant horizontal force F. Sand spills on to the car from a stationary hopper placed a little above the car at a constant rate μ kg/s. Find the velocity and acceleration of the car t seconds after the process of loading starts. Assume that loading and pulling the car by the force starts simultaneously. There is no friction.

$$\left[\text{Ans. } v=\frac{Ft}{m_0+\mu t}\,;\; a=\frac{m_0F}{(m_0+\mu t)^2}\right]$$

31. Calculate the downward acceleration of the mass m_1 of the figure (Fig. G), assuming that the surface and pulleys are frictionless and pulleys are light.

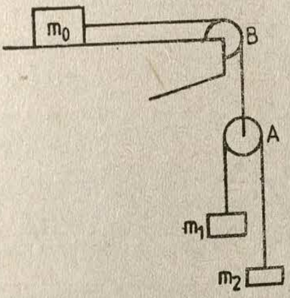


Fig. G

$$\left[\text{Ans. } a=\frac{4m_1m_2+m_0(m_1-m_2)}{4m_1m_2+m_0(m_1+m_2)}\cdot g\right]$$

32. A convexo-concave lens of crown glass has the radii of curvature equal to 1 m and 12 cm. The lens is placed horizontally on a plane mirror and water is introduced between plane mirror and the lower surface of the lens and the concave surface is filled with water. What is the power of this system ? μ of glass=1·5 and of water=$\frac{4}{3}$. (Ans. +0·8 D)

[*Hint* : Since light traverses twice, power is doubled.]

33. A chain of length l is placed on a smooth spherical surface of radius R with one of its ends fixed at the top of the sphere. What will be the tangential acceleration of each element of the chain when its upper end is released ? It is assumed that the length of the chain $l<\frac{1}{2}\pi R$.

$$\left|\text{Ans. } a=\frac{Rg}{l}\left(1-\cos\frac{l}{R}\right)\right|$$

[*Hint* : Consider an element of angular width $d\theta$ at an angular distance θ from the fixed end. The tangential component of the gravitational pull on the element is $(Rd\theta m)\, g \sin\theta$. Integrate from 0 to α where α is the angle subtended by the chain at the centre of the sphere. Now apply force=mass×acceleration.

Here the radial acceleration=0 as the body is just released. Hence tangential acceleration (a_t) is the total acceleration.]

34. A particle of mass m moves in a circle of radius R and the distance s described by it varies with time as $s=at^2$ where a is a constant. Calculate the force acting on the particle.

$$\left(\text{Ans. } F=2am\sqrt{1+\frac{4s^2}{R^2}}\right)$$

[*Hint* : $F=ma$ and $a=\sqrt{a_r^2+a_t^2}$.]

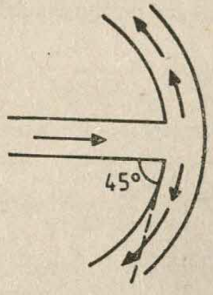


Fig. H

35. Water from the main pipe of municipal water supply branches off as shown in the figure H. If the diameter of the pipe is 1 m and water current in the main pipe is of speed 10 ms^{-1} and the current is evenly diverted, find the force exerted by the bend on the stream.

[*Hint* : Thrust=rate of change of momentum.] (Ans. $1{\cdot}06\times10^5$ N)

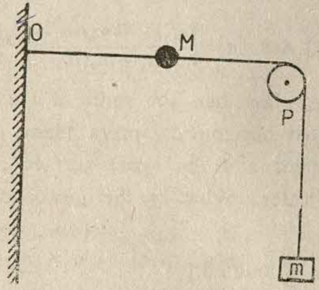


Fig. I

36. A string with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 m from the wall, has a point mass M=2 kg attached to it at a distance of 1 m from the wall. A mass m=0·5 kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released ?

(I. I. T. 1985)

(Ans. 3·36 ms^{-1})

37. A table of 15 kg can move without friction over a level floor. A block of mass 10 kg is placed on the table with string attached as shown in the figure (Fig. J). The coefficient of friction between the table and block is 6. What is the acceleration of the table if a constant force 8 kgf is applied to the free end of the string ? Consider two cases : (i) the force directed horizontally, (ii) the force directed vertically upward.

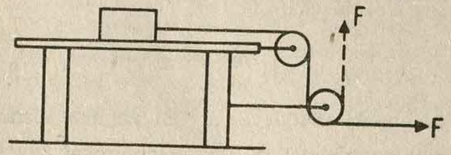


Fig. J

$$\left(\text{Ans. } a_1=a_2=\frac{8g}{25};\ \frac{2g}{15} \text{ to the left}\right)$$

* * *

LORARITHMIC TABLES

LOGARITHMS

											Mean Differences								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

Logarithms

											Mean Differences								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9653	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9988	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTI-LOGARITHMS

											Mean Differences								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

Anti-logarithms

											Mean Differences								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4750	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences				
											1	2	3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0
90	1·000														

NATURAL TANGENTS

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences				
											1	2	3	4	5
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970	Mean differences cease to be sufficiently accurate.				
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞														

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Numerical Examples in Physics (9CA 80)

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